

CSE140 Midterm 1 Solution

(I) (Problem Formulation) Convert the following statement to Boolean equations. An antilock brake system will automatically keep the brake pedal down when the driver keeps pumping the brake. List needed sensors for inputs and output control signals. Express the outputs as Boolean equations of the inputs. (15 points)

Sensors:

Let A = Whether breaks were applied at current time.

Let B = Whether breaks were applied at current time - t , where t is the threshold time for pumping.

Let ABS = Output for ABS system, holding break pedal down.

$$ABS = A + B$$

Note, any working solution would have been accepted, with emphasis on working. The keys here were that you recognized that a temporal component was needed, as well as some logic that can determine if the breaks were being pumped. I looked at all of your solutions and read them to see if they would work, and allotted points accordingly. Since this was a very difficult problem, I gave a lot of partial credit just for writing boolean variables and a boolean equation. People who simply wrote "Let A = Pumping, $ABS = A$ " got 10 points just for putting that. Majority of scores were between 7-10, with very few above that and very few below that. Therefore, if you got between a 7-10, you did fairly well on this problem. People who received 4-7 probably were missing the boolean equation entirely, or had nonsensical variables. People who received above 10 almost had it, with some minor things. Getting above a 10 was very high, so if you got above a 10, congratulations!

(II) (Laws and Theorems of Boolean Algebra)

i. Prove using Boolean algebra that $a'b' + a'c + bc = a'b' + bc$. (10 points in total. 2 points for each step of correct derivation)

$$\begin{aligned} & a'b' + a'c + bc \\ = & a'b' + a'c * 1 + bc \\ = & a'b' + a'c * (b + b') + bc \\ = & a'b' + a'bc + a'b'c + bc \\ = & (a'b' + a'b'c) + (a'bc + bc) \\ = & a'b' * (1 + c) + bc * (a' + 1) \\ = & a'b' * 1 + bc * 1 \\ = & a'b' + bc \end{aligned}$$

ii. Prove using Boolean algebra that $(a' + b)(a' + c)(b' + c) = (a' + b)(b' + c)$. (10 points in total. 2 points for each step of correct derivation)

$$\begin{aligned} & (a' + b)(a' + c)(b' + c) \\ = & (a' + b)(a' + c + 0)(b' + c) \\ = & (a' + b)(a' + c + bb')(b' + c) \\ = & (a' + b)(a' + c + b)(a' + c + b')(b' + c) \\ = & (a' + b)(a' + b + c)(a' + b' + c)(b' + c) \\ = & (a' + b)(1 + c)(b' + c)(a' + 1) \\ = & (a' + c) * 1 * (b' + c) * 1 \\ = & (a' + c)(b' + c) \end{aligned}$$

(III) (Karnaugh Map) Use Karnaugh map to simplify function $f(a, b, c) = \sum m(1, 4, 7) + \sum d(0, 6)$. List all possible minimal two-level sum of products expressions. Show the switching functions. No need for the logic diagram. (15 points in total. 4 points for correct K-Map; 5 points for correct prime implicants and essential prime implicants; 6 points for solutions)

points for each))

Possible Solution 1:

Prime Implicants: $\sum m(0, 1), \sum m(6, 7), \sum m(0, 4)$

Essential Prime Implicants: $\sum m(0, 1), \sum m(6, 7)$

Function 1: $f(a, b, c) = a'b' + ab + b'c'$

Possible Solution 2:

Prime Implicants: $\sum m(0, 1), \sum m(6, 7), \sum m(4, 6)$

Essential Prime Implicants: $\sum m(0, 1), \sum m(6, 7)$

Function 2: $f(a, b, c) = a'b' + ab + ac'$

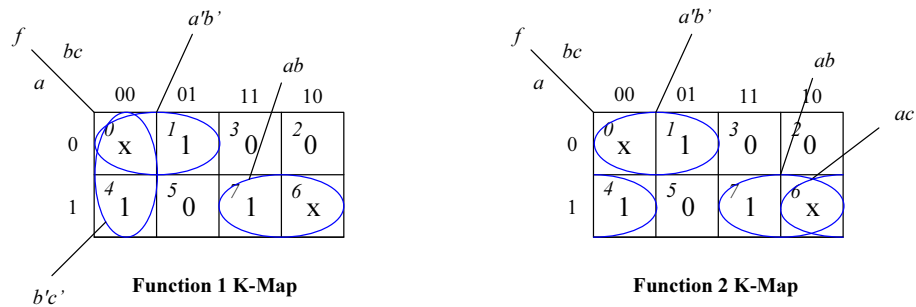


Figure 1: Problem III Karnaugh Maps

(IV) (Quine-McCluskey Method) Use the Quine-McCluskey method to find the minimum sum-of-products expression of function $f(a, b, c, d, e, f) = \sum m(0, 8, 40) + \sum d(10, 32)$. Show the implication chart, and give the result in Boolean expressions. (20 points in total. 12 points for implication charts (4 points for each); 4 points for correct prime implicants and essential prime implicants; 4 points for final solution)

id	a	b	c	d	e	f	F
0	0	0	0	0	0	0	1
8	0	0	1	0	0	0	1
32	1	0	0	0	0	0	-
10	0	0	1	0	1	0	-
40	1	0	1	0	0	0	1

id	a	b	c	d	e	f	F
(0,8)	0	0	-	0	0	0	1
(0,32)	-	0	0	0	0	0	1
(8,10)	0	0	1	0	-	0	1
(8,40)	-	0	1	0	0	0	1
(32,40)	1	0	-	0	0	0	1

id	a	b	c	d	e	f	F
(0,8,32,40)	-	0	-	0	0	0	1

Step 1: Find prime implicants: $\sum m(0, 8, 32, 40), \sum m(8, 10)$

Step 2: Find essential prime implicants: $\sum m(0, 8, 32, 40)$

Step 3: Select remaining prime implicants to cover all minterms: None

	0	8	40
(0,8,32,40)	1	1	1
(8,10)		1	

$$f(a, b, c, d, e, f) = \sum m(0, 8, 32, 40) = b'd'e'f'$$

(V) Universal Set of Gates: Check if the set in the following list is universal and explain your decision. Assuming constants 0 and 1 are available as inputs. (20 points in total. 5 points for each problem (2 points for Yes/No answer, 3 points for explanation))

i. {OR, NOT}

Yes. OR and NOT are already there. AND is implemented as $(x' + y')' = x''y'' = xy$.

ii. {NOR}

Yes. NOT is implemented as $NOR(x, 0) = (x+0)' = x'$. OR is implemented as $NOT(NOR(x, y)) = ((x+y)')' = x+y$. According to (i), AND can also be implemented.

iii. $\{f(x, y)\}$, where $f(x, y) = x + y'$

Yes. NOT is implemented as $f(0, x) = 0 + x' = x'$. OR is implemented as $f(x, f(0, y)) = f(x, y') = x + y'' = x + y$. According to (i), AND can also be implemented.

iv. $\{f(x, y, z)\}$, where $f(x, y) = x + yz$

No. NOT cannot be implemented.

(VI) Flip-Flops: Construct a D flip-flop from a JK flip-flop. Show the logic diagram. (10 points)

The logic diagram is shown in the figure below.

- $D = 1$ implies $J = 1$ and $K = 0$, then $Q_{i+1} = 1 = D$;
- $D = 0$ implies $J = 0$ and $K = 1$, then $Q_{i+1} = 0 = D$;

Thus $Q_{i+1} = D$.

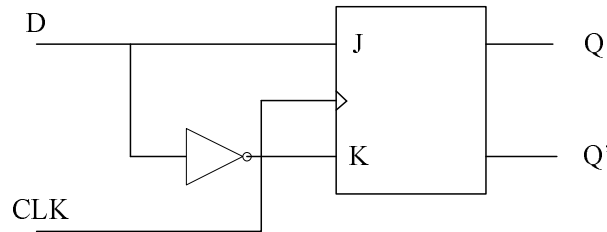


Figure 2: Problem VI Logic Diagram