

CSE252B – Computer Vision II – Assignment #3

Instructor: Prof. Serge Belongie.

Teaching Assistant: Satya Mallick.

<http://www-cse.ucsd.edu/classes/sp06/cse252b>

Due Date: Tue. May. 16, 2006.

1. Show that an affine transformation can map a circle to an ellipse, but cannot map an ellipse to a hyperbola or parabola.
2. Consider the Lyapunov map from MaSKS Equation (6.42), p. 195:

$$L : \mathbb{C}^{3 \times 3} \rightarrow \mathbb{C}^{3 \times 3}; \quad X \mapsto X - CXC^T$$

Assume C has n independent eigenvectors $\{\mathbf{u}_i \in \mathbb{C}^n\}_{i=1}^n$, with eigenvalues given by $C\mathbf{u}_i = \lambda_i\mathbf{u}_i$.

- (a) Show that $X_{ij} = \mathbf{u}_i\mathbf{u}_j^* \in \mathbb{C}^{3 \times 3}$ is an eigenvector of L . What is the corresponding eigenvalue?
 - (b) Assuming $\det(C) = 1$, which eigenvectors are in $SRker(L)$? In other words, for which values of i and j does L map the symmetric real X to the zero matrix?
 - (c) Explain the significance of $SRker(L)$ if we interpret X as the coefficient matrix for a conic.
3. 2D Upgrade from Affine to Euclidean via Orthogonal Lines.
 - (a) Load in the affine-rectified image `affine_tile.gif`, identify two pairs of imaged orthogonal lines, and plot them on the raw image.
 - (b) Implement the algorithm to solve for $K \in SL(2)/SO(2)$ from two imaged right angles on a plane as described in H&Z Example 2.26 (Metric Rectification I), p. 56.
 - (c) Demonstrate your code on the tile image. Display the Euclidean-rectified image and plot the transformed line pairs on it.
 4. Implement the algorithm described in H&Z Example 8.18 (A Simple Calibration Device), p. 211. Use it to estimate K from the image `squares.gif` depicting three metric planes. Compare your K to the K from H&Z.
 5. Derive the solution for the homography $H \in GL(4)$ relating $n \geq 5$ corresponding 3D points $(\mathbf{X}_1^j, \mathbf{X}_2^j)$, $j = 1, 2, \dots, n$, in general position.
 6. Uncalibrated 3D Reconstruction.
 - (a) Run the script `house_views.m` to produce two uncalibrated views of a wireframe house.
 - (b) Recover F using the 8-pt. algorithm and compute the canonical camera matrices (Π_{1p}, Π_{2p}) .
 - (c) Triangulate to produce the projective structure \mathbf{X}_p .
 - (d) Find the homography $H \in GL(4)$ to upgrade \mathbf{X}_p directly to the Euclidean structure \mathbf{X}_e using 5 ground truth points.
 - (e) Solve for the image of the plane at infinity $(\mathbf{v}^\top, v_4)^\top$ from three vanishing points and use it to upgrade \mathbf{X}_p to the affine structure \mathbf{X}_a .