

CSE 21: Quiz 2 Solutions

April 19, 2006

1. How many subsets of seven integers chosen (without repetition) from $1, 2, \dots, 30$ are there with no consecutive integers (e.g. if 5 is in the subset, then 4 and 6 cannot be in it)? We consider an equivalent problem of the number of binary sequences of length 30 that contain exactly 7 non-consecutive 1s (a 1 in the i th position of the string will indicate that i is in the subset). So we first place the 7 ones into our string and pad them with zeros around them giving us the string " $_1_01_01_01_01_01_01_$ " where each $_$ can be filled with any non-negative amount of zeros. We've already put 13 characters in the string leaving us with 17 zeros to place indicating 17 stars. There are 8 blanks indicating that we need 7 bars. So we get **Ans:** $\binom{24}{7}$.
2. How many ways are there to pick a selection of coins from \$2 worth of identical pennies, \$4 of identical quarters and \$9 of identical dollar coins if you select a total of 14 coins? We have 200 pennies, 16 quarters and 9 dollar coins. We have 14 coins to choose and 3 types of coin, so without any restrictions on the number of coins we choose of each type, we have 14 stars, 2 bars giving us $\binom{16}{2}$. We now must subtract out the cases that choose more than 9 of the dollar coins. We throw in 10 dollar coins into our chosen group of coins leaving us with 4 left to choose (4 stars) and 3 types of coins (2 bars) $-\binom{6}{2}$. **Ans:** $\binom{16}{2} - \binom{6}{2}$.