

Useful References 1

- Cormen, Leiserson, Rivest & Stein
- T.C. Hu and M.T. Shing
Enlarged Second Edition, Dover
paperback
- Cook, Cunningham, Pulleyblank,
Schrijver
- Korte & Vygen
- Kleinberg & Tardos

Useful References 2

- Graphs, Networks & Algorithms
by Dieter Jungnickel
- Combinatorial Optimization
by Alexander Schrijver
Vol A, B, C
Pages 1 – 1882

0-3

0-3

BFS (G,s)

```
1  for each vertex  $u \in V[G] - \{s\}$ 
2      do color[u]  $\leftarrow$  WHITE
3          d[u]  $\leftarrow \infty$ 
4           $\pi[u] \leftarrow \text{NIL}$ 
5  color[s]  $\leftarrow$  GRAY
6  d[s]  $\leftarrow$  0
7   $\pi[s] \leftarrow \text{NIL}$ 
8   $Q \leftarrow \{s\}$ 
9  While  $Q \neq \emptyset$ 
10     do  $u \leftarrow \text{head}[Q]$ 
11         for each  $v \in \text{Adj}[u]$ 
12             do if color[v] = WHITE
13                 then color[v]  $\leftarrow$  GRAY
14                     d[v]  $\leftarrow$  d[u] + 1
15                      $\pi[v] \leftarrow u$ 
16                     ENQUEUE (Q, v)
17     DEQUEUE (Q)
18     color[u]  $\leftarrow$  BLACK
```

1-1

DFS (G)

```
1  for each vertex  $u \in V[G]$ 
2      do color[ $u$ ]  $\leftarrow$  WHITE
3           $\pi[u] \leftarrow$  NIL
4  time  $\leftarrow$  0
5  for each vertex  $u \in V[G]$ 
6      do if color[ $u$ ] = WHITE
7          then DFS-Visit ( $u$ )
```

DFS-Visit (u)

```
1  color[ $u$ ]  $\leftarrow$  GRAY     $\triangleright$  White vertex  $u$  has just been discovered.
2  d[ $u$ ]  $\leftarrow$  time  $\leftarrow$  time + 1
3  for each  $v \in \text{Adj}[u]$      $\triangleright$  Explore edge ( $u, v$ ).
4      do if color[ $v$ ] = WHITE
5          then  $\pi[v] \leftarrow u$ 
6              DFS-Visit ( $v$ )
7  color[ $u$ ]  $\leftarrow$  BLACK     $\triangleright$  Blacken  $u$ ; it is finished.
8  f[ $u$ ]  $\leftarrow$  time  $\leftarrow$  time + 1
```

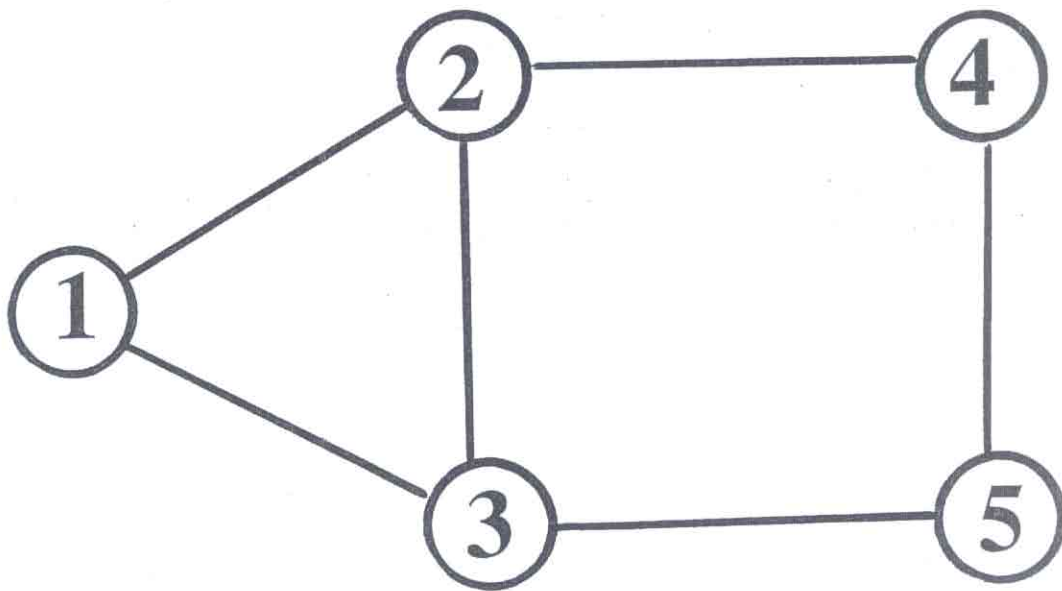
1-2

2

Traversing Graphs

Visiting all nodes of

G in some order

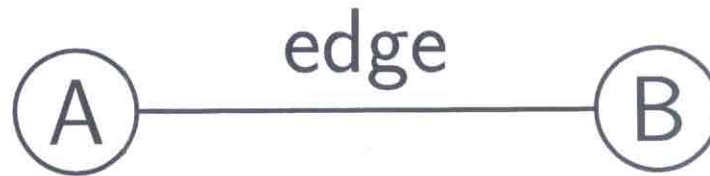


Two nodes are neighbors

if they share an edge

1-3
1-2

Edge & Arc



Node A and Node B are neighbors



A is not a neighbor of B

B is a neighbor of A

1-4

BFS

Let $1, 2, \dots, i, \dots, j, \dots, k$ be
the assigned labels

& V_i has the smallest label
and unlabeled neighbors.

Label the neighbor $K+1$

1-5
1- (2)

BFS

0. Label a vertex V_1 . $I = \{1\}$

1. Index $I = \{1, 2, \dots, k\}$

x be a neighbor of the
smallest index vertex

2. Label x with $k+1$

Return to step 1

DFS

Let $1, 2, \dots, i, \dots, j, \dots, k$

be the assigned labels

& V_j has the largest label

and unlabeled neighbors

Label the neighbor $K+1$

DFS

7,2

Step 0. Same

Step 1. Smallest $\leftarrow$ largest

Step 2. Same

1-8

(5) 2

Jan 10, 06

BFS

DFS

**Shortest Path
(Dijkstra)**

**Minimum Spanning Tree
(Prim)**

**Shortest Paths
Floyd & Warshall**

1-9
(6)
(0)

01 $| \text{path} | \geq | \text{subpath} |$

02 Subpath of shortest path must itself be shortest

03 Any shortest path contains at most $n-1$ arcs

$$P_1 \leq P_2 \leq P_3 \leq \dots \leq P_{n-1}$$

Shortest Paths (Dijkstra)

0. Vertex V_0 with $\ell_0^* = 0$

V_i get temp labels

$\ell_i = d_{0i}$ (direct arc)

$\ell_i = \infty$ (if no direct arc)

1. Pick $\ell_K = \min_i \ell_i$

$\ell_K \leftarrow \ell_K^*$

2. $\ell_i \leftarrow \min [\ell_i, \ell_K^* + d_{KI}]$

Shortest Path

$$0. \ l_{oi} = d_{oi}$$

$$1. \ l^*_{ok} = \min_i l_{oi}$$

$$2. \ l_{oi} = \min (l_{oi}, l^*_{ok} + d_{ki})$$

Minimum Spanning Tree (Prim)

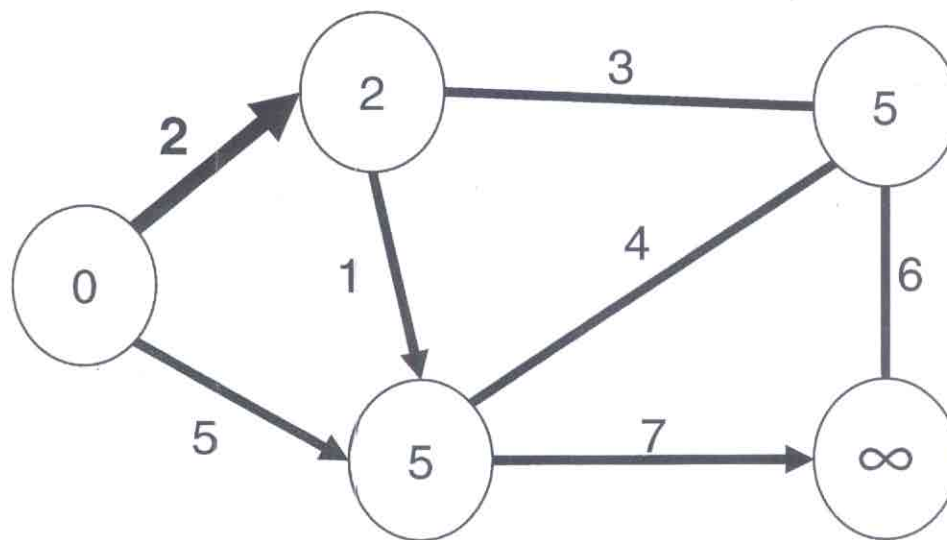
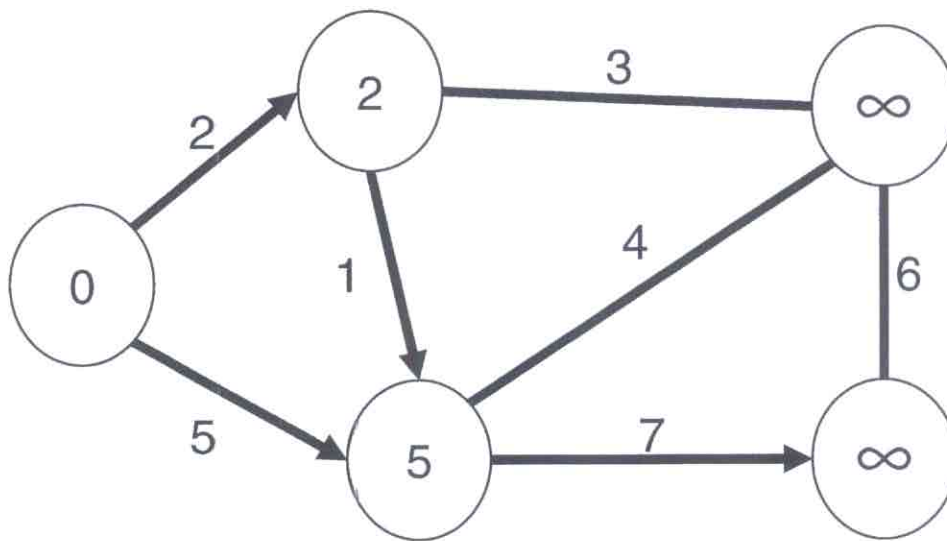
0. Same

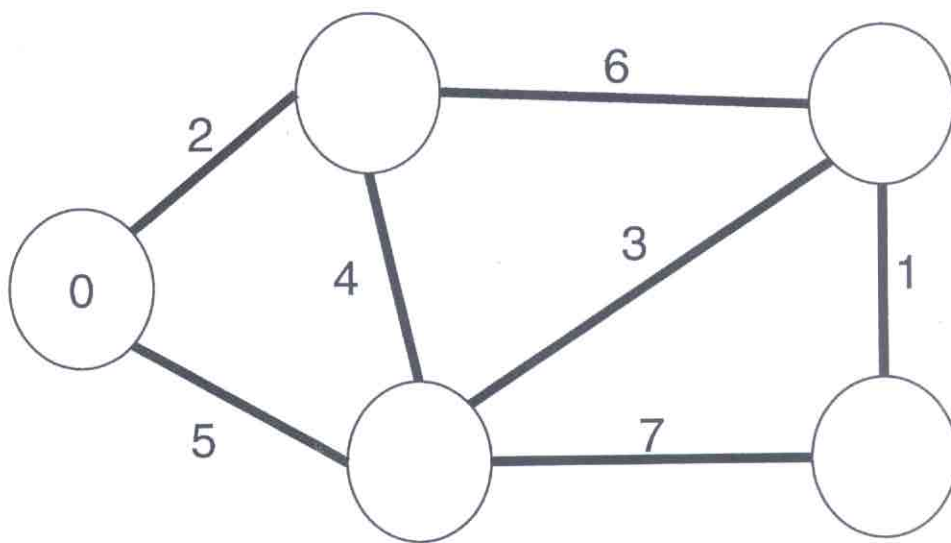
1. Same

2. $l_i \leftarrow \min [l_i, d_{Ki}]$

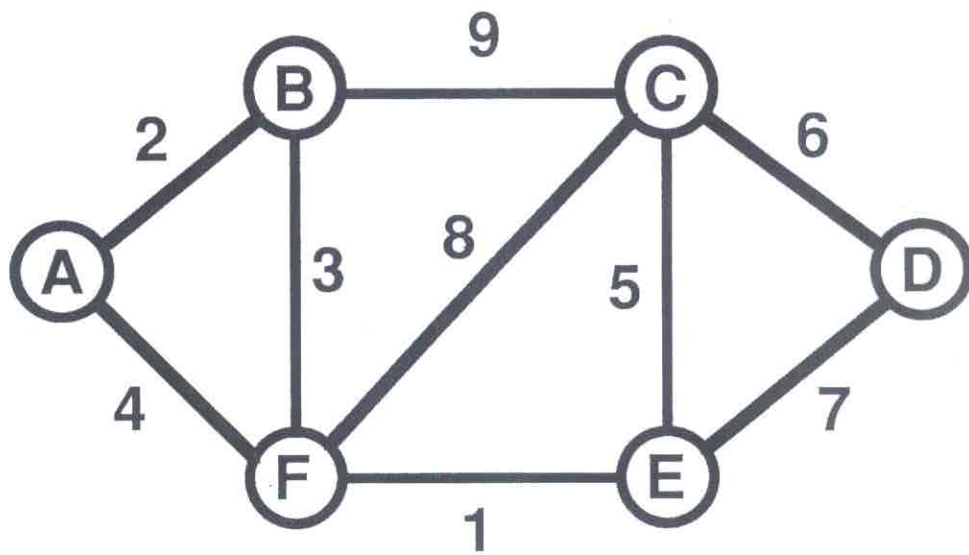
In general

$$l_i \leftarrow \min [l_i, \alpha l_K^* + \beta d_{Ki}]$$

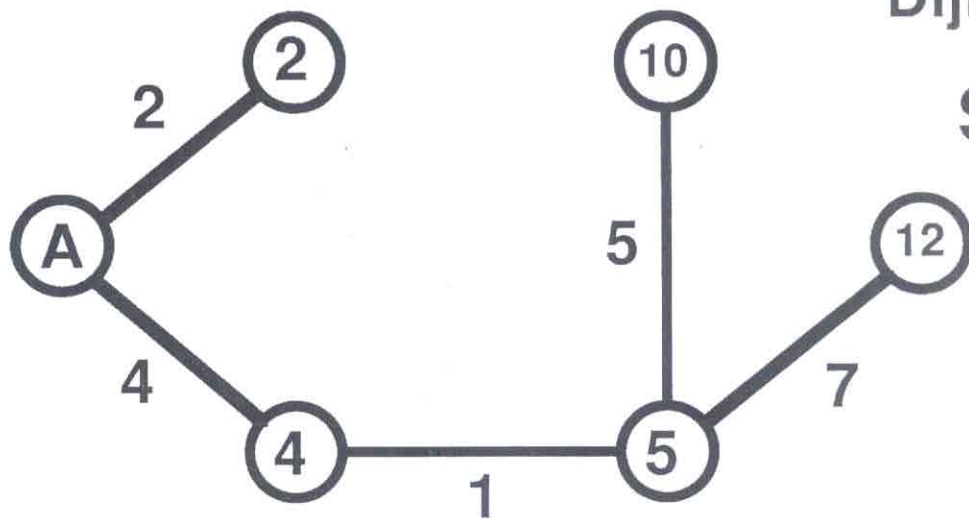




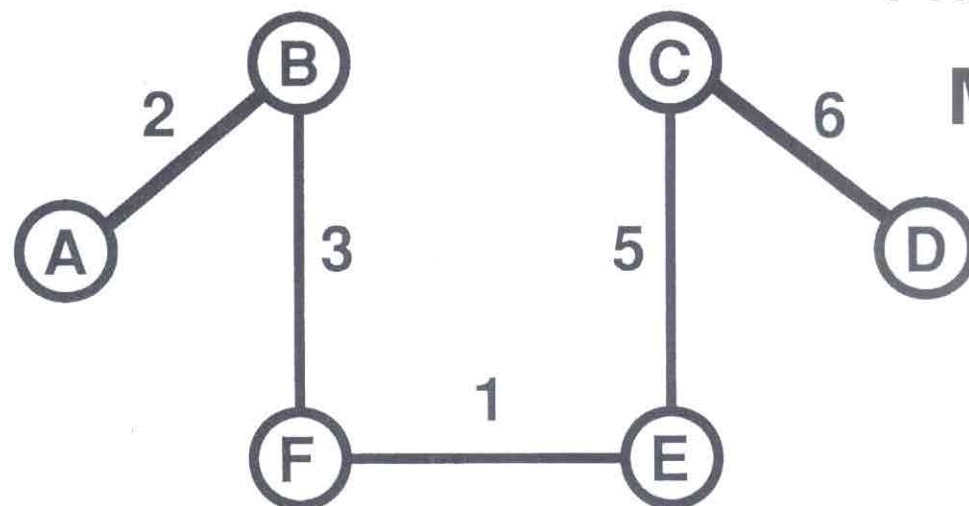
1-15

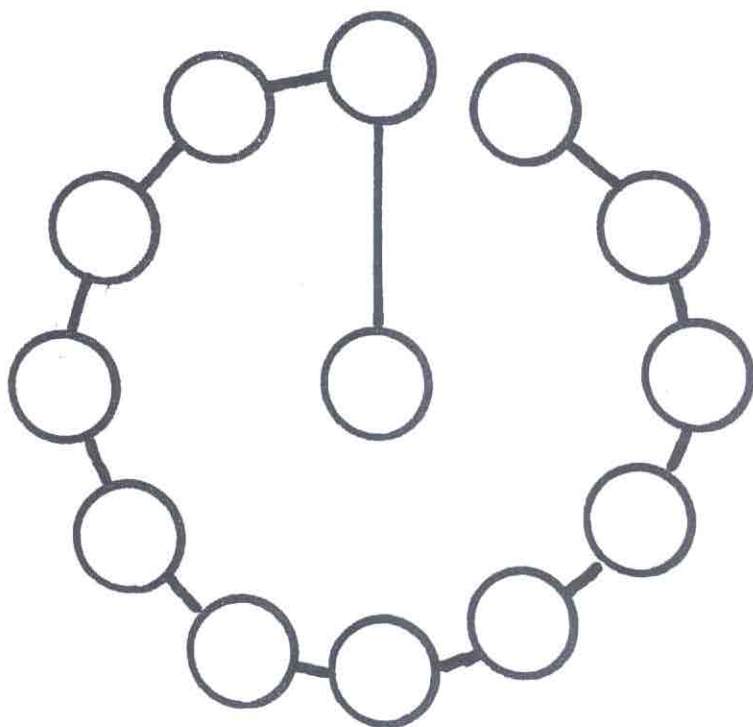
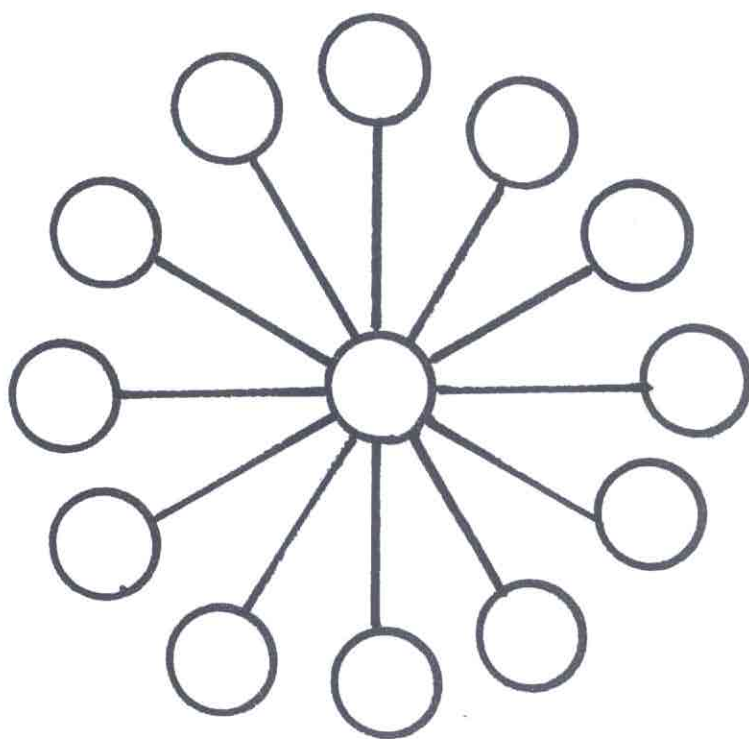


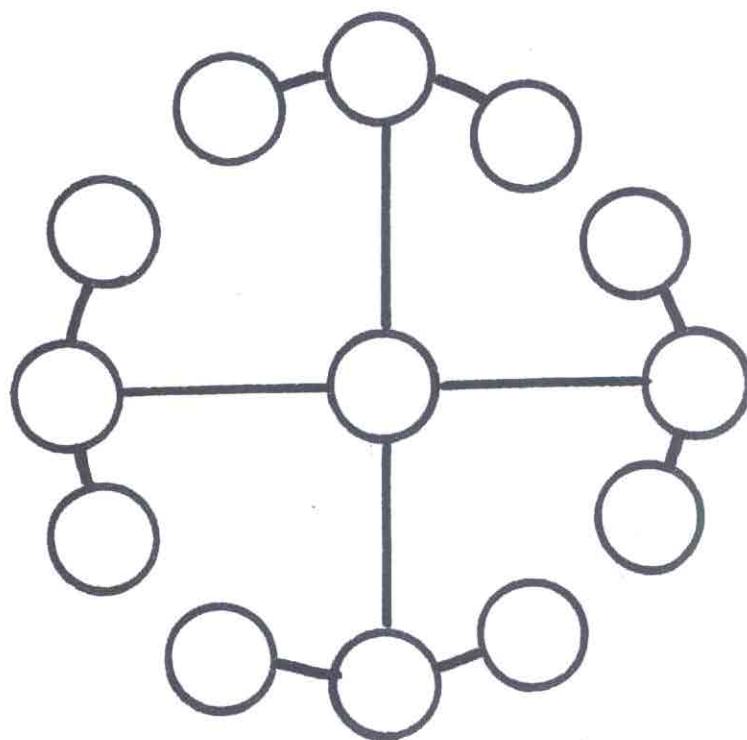
Dijkstra
SPT



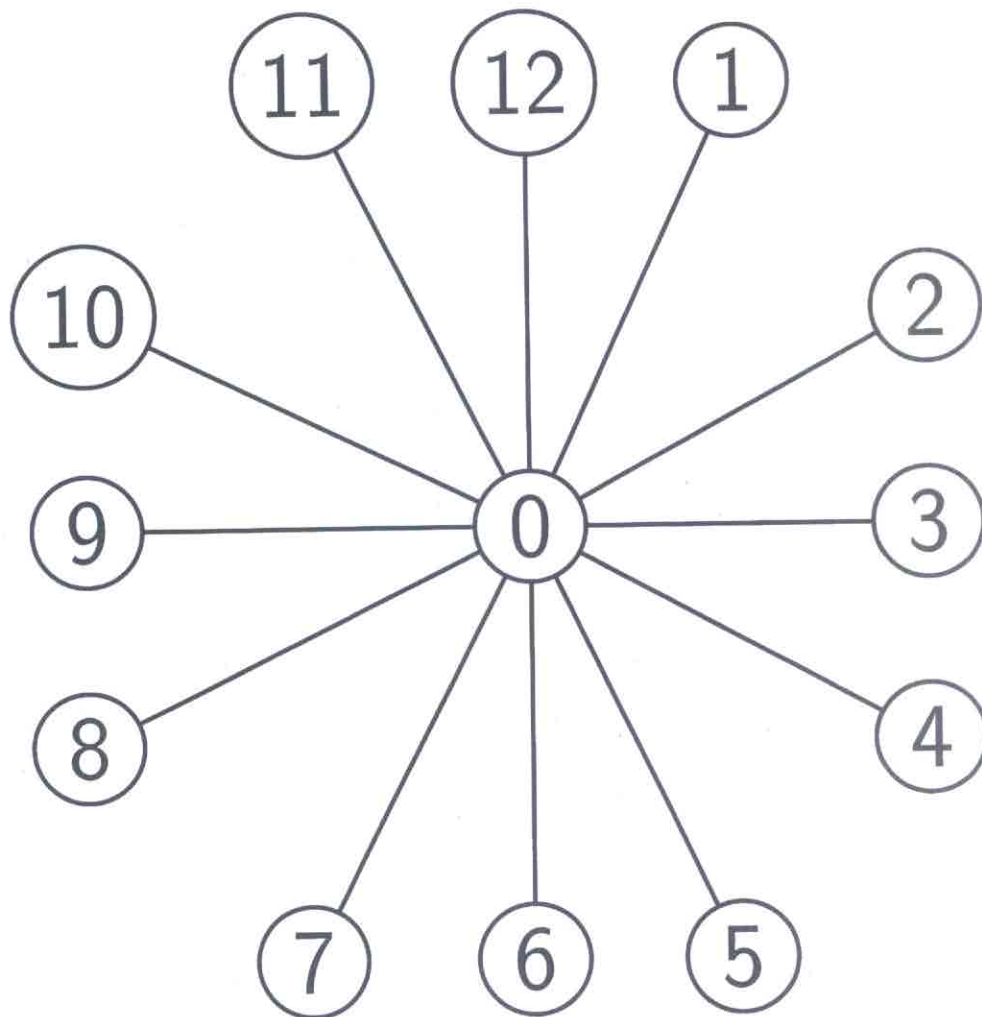
Prim
MST





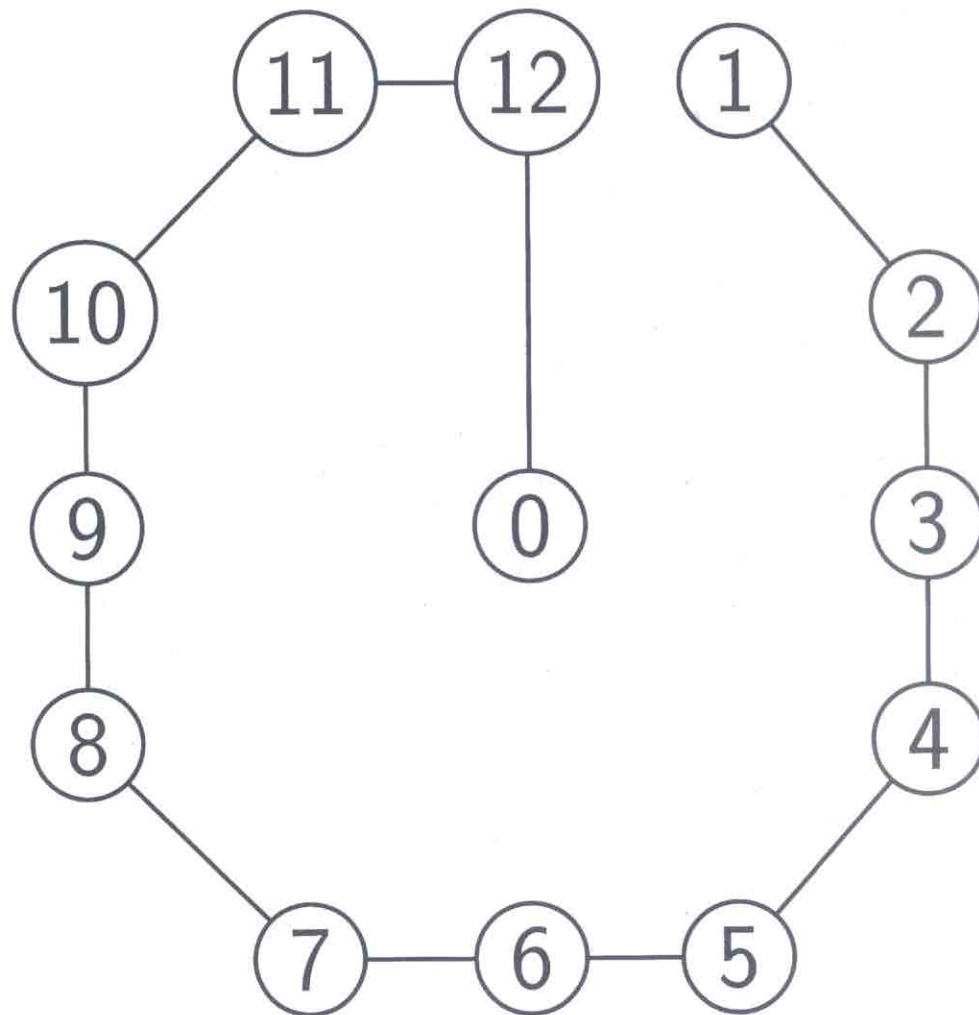


Shortest Paths From 0 To All



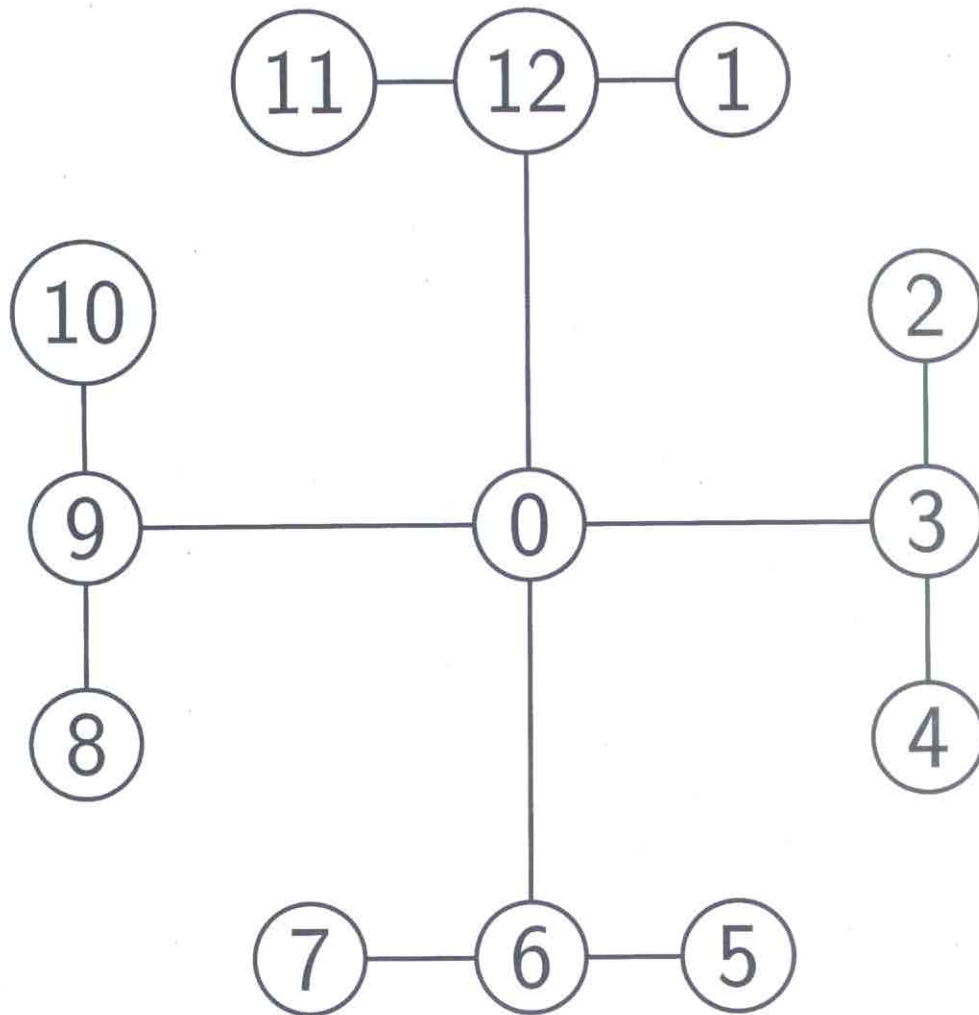
1-20

Minimum Spanning Tree



1-2

Minimum Clock Tree



1-22