

CSE 21—Mathematics for Algorithm and System Analysis

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Lecture 14

Solving recurrence relations with:
Change of Variable,
Induction

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Adjustment for Multiple Roots

Given a recurrence relation of form

- $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_r a_{n-r}$

...

Solve for r roots, $\alpha_1, \dots, \alpha_r$

If we have a root α_i of multiplicity 2, general form of equation is

- $a_n = A_1 \alpha_1^n + A_2 \alpha_2^n + \dots + A_i \alpha_i^n + A_{i+1} n \alpha_i^n + \dots + A_r \alpha_r^n$

Example with Multiple Roots

Given

- $a_n = 4a_{n-1} - 4a_{n-2}$
- $a_0 = 1$
- $a_1 = 3$

Find a closed form for a_n

Change of Variable

$$T_n = 2T_{n-1} + 1$$

$$T_0 = 0$$

- Repeated Substitution
- Change of Variable
- $S_n = T_n + 1$

Change of Variable

$$T_n = 2T_{n-1} + 4$$

$$T_0 = 0$$

- Change of Variable
- $S_n = T_n + 4$

Change of Variable

$$T_n = 2T_{\sqrt{n}} + \lg n$$

$$T_1 = 1$$

- Change of Variable
- Let $m = \lg n$

By Induction

Guess answer

Prove guess by induction

Example (by induction)

$$T_n = T_{n/5} + T_{7n/10} + 12n$$

$$T_i = 7 \quad (i \leq 5)$$

- Guess $T_n \leq 20n$

Josephus problem

Assume n people (numbered 1- n) around a circle

Eliminate every other second remaining person (starting with person number 2)

What is the number of the person who remains?

n	1	2	3	4	5	6	7
$J(n)$	1	1	3	1	3	5	7

Josephus Problem

Create recurrence relation: J

Base:

- $J(1)=1$

Even values

Josephus Problem

Odd values

Josephus problem

Summary:

$$J(1) = 1$$

$$J(2n) = 2J(n) - 1$$

$$J(2n+1) = 2J(n) + 1$$

Solve?

- Guess solution of form:
- Prove by induction

Josephus problem

Look at numbers in binary

n	J(n)