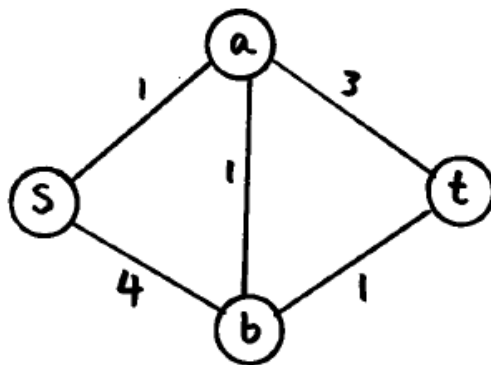


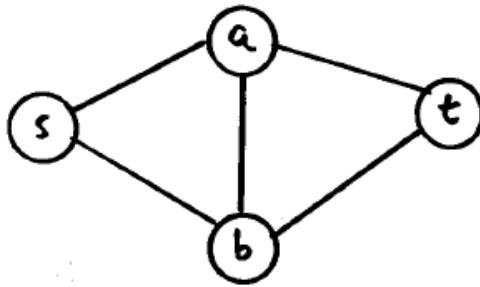
CSE 202: Max Flow and Min Cut

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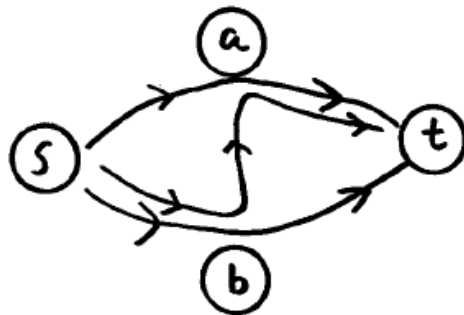
Network



Underlying Graph

Edge: weight, capacity, value

Vertex, node



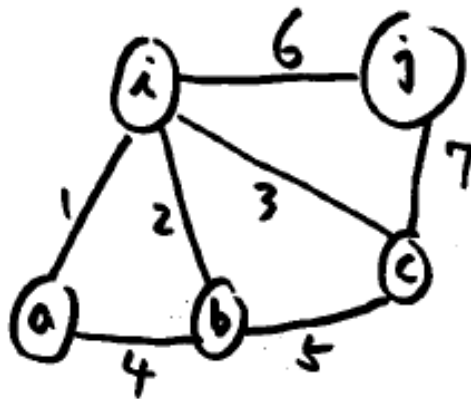
Oil Pipeline, Railroad, Traffic on Highway

Data in Internet (New Model needed!)

What is the Max Flow from s to t in the steady state?

Notations:

- Vertices by lower case (a, b, i, j, x)
- Sets of vertices by upper case (A, I, X)
- Edge connecting i and j by (i, j)



$$X = \{a, b, c\}, Y = \{i, j\}$$

$$(i, X) = 1 + 2 + 3 = 6$$

$$(X, Y) = 1 + 2 + 3 + 7 = 13$$

A cut separating two vertices: an (s, t) cut

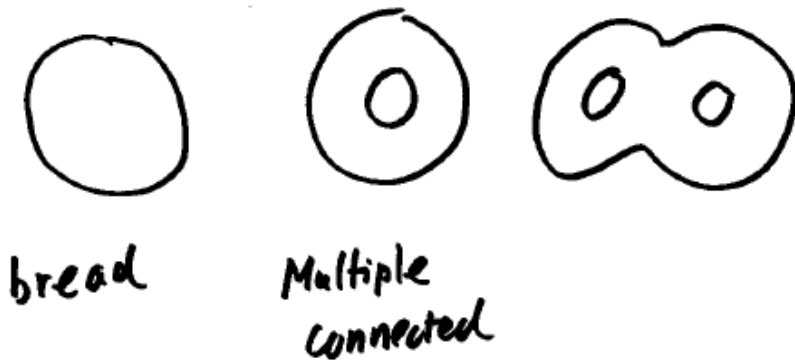
Cuts are denoted by $[s, t]$: a cut separating two sets of vertices

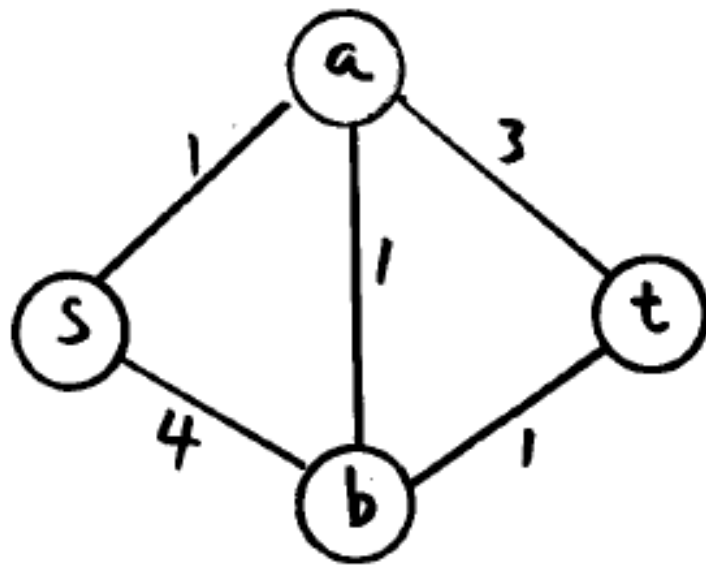
The cut is the set of edges whose deletion breaks all connection between _____

The capacity of the cut is the sum of all edge weights.

Digital Topology

- Connection
- Separation



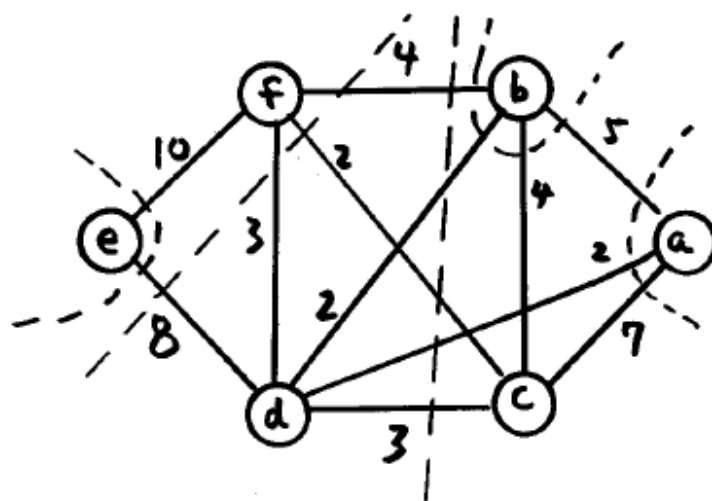
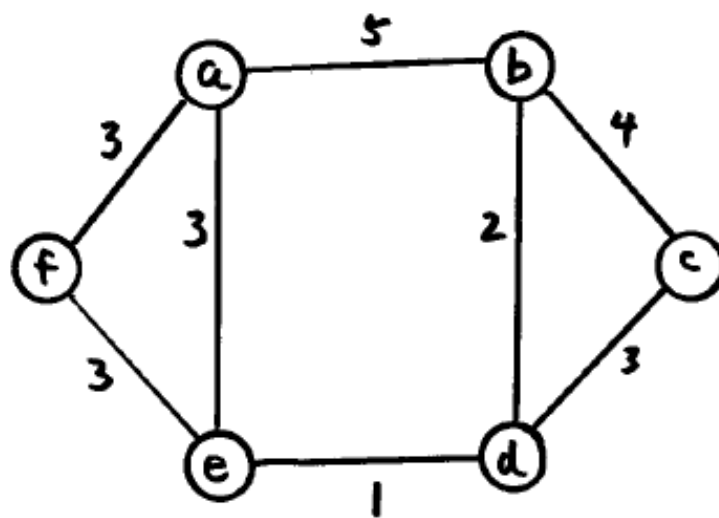


$$F^*(s, t) = 3 \qquad F^*(a, b) = 3$$

$$F^*(s, b) = 5 \qquad F^*(b, t) = 4$$

$$C^*[s, t] = 3 \qquad C^*[a, b] = 3$$

MAX FLOW = Min Cut



Global Min Cut Algorithm by Nagamochi and Ibaraki

SIAM Journal on Discrete Math, Vol 5, No. 1 (Feb 1992), pp 54-66.

Max Adjacency algorithm

Max Adjacency algorithm on an n -vertex network has $n - 2$ phases.

Every phase produces a candidate for global min cut.

The first phase works on n vertices.

The second phase works on $n - 1$ vertices.

The third phase works on $n - 2$ vertices.

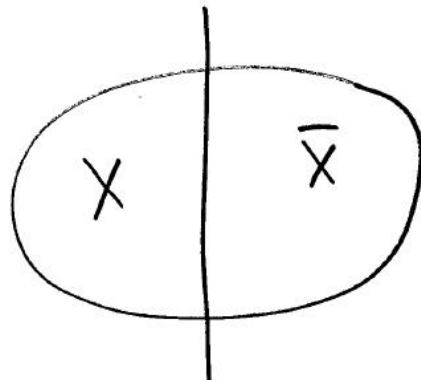
It is a local index algorithm with $w_1 = w_2 = \dots = w_n$.

Step 0 Arbitrary vertex as v_1 in G_n .

Step 1 Let $V^k = v_1, v_2, \dots, v_k$ be the index set. Find the vertex x which has max connection with V^k , i.e. $c(x, V^k) \geq c(y, V^k) \forall y \notin V^k$.

Index v_x as v_{k+1} . Repeat until we have v_n .

Step 2 Record $[v_n]$. Shrink v_{n-1} and v_n and get new network G_{n-1} . Return to Step 0 with G_{n-1} .



Global Min Cut

v_{n-1} and v_n both in X or X'

The min cut separating v_{n-1} and v_n is:

