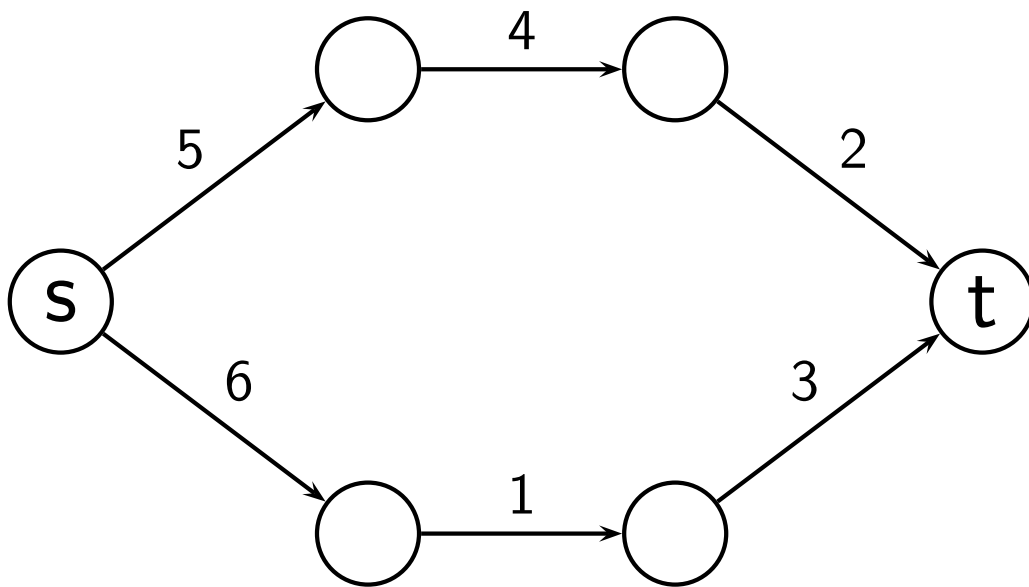


Shortest Path Problem

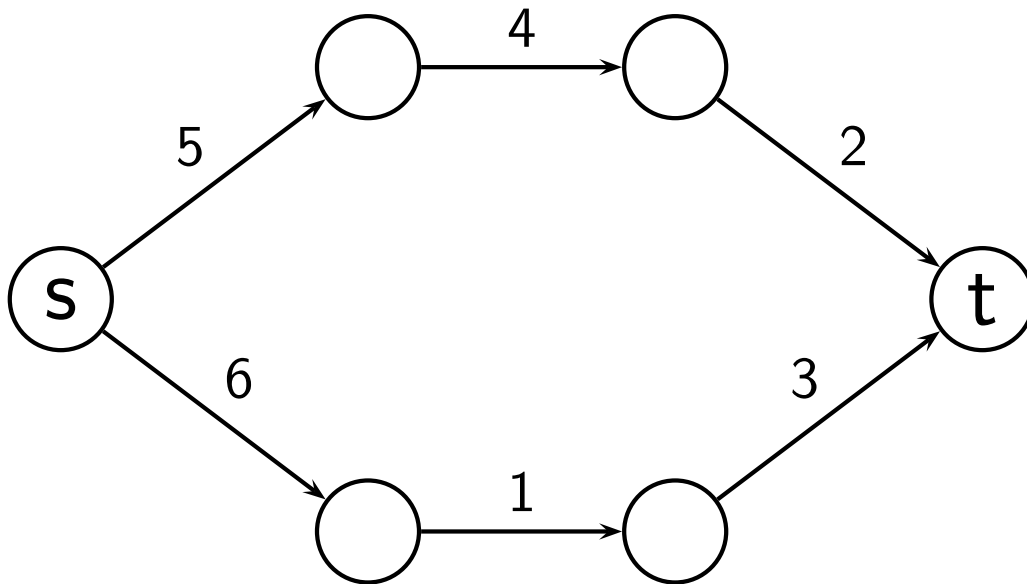
Every arc has a length



The bottom path is better

Maximum Flow Problem

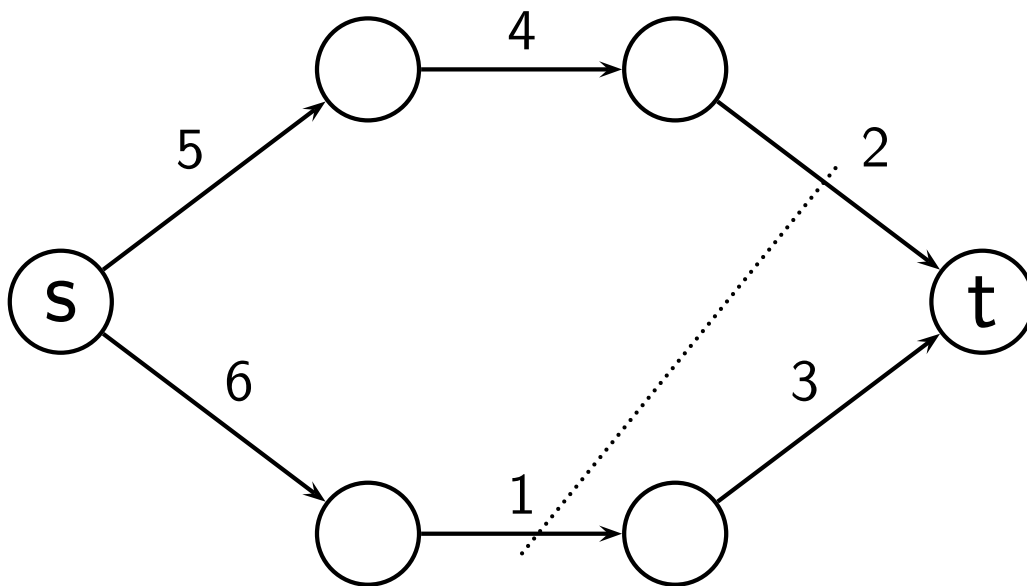
Every arc has a length



The max flow in the bottom path is less than the top path

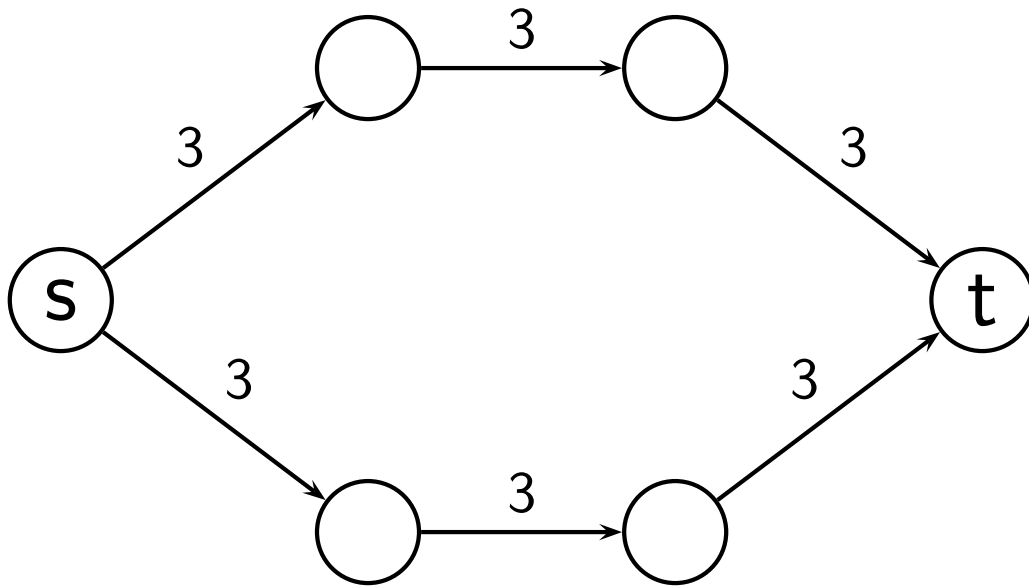
Maximum Flow Problem

Concept of bottleneck in a path is generalized to a network



Max flow is $(2 + 1) = 3$

Example



Max flow is $(3 + 3) = 6$

Maximum Flow Problem

Given a network

with a source V_S , a sink V_T

arcs with capacity b_{ij}

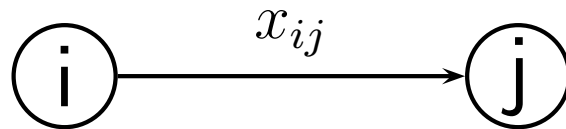
What is the Max Flow from V_S to V_T

if flow is conserved in

all nodes except V_S & V_T

Constraints

Let x_{ij} be the amount of arc flow in



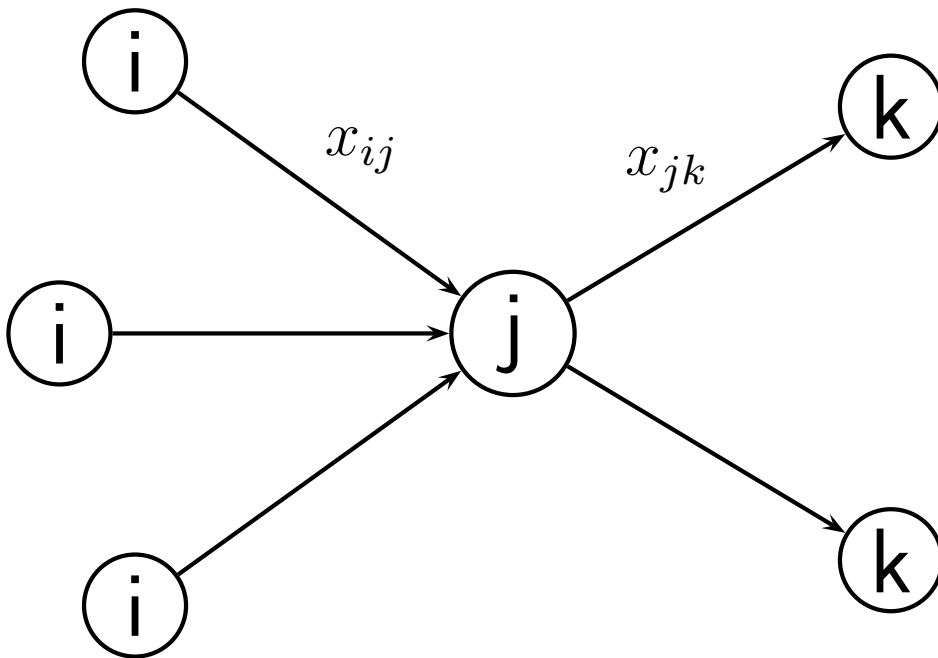
The arc flow cannot exceed the capacity b_{ij} :

$$x_{ij} \leq b_{ij}$$

Constraints

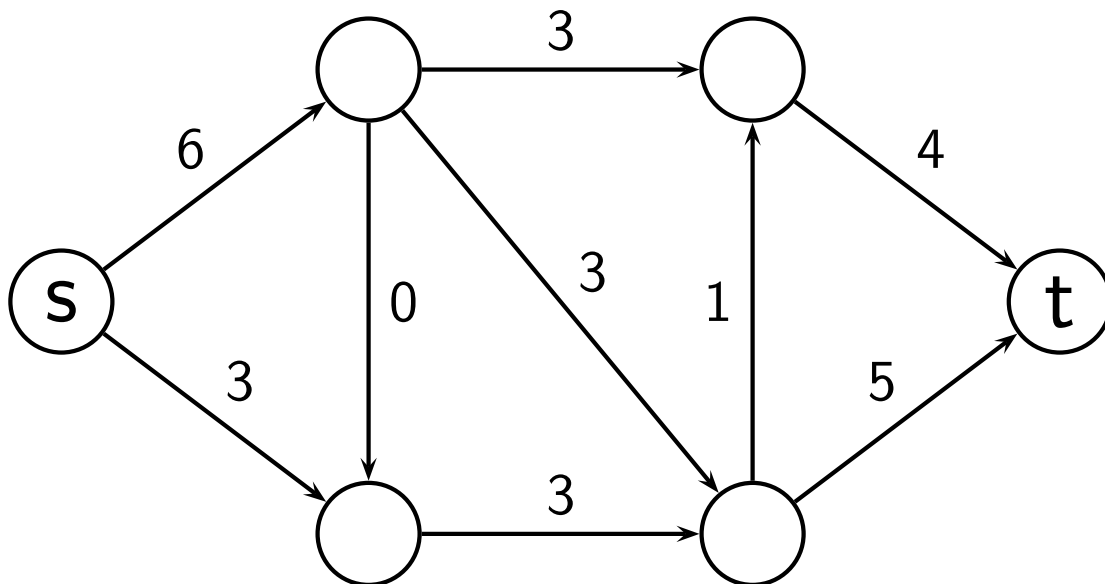
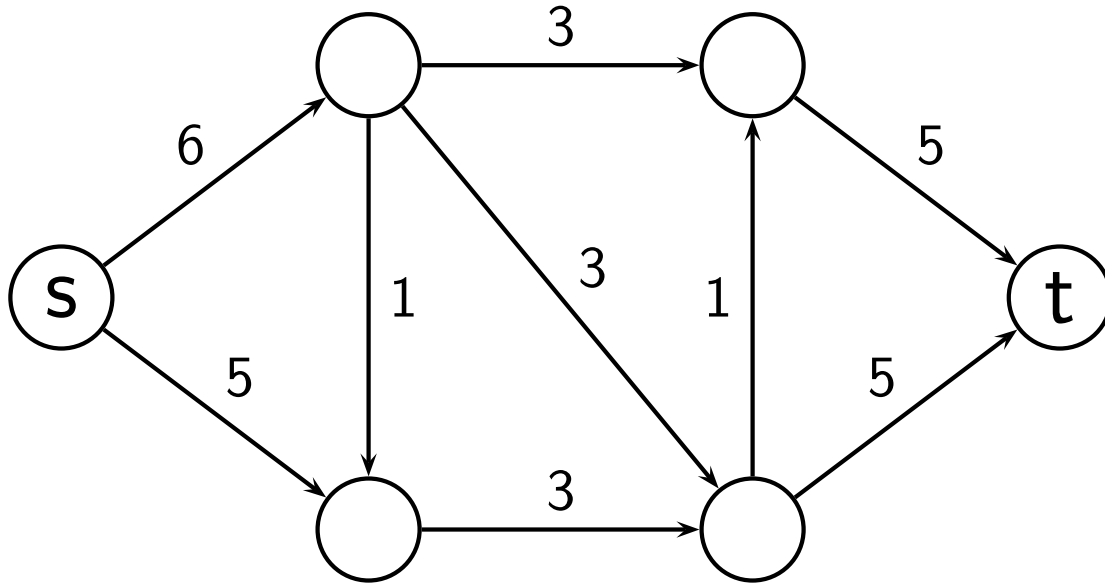
The total flow into a node j must be equal to the total flow out of a node

$$\sum_i x_{ij} - \sum_k x_{jk} = 0$$



Example

capacity b_{ij}



Arc flow x_{ij}

Maximum Flow

Max V

$$\begin{aligned} \text{Sub } & \sum_i x_{ij} - \sum_k x_{jk} \\ &= \begin{cases} -v & j = s \\ 0 & j \neq s, t \\ v & j = t \end{cases} \end{aligned}$$

$$0 \leq x_{ij} \leq b_{ij}$$

Max flow Min cut

The MAX FLOW value in a network is
equal to the value of a min cut
separating the source and the sink

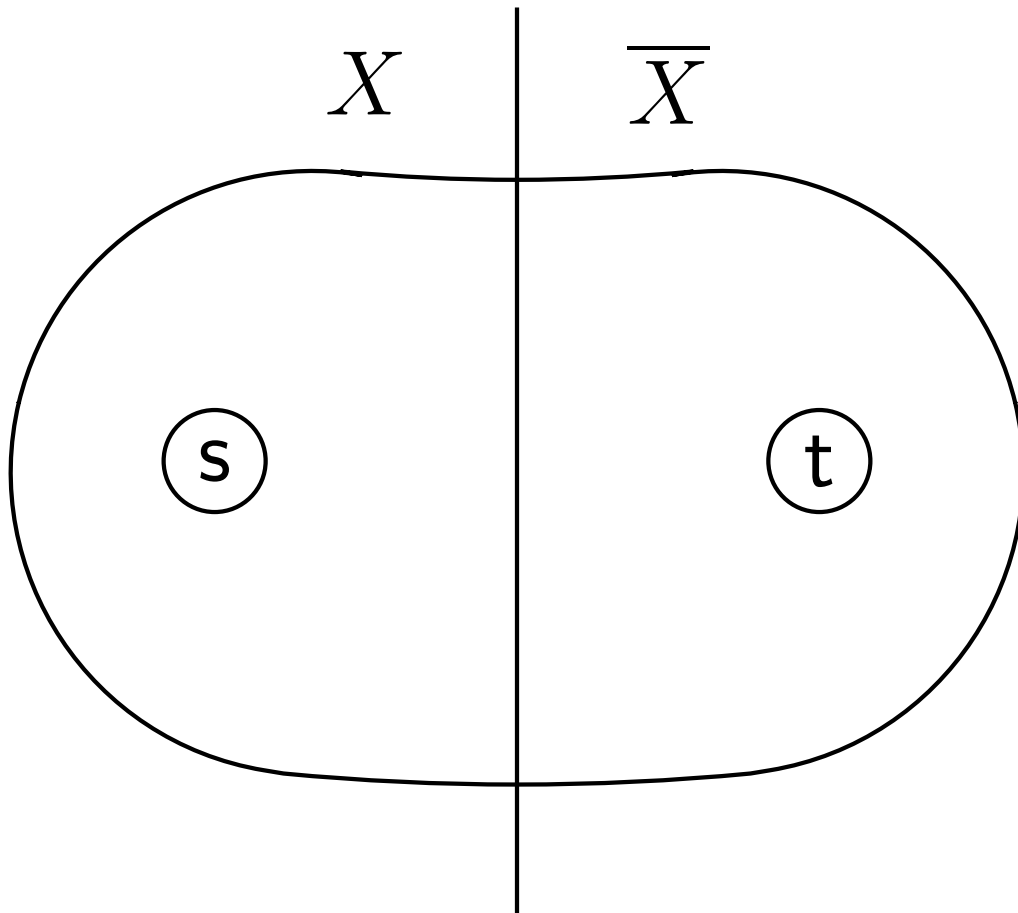
$$\text{MAX FLOW} = \text{MIN CUT}$$

Max flow Min cut

MAX FLOW VALUE is unique,
but flow pattern is not.

There could be many min cuts
in a network.

Max flow Min cut



$$F(s, t) = c(X, \overline{X})$$

$$s \in X, t \in \overline{X}$$