

CSE 21—Mathematics for Algorithm and System Analysis

April 11, 2006
Day 3

Multinomial Coefficients (Bars and Stars)

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Arrangements

How many arrangements of the letters 'l', 'a', 'j', 'o', 'l', 'l', 'a'?

- We have 3 l's to place
 - Choose 3 places from 7 total
- We have 2 a's to place
 - Choose 2 places from 4 remaining
- We have one 'j' to place
 - Choose 1 place from 2 remaining
- We have one 'o' to place
 - Choose 1 place from 1 remaining

- $C(7, 3) \cdot C(4, 2) \cdot C(2, 1) \cdot C(1, 1)$

Arrangements (multinomial coefficients)

If there are n objects with r_1 of type 1, r_2 of type 2, ..., r_m of type m , where $r_1 + r_2 + \dots + r_m = n$, then the number of arrangements of these n objects is:

$$\begin{aligned} P(n; r_1, r_2, \dots, r_m) &= \binom{n}{r_1, r_2, \dots, r_m} \\ &= \binom{n}{r_1} \binom{n - r_1}{r_2} \binom{n - r_1 - r_2}{r_3} \dots \binom{n - r_1 - r_2 - \dots - r_{m-1}}{r_m} \\ &= \frac{n!}{r_1! r_2! \dots r_m!} \end{aligned}$$

Arrangements (Bars and Stars)

How many ways to select six ice cream cones from three flavors of ice cream?

- If the three flavors of ice cream are vanilla (V), chocolate (C), and Strawberry (S), then we can look at an order as # V followed by # C followed by #S. Imagine writing down the order in three columns:

- V		C		S
**		***		*

- How many way to create an arrangement of six stars, and two bars?
- $P(8;6,2) = 8!/(6!2!) = 28$

Selection with repetition

The number of selections with repetition of r objects from n types is $C(r+n-1, r)$

- Number of ways to create an arrangement of r stars and $n-1$ bars is:
- $P(r+n-1; r, n-1) = (r+n-1)! / (r!(n-1)!) = C(r+n-1, r)$

Equivalent to the number of ways to distribute r identical objects into n distinct boxes

Equivalent to the number of non-negative integer solutions to $x_1 + x_2 + \dots + x_n = r$

Arrangements

How many ways to select nine ice cream cones from three flavors of ice cream if we have to have at least one of each variety?

- Get one of each flavor
- Now, select six ice cream cones from three different flavors
 - If the three types of ice cream are vanilla (V), chocolate (C), and Strawberry (S), then we can look at an order as # V followed by # C followed by #S. Imagine writing down the order in three columns:

- V		C		S
**		***		*

- How many way to create an arrangement of six stars, and two bars?
- $P(8;6,2) = 8!/(6!2!) = 28$

Example (from 5.3 example 3)

Nine students (three from class A, three from class B and three from class C) bought a block of nine seats for homecoming game. What is the probability that the three A students, three B students and three C students will each be seated consecutively?

Example (from 5.3 example 4)

DNA string consists of A, C, G, T.

Suppose a C-enzyme (breaks up DNA after each appearance of C) breaks a 20-letter DNA string into 8 fragments: AC, AC, AAATC, C, C, C, TATA, TGGC (all fragments except last must end in C). How many different strings could have caused these fragments?

Example (from 5.3 example 5)

How many ways are there to form a sequence of 10 letters from 4 **a**s, 4 **b**s, 4 **c**s, and 4 **d**s if each letter must appear at least twice?

Example (from 5.3 example 6)

How many ways are there to fill a box of a dozen doughnuts chosen from 5 varieties if there must be at least one doughnut of each variety.

If there must be at least two of each variety?

Example (from 5.3 example 7)

How many ways to pick a collection of exactly 10 balls from a pile of red balls, blue balls, and purple balls if there must be at least 5 red balls?

If at most 5 red balls?

Example (from 5.3 Example 8)

How many arrangements of **b, a, n, a, n, a** such that:

- The **b** is immediately followed by an **a**?
- The pattern **bnn** never occurs.
- The **b** occurs before any of the **as**

Example (from 5.3 exercise 22)

How many ways are there first to pick a subset of r people from 50 people (each of a different height) and next to pick a second subset of s people such that everyone in the first subset is shorter than everyone in the second subset?