CSE252B – Computer Vision II – Final Exam

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`http://www-cse.ucsd.edu/classes/sp07/cse252b`

3:00pm-6:00pm Wed. June 13, 2007.

On this exam you are allowed to use a calculator and two 8.5" by 11" sheets of notes. The total number of points possible is 42. In order to get full credit you must show all your work. Good luck!

1. Consider the homogeneous transformation $H \in GL(2)$.
   (a) (1 pt) How many degrees of freedom does H have?
   (b) (1 pt) Given a set of points in $\mathbb{P}^1$, what does H represent?
   (c) (2 pts) How many ground truth coordinates are needed to estimate H? What condition must these points satisfy?
   (d) (3 pts) Given 3 points in $\mathbb{P}^1$ in general position, show that there exists a mapping H that leaves the outer two points fixed while moving the inner point to any desired location.
2. (2 pts) Prove that all epipolar lines in an image plane intersect at the epipole.
3. (3 pts) Provide three different definitions of an epipole.
4. Both the Lucas-Kanade optical flow method and the Förstner operator require the computation of a special 2×2 symmetric matrix in a window around each pixel as an intermediate step.
   (a) (3 pt.) What are the entries of this matrix?
   (b) (2 pts.) Prove that this matrix is positive semidefinite.
   (c) (3 pts.) How does one interpret this matrix in terms of different types of image neighborhoods?
5. (3 pts) Show that an affine transformation can map a circle to an ellipse, but cannot map an ellipse to a hyperbola or parabola.
6. (3 pts) What is the coefficient matrix C for a conic whose points of intersection with $\boldsymbol{l}_\infty$ have homogeneous coordinates $(1, 1, 0)^\top$ and $(1, -1, 0)^\top$? What kind of conic is C?
7. Rectified stereo.
   (a) (2 pts.) Write down the normalized essential matrix (E) for a rectified stereo rig.
   (b) (2 pts.) What are the epipoles ($\boldsymbol{e}_{1,2}$) in this case?
   (c) (1 pt.) Why is this configuration desirable for computing stereo disparity?
8. (3 pts) What is the normal vector of the plane at infinity? What is the motivation for identifying its image under perspective projection? Show how to estimate the image of the plane at infinity by solving a null space problem.
9. (2 pts.) Referring to the expression for a 3D point imaged by a general camera, i.e., $\lambda \boldsymbol{x} = \Pi \boldsymbol{X}$, with $\Pi = KR[I, \boldsymbol{T}]$, explain why distant points (e.g., on the moon or a mountain) appear stationary when viewed from a translating vehicle.
10. (1 pt) What is the significance of the eigenvectors of a 2D similarity transform?
11. (5 pts) This problem is a 2D version of the sphere silhouette problem of HW1. Consider a unit disk in $\mathbb{R}^2$ centered at $(2, 3)^\top$ and the line $y = 1$. (The line represents a forward image 'plane' with $f = 1$.) Solve for the left and right edges of the silhouette of the disk on the image plane.