CSE 200 Final Exam
Due Wed. June 13 at 6 PM

Answer four out of five questions with a complete proof. You may be informal, if you are precise. You may not discuss this exam with anyone except myself and Chris, whether taking the course or not. Each question has equal weight. You may cite without proof any result from the text or proved in class. In particular, you can use without proof the NP-completeness of any problem proved NP-complete in class, in the Sipser text, or on the homeworks.

Computability For each TM with two inputs, $M(x, y)$, we can construct an infinite directed graph G_M where the nodes are all strings, and there is an edge from x to y if M accepts the pair (x, y). Note that, while the graph is infinite, a path from from u to v is a *finite* length sequence of edges in G_M starting at u and ending at v. Consider the language $L = \{(M, u, v)|$ there is a path in G_M from u to $v\}$. Is L recursive? R.E.? Co-R.E.?

NP-Completeness Prove that the 3SAT problem remains NP-complete when restricted to formulas where each variable appears in at most 3 clauses. Remember that the input to 3SAT is a CNF formula with *at most* three variables per clause, so clauses of size 2 and 1 are also permissible.

NP-Completeness Consider a size-restricted version of the Tiling problem shown undecideable in class. $BoundTiles = \{(S, 2^B)|S = \{T_1, ..T_k\}$ is a set of tiles and there is a tiling of the $B \times B$ square with tiles from $S\}$ Prove that $BoundTiles$ is NP-complete. (Hint: it may be easier to give a direct reduction from an arbitrary language in NP than from a specific NP complete language.)

Polynomial-time hierarchy and circuits Remember $P_{circuit}$ is the class of all Boolean functions f, where there is a (non-uniform) family of polynomial-sized circuits $C_1, ...C_n, ..$ so that C_n decides f restricted to inputs of size n. (The standard notation for $P_{circuit}$ is *P/poly*.) Assume that $NP \subseteq P_{circuit}$. Prove that $PH = \cup_i \Sigma_i \subseteq P_{circuit}$. (Remember that $A \subseteq B$ does NOT imply that $A^C \subseteq B^C$.)

Probabilistic Complexity Consider the following additive error approximate circuit probability problem (AEACP): given a circuit C with n inputs, compute $Pr_{x \in \{0,1\}^n}[C(x) = 1]$ to within an additive error of $1/4$. In other words, an algorithm A solves the problem if for every circuit C, $|A(C) - Pr_x[C(x) = 1]| \leq 1/4$. First, give a probabilistic polynomial-time algorithm A that solves AEACP with probability at least 7/8 on every circuit. (Hint: use standard tail bounds from probability, such as Chebyshev bounds or Chernoff bounds. You can look these up in a probability textbook, if you cite the textbook.) Second, show that if there is a deterministic polynomial time algorithm that solves $AEACP$, then $P = BPP$.