

Midterm Exam

CSE 101

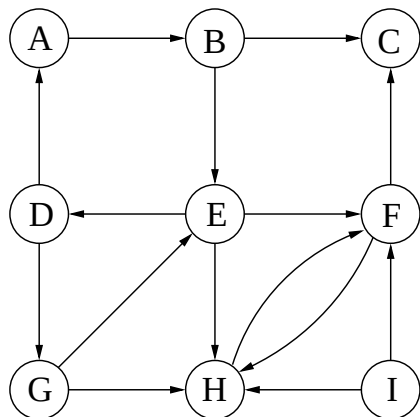
Name: _____

ID: _____

Be clear and concise. Write your answers in the space provided. Use the backs of pages for scratchwork.

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1. (10 points) Consider the following graph.



- (a) Do a depth-first search of the above graph, processing nodes (and edges out of a node) in alphabetical order. Show the DFS search tree/forest, and write in the pre and post numbers.

- (b) Draw the DAG (metagraph) of strongly connected components.

- (c) What is the minimum number of edges you have to add to make the graph strongly connected?

2. **(10 points)** Give an efficient algorithm which takes as input a directed graph $G = (V, E)$, and determines whether or not there is a vertex $s \in V$ from which all other vertices are reachable. [*Hint*: your algorithm should take linear time.]

Algorithm:

Briefly justify its correctness:

Briefly justify its running time:

3. **(10 points)** Often there are multiple shortest paths between two nodes of a graph. Give a linear-time algorithm for the following task.

Input: Undirected graph $G = (V, E)$ with unit edge lengths; nodes $u, v \in V$.

Output: The number of distinct shortest paths from u to v .

Algorithm:

Briefly justify its correctness:

Briefly justify its running time:

4. (10 points) The following statements are **false**. Give a counterexample in each case.
- (a) When depth-first search is run on a directed acyclic graph, the node with highest PRE number is always a sink.
 - (b) If G is a connected, undirected graph, then its BFS search tree contains the shortest path between every pair of nodes.
 - (c) Any directed acyclic graph with n nodes can have at most n distinct topological orderings.
 - (d) If $f(n)$ and $g(n)$ are functions for which $f = O(g)$, then $2^f = O(2^g)$.
 - (e) If a graph is not strongly connected, then its depth-first search will necessarily have a cross edge.