

UCSD CSE 101 Final, Winter 2008

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CSE 101

Name: _____

Student ID: _____

Read all of the following information before starting the exam:

- This test has 5 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- You have 180 minutes to work on the exam. If you finish early, please be quiet when you leave. *Out of courtesy to those still working on the exam, no one will be allowed to turn in their exam during the last 10 minutes.*
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Circle or otherwise indicate your final answers.
- Be clear and to the point. You will lose points for rambling and for **incorrect or irrelevant** statements.
- Make sure you have all 13 pages. Use the scratch page and/or the reverse sides of pages as necessary - and ask if you need extra sheets of paper. Good luck!

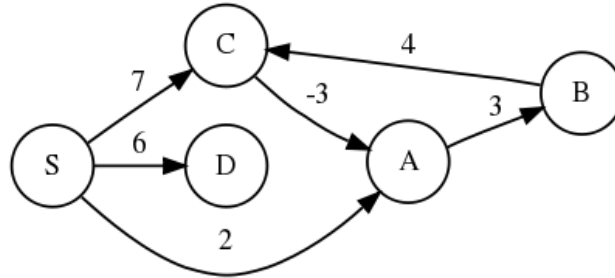
1. (20 points) Graph Questions.

a. (5 pts) Dijkstra. Give an example of a graph of at most four vertices (including S), with an identified source vertex S , for which Dijkstra's algorithm does not compute a correct shortest path tree. Your graph must have no negative cycles, but of course it can have one or more negative edges.



b. (5 pts) Graph search/ordering. Given a DAG G , give an algorithm that determines in linear time if G contains a Rudrata path. Briefly argue that it's correct.

c. (10 pts) **Trees.** Consider the graph below. For this graph give a 1) breadth first search tree, 2) depth first search tree, 3) shortest path tree, and 4) a minimum spanning tree. Note: For the BFS tree and the DFS tree *ignore the edge weights* but consider the direction of the edges. For the MST, *ignore the direction of the edges*, but consider the edge weights. Use “s” wherever a start vertex is appropriate.



2. (20 points) Divide and Conquer. You are given an array P of length n , where $P[i]$ denotes the closing price of Google's stock on day i .

a. (10 pts) Give a divide and conquer algorithm that outputs an ordered pair (j, k) , with $1 \leq j < k \leq n$, where j is the day on which you should have bought Google stock and k is the day on which you should have sold the stock in order to maximize your profit. Your algorithm must be asymptotically more efficient than the trivial $O(n^2)$ algorithm that simply tests all pairs of days $j < k$ and outputs the largest profit.

Example 1: $P = [9, 1, 5]$ - your algorithm should return $(j, k) = (2, 3)$ (= a profit of $P[3] - P[2] = \$4$ per share).

Example 2: $P = [15, 2, 6, 3, 4, 14, 18, 1]$ - your algorithm should return $(j, k) = (2, 7)$ (= a profit of $P[7] - P[2] = \$16$ per share).

b. (5 pts) Prove that your algorithm is correct.

c. (5 pts) Analyze the running time of your algorithm.

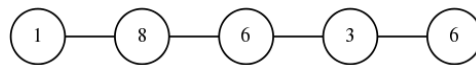
3. (20 points) Dynamic Programming.

Definition: A graph is called a *path* if its nodes can be written down as $v_1, v_2, \dots, v_n$ where there's an edge between v_i and v_j if and only if i and j differ by exactly 1.

Definition Given graph $G = (V, E)$, a subset of vertices $U \subseteq V$ is an *independent set* if no two vertices in U are adjacent to each other.

Maximum-Weight Independent Set in a Path (MWISP) Problem: Given a graph $G = (V, E)$ that is a path, and a weight w_i for each vertex v_i , find the value of the maximal weight independent set in G .

For example, the path below has a independent set of maximal weight $= 14 = 8 + 6$.



a. (6 pts) Write a recurrence to solve the MWISP problem. Don't forget the base cases!

b. (6 pts) Write down what your recurrence relation means in English and argue briefly why it's correct .

c. (4 pts) Convert your recurrence into a dynamic programming algorithm.

d. (2 pts) What is the space complexity of your algorithm (i.e., how big is the table in terms of $|V|$)?

e. (2 pts) What is the time complexity of your algorithm in terms of $|V|$?

4. (20 points) NP-Completeness Reduction.

Consider the problem $\text{DENSE_SUBGRAPH}(G, j, k)$, where $G = (V, E)$ is an undirected, unweighted graph and j, k are non-negative integers.

$\text{DENSE_SUBGRAPH}(G, j, k)$: Is there a set of j vertices in G with at least k edges between them?

- a. (6 pts)** Show that DENSE_SUBGRAPH is in NP.

b. (14 pts) Finish the proof that DENSE_SUBGRAPH is NP-complete. Specifically, you must show a polynomial-time reduction from **one** of the following known NP-complete problems to DENSE_SUBGRAPH(G, j, k).

- **CLIQUE(G, k)**: Given undirected, unweighted graph $G = (V, E)$ and non-negative integer k , does G contain a clique of size k ?
- **INDEPENDENT_SET(G, k)**: Given undirected, unweighted graph $G = (V, E)$ and non-negative integer k , does G contain an independent set of size k ? (Remember: a set of vertices in G is independent if no two of the vertices have an edge between them.)
- **LONGEST_PATH(G, k)**: Given an undirected, edge-weighted graph $G = (V, E)$ and non-negative integer k , is there a path in G with no repeated vertices that has total edge weight at least k ? (You can use the notation: $w(e)$ denotes the weight of edge $e \in E$.)

5. (20 points) True/False and Feedback. Are the following statements True or False? Briefly explain (one or two sentences).

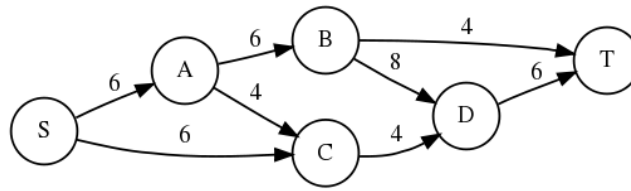
a. (2 pts) If the feasible region for a linear program is unbounded, then there is no solution.

b. (2 pts) Forward edges do not exist in breadth first search.

c. (2 pts) The weight of a minimum spanning tree in a positively weighted undirected graph is always less than the weight of the minimum Rudrata cycle of the graph.

d. (2 pts) The recurrence $T(n) = 2T(n/2) + O(n)$ is $O(n \log n)$.

e. (2 pts) $f(n) = 16^n$ is $\theta(2^n)$.



f. (2 pts) In the flow network shown above, the minimum (s, t) -cut is unique.

g. (2 pts) If all edge weights are unique, then the flows along each edge in a maximum flow will be unique.

h. (2 pts) Given a number x , x^{21} can be computed with 6 or fewer multiplications.

i. (4 pts) **Feedback.** If Prof. Kahng and/or Evan were to conduct this course again, give three constructive suggestions for improvement (use the back if necessary).

Scrap Page

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