

Disc Week 10

Agenda

- ① Finish Ind Set / 3SAT
- ② Sample Final

1] 3SAT

Input: φ a 3CNF
output: Is φ satisfiable?

Ex:

$$\varphi = (x \vee y \vee z) \wedge (\bar{x} \vee y) \\ \wedge (\bar{x} \vee \bar{y} \vee \bar{z}) \wedge (\bar{z} \vee x)$$

$$x = T$$

$$y = T$$

$$z = F$$

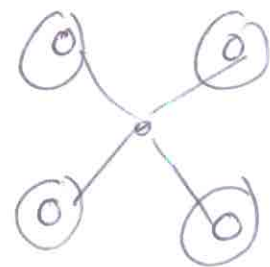
satisfies φ

Ind Set

Input: G undirected, unweighted and k

Output: Is there an independent set of size k

Ex: in G ?



Size 4
ind. set

Claim: 3SAT reduces to Ind. Set.

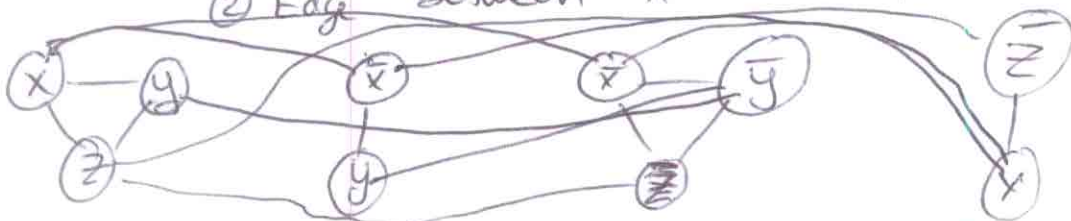
① How to transform φ to G, k :

Let there be k clauses in φ . Construct G as follows

① Make a clique for each clause

② Edge between x and $\bar{x}$ for all var's x

Ex:



Want to show: G has an ind. set of size k
iff φ is satisfiable.

"if": Assume G has an ind. set of size k . Since each clause is a clique we can only choose 1 vertex per clause. Moreover, since x connects to $\bar{x}$ everywhere, we can only choose one of them. Therefore, if x is chosen, $x=T$ is part of the satisfying assignment for φ , and if $\bar{x}$ is chosen $x=F$ would do it. We satisfy φ since exactly 1 variable is chosen from each clause.

$\{x, y, \bar{z}, \bar{z}\}$ is ~~an~~ an ind set of size 4
in our examples

"only if": Assume φ is satisfiable, then there is some choice of vars that satisfy each clause. This is an independent set in G ^{of size k} by how we constructed G .