Dynamic Programming

$$\text{Max} \quad \sum v_j x_j$$

$$\text{Sub} \quad \sum w_j x_j \leq b$$

$$x_i \geq 0, \text{ integers}$$

v_j, w_j, b are given

Find x_j

Max Return Function

$$F_k(y) = \text{ Max Return}$$

When k types of items are allowed

and $b = y$

$$F_0(y) = 0$$

$$F_k(0) = 0$$

$$F_1(y) = v_1 \left\lfloor \frac{y}{w_1} \right\rfloor$$

Two Types

Given two types,

Either let $x_2 = 0$

or use second type $x_2 \geq 1$

$$F_2(y) = \max\{F_1(y), F_2(y - w_2) + v_2\}$$

$$F_3(y) = \max\{F_2(y), F_3(y - w_3) + v_3\}$$

Generalize

$$F_k(y) = \max\{F_{k-1}(y), F_k(y - w_k) + v_k\}$$

for $k = 1, 2, \ldots, n, \quad y = 0, 1, 2, \ldots, n$

Example:

Max $1x_1 + 3x_2 + 6x_3$

Sub $2x_1 + 4x_2 + 6x_3 \leq 8$

$x_i \geq 0$, integers

Values of $F_k(y)$

$k \backslash y$	0	1	2	3	4	5	6	7	8
1	0	0	1	1	2	2	3	3	4
2	0	0	1	1	3	3	4	4	6
3	0	0	1	1	3	3	6	6	7

Example

$$\text{Max } V = 10X_1 + 7X_2 + X_3$$

$$\text{Sub} \qquad 7X_1 + 5X_2 + X_3 \le b$$

$$X_j \ge 0, \text{ integers}$$

Homework

Due April 20:

$$\text{Max } V = 10x_1 + 7x_2 + x_3$$

$$\text{Sub} \qquad 7x_1 + 5x_2 + x_3 \leq b$$

$$x_j \geq 0, \text{ integers}$$

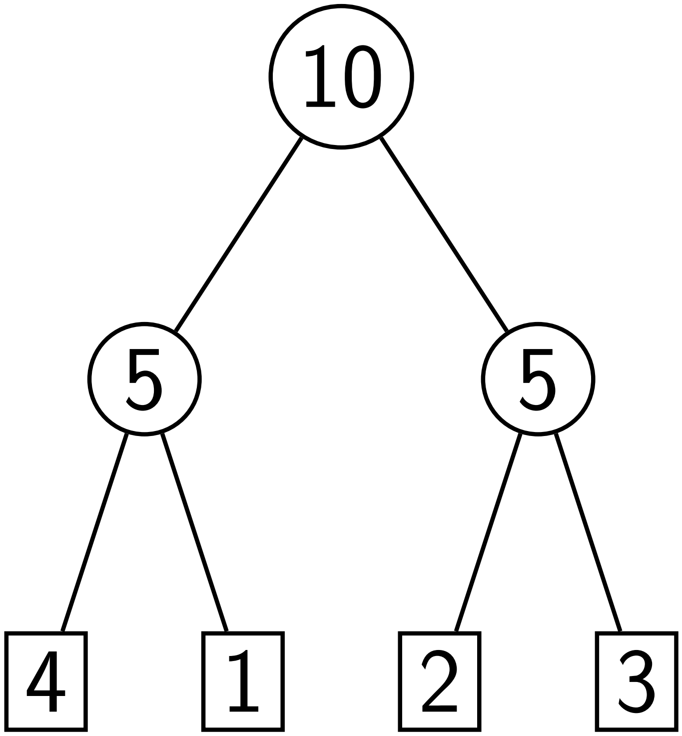
Example 2

Given $\boxed{w_1}$ $\boxed{w_2}$ $\boxed{w_3}$ $\boxed{w_4}$

Find a binary tree

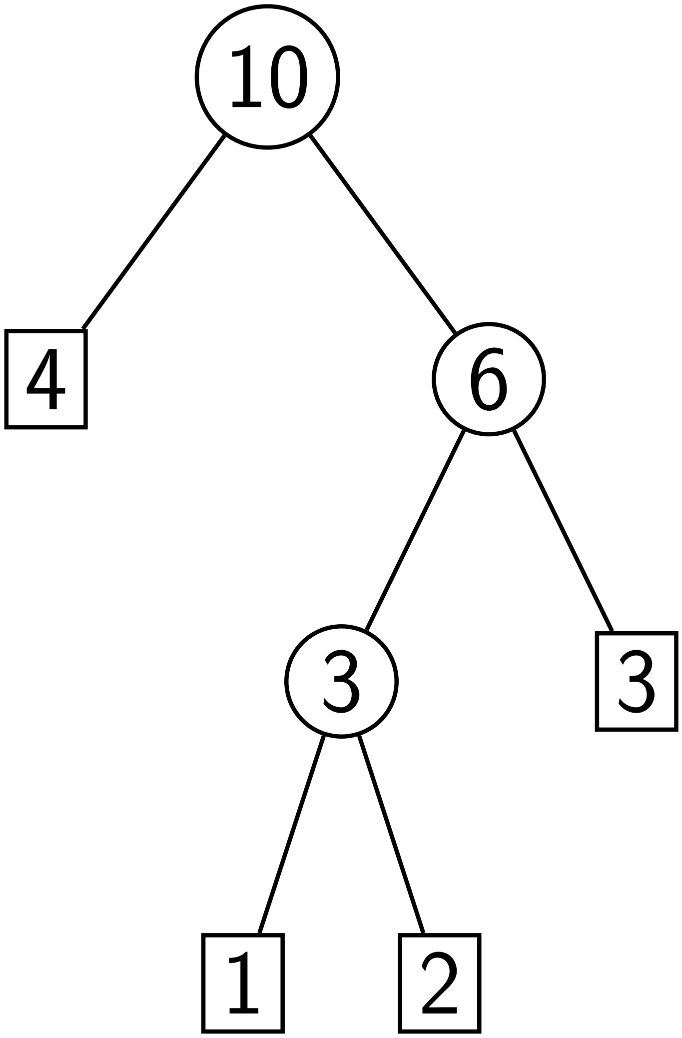
$$\min \quad \sum w_i l_i$$

Tree 1



$$\sum w_i l_i = 20$$

Tree 2



$$\sum w_i l_i = 19$$