

CSE 21—Mathematics for Algorithm and System Analysis

Spring, 2006

May 11, 2006

Day 12

Probability

Recurrence Relations

Instructor: Neil Rhodes

Recurrence Relations section based on material from Ars Digita University
Released under Creative Commons Attribution-ShareAlike license

Yet Another Decision Tree Example

Team A plays Team B in a tournament

- The probability of team A winning any game is $1/3$.
- If any team wins 2 games in a row, that team is the champ.
- At most four games are played
 - If no champion, it's a draw

Given that the tournament lasts more than 2 games, what is the probability that A is the winner?

Example

In a certain college 4% of the men and 1% of the women are over 6 ft. tall. 60% of the students are women. A randomly selected student is over 6 ft. tall. What's the probability the student is a woman?

Independence

A and B are independent events (in (S, p)) if:

- $P(A \cap B) = P(A)P(B)$

Independence Example

Consider the following events for a family with children:

- $A = \{\text{children of both sexes}\}$ $B = \{\text{at most one boy}\}$

Show that A and B are independent events if a family has 3 children

Show that A and B are dependent events if a family has 2 children

Example

Let $S = \{a, b, c, d\}$ with uniform probability. Let:

- $A = \{a, d\}$ $B = \{b, d\}$, $C = \{c, d\}$

Show that A, B, C are pairwise independent

Show that A, B, C, are not independent

Expected value (Expectation)

Expected value = sum of $P(e)$ * value of outcome e

Flip a coin. Heads, you pay \$5, tails, you win \$6. What is the expected value of amount you win?

7

Recurrence Equations

Like induction or recursion

Compound Interest

- $M_0 = \$1000$ (starting balance)
- $M_i = 1.06 \cdot M_{i-1}$ (6% interest. After 1 year, you have 6% more)

Want M_i in closed form (no dependence on M on the RHS)

8

Binary Search

Pick a number between 1 and 1000

- I'll guess a number, you tell me correct, too high, or too low.
- How many guess necessary to count the number?
- $T_0 =$
- $T_n =$

Towers of Hanoi

Rules:

- Rings can only be on larger rings
- Can only move topmost ring
- Want to move all rings from one peg to another peg