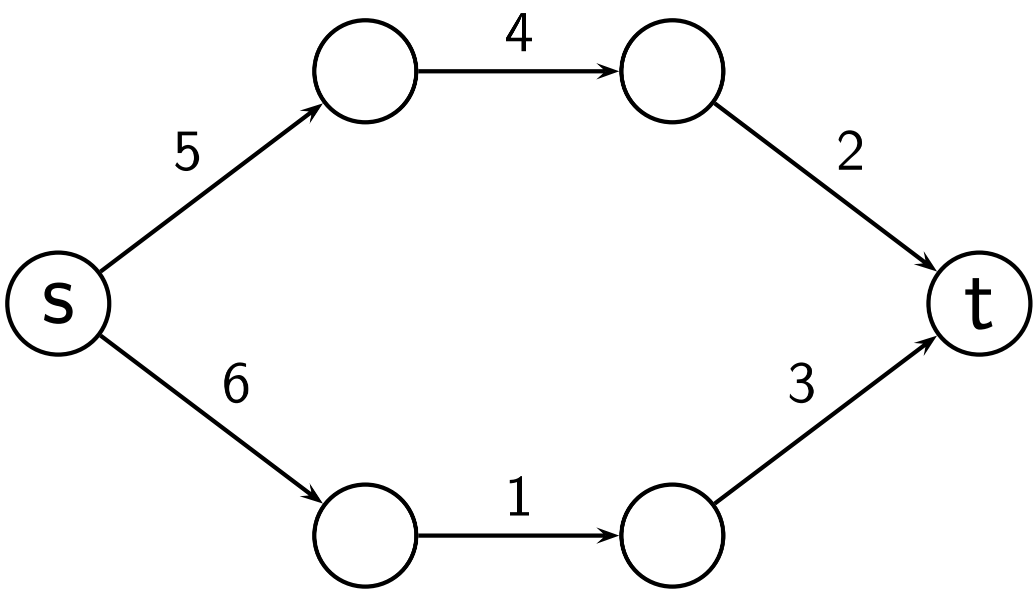
Shortest Path Problem

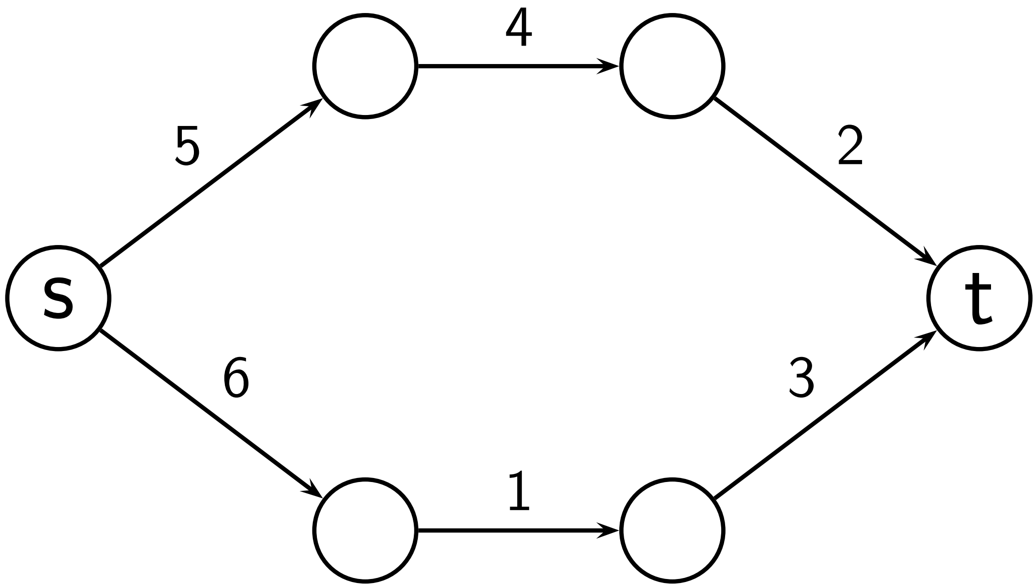
Every arc has a length



The bottom path is better

Maximum Flow Problem

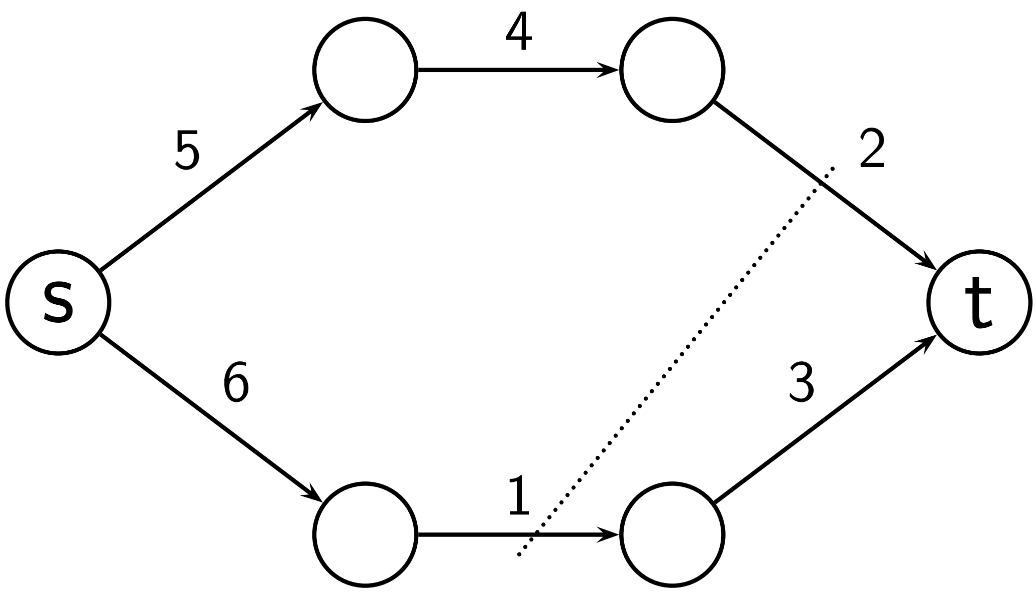
Every arc has a length



The max flow in the bottom path is less than the top path

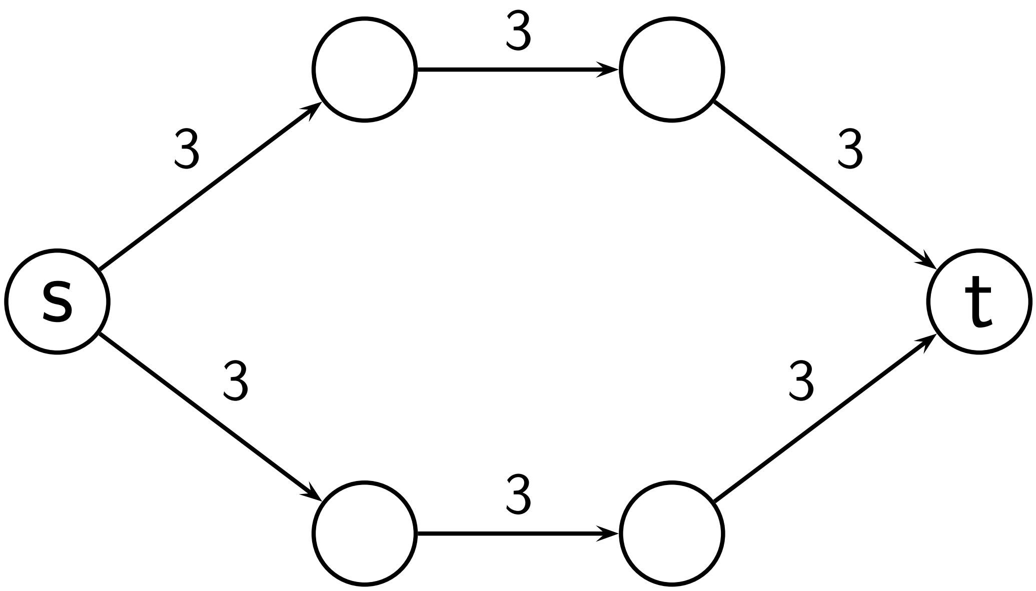
Maximum Flow Problem

Concept of bottleneck in a path is generalized to a network



Max flow is $(2 + 1) = 3$

Example



Max flow is $(3 + 3) = 6$

Maximum Flow Problem

Given a network

with a source V_S, a sink V_T

arcs with capacity b_{ij}

What is the Max Flow from V_S to V_T

if flow is conserved in

all nodes except V_S & V_T

Constraints

Let x_{ij} be the amount of arc flow in

$$\text{i} \xrightarrow{x_{ij}} \text{j}$$

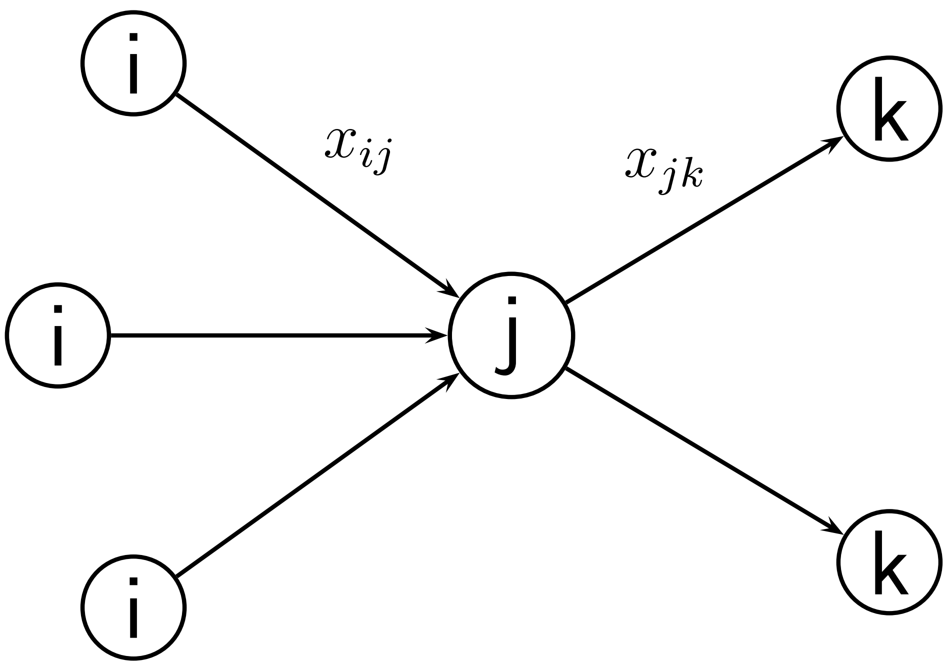
The arc flow cannot exceed the capacity b_{ij}:

$$x_{ij} \leq b_{ij}$$

Constraints

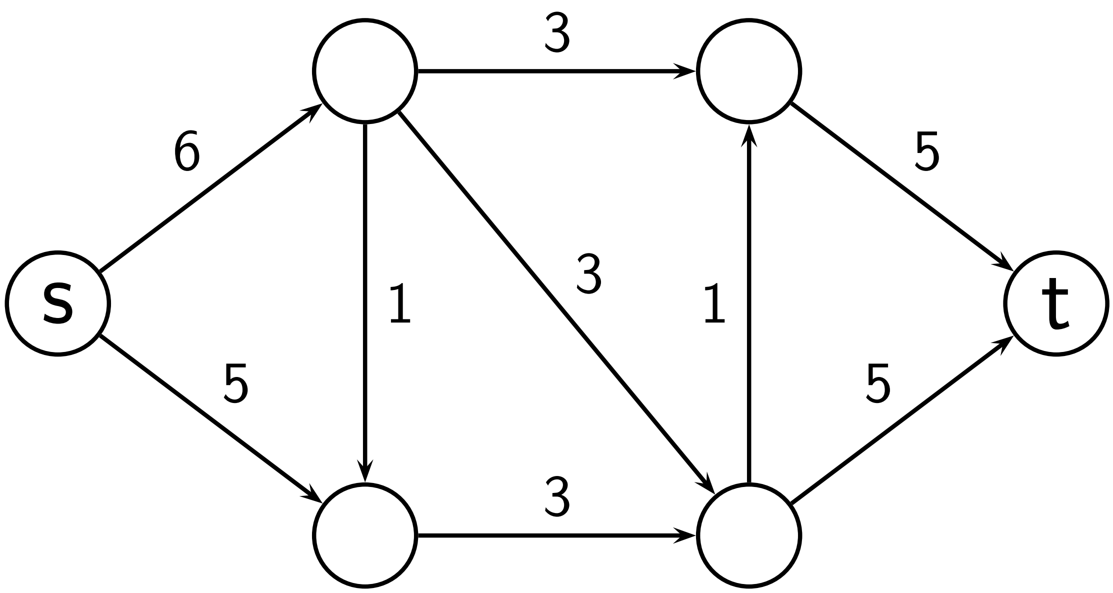
The total flow into a node j must be equal to the total flow out of a node

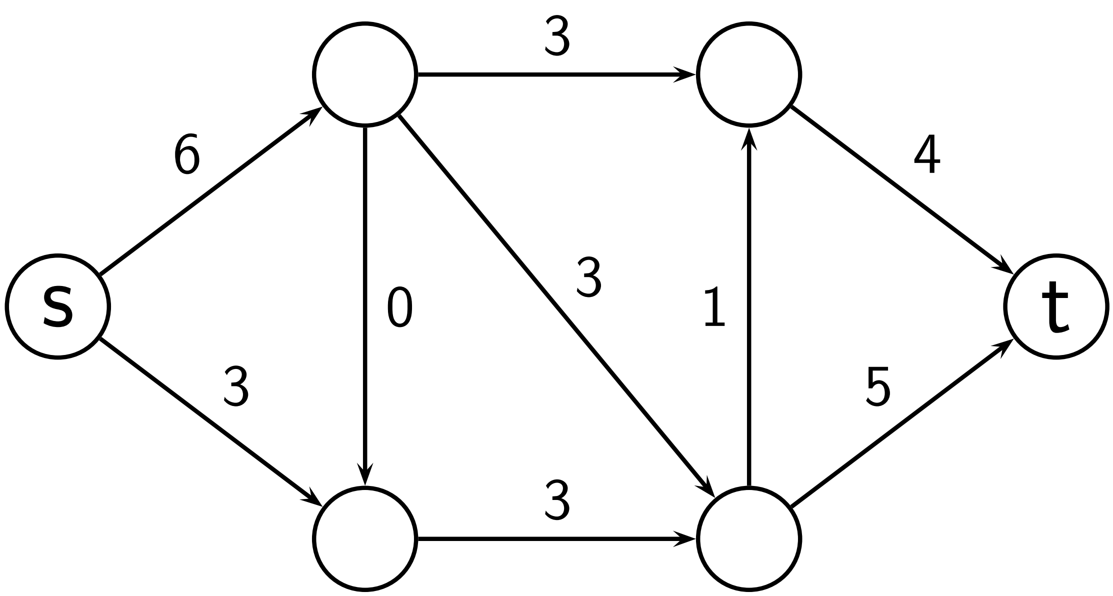
$$\sum_i x_{ij} - \sum_k x_{jk} = 0$$



Example

capacity b_{ij}





Arc flow x_{ij}

Maximum Flow

Max V

Sub $$\sum_i x_{ij} - \sum_k x_{jk} = \begin{cases} -v & j = s \\ 0 & j \neq s, t \\ v & j = t \end{cases}$$

$$0 \leq x_{ij} \leq b_{ij}$$

Max flow Min cut

The MAX FLOW value in a network is

equal to the value of a min cut

separating the source and the sink

MAX FLOW $=$ MIN CUT

Max flow Min cut

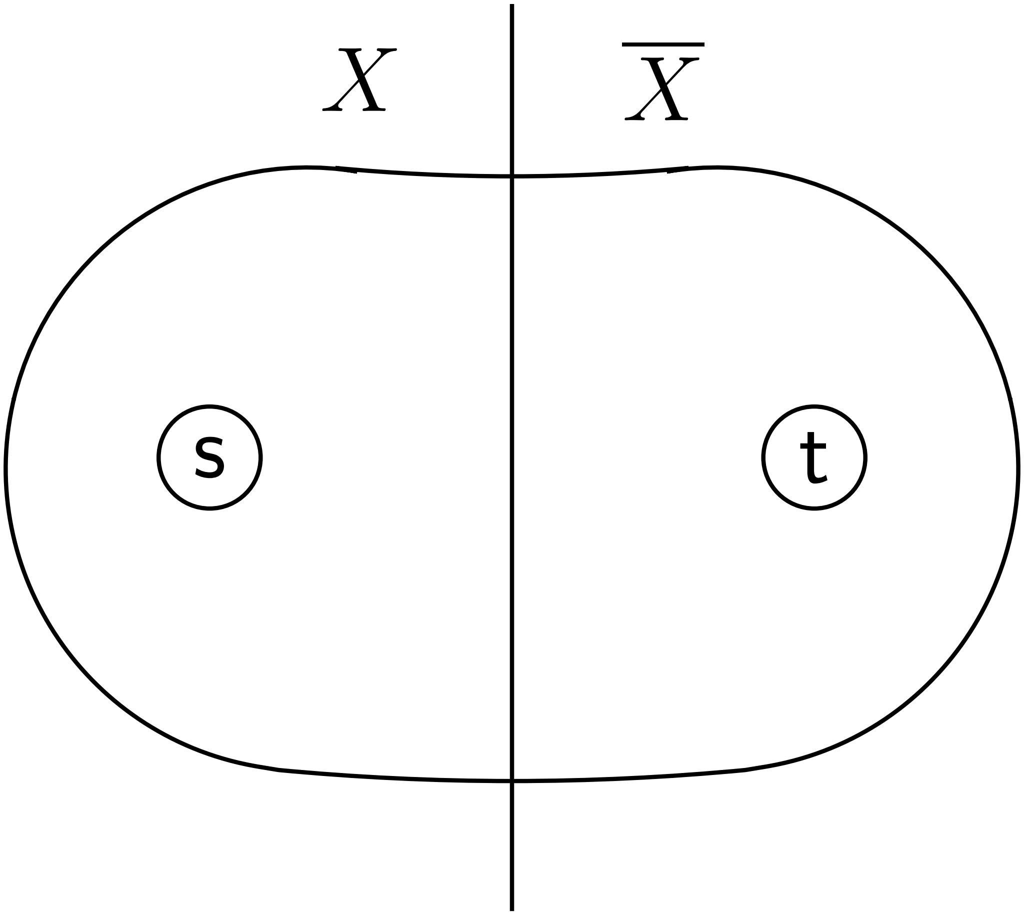
MAX FLOW VALUE is unique,

but flow pattern is not.

There could be many min cuts

in a network.

Max flow Min cut



$$F(s,t) = c(X, \overline{X})$$

$$s \in X, t \in \overline{X}$$