

$$\lceil x \rceil = \lfloor x \rfloor + 1$$

P39

Knuth I

$$\lfloor -x \rfloor = -\lceil x \rceil$$

$$x-1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x+1$$

For x, y real

$$x \bmod y = x - y \left\lfloor \frac{x}{y} \right\rfloor, \quad y \neq 0$$

$$x \bmod 0 = x$$

$$n! = 1 \cdot 2 \cdot \dots \cdot n$$

$$0! = 1$$

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

$$p_{nk} = n(n-1) \dots (n-k+1)$$

$$\binom{n}{k} \quad n \text{ chose } k$$

P53

$$\binom{n}{k} = \frac{n(n-1) \dots (n-k+1)}{k(k-1) \dots 1}$$

$$\binom{5}{3} = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10$$

$O(f(n))$

P107

$\exists$ Positive constants M & n_0
such that the no. x_n represented
by $O(f(n))$

$$|x_n| \leq M |f(n)| \text{ for all } n \geq n_0$$

$$1^2 + 2^2 + \dots + n^2 = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

$$1^2 + 2^2 + \dots + n^2 = \frac{1}{3}n^3 + O(n^2)$$

$$1^2 + 2^2 + \dots + n^2 = O(n^3)$$

$$1^2 + 2^2 + \dots + n^2 = O(n^4)$$

Traversal Binary Trees

P318

O denotes the root

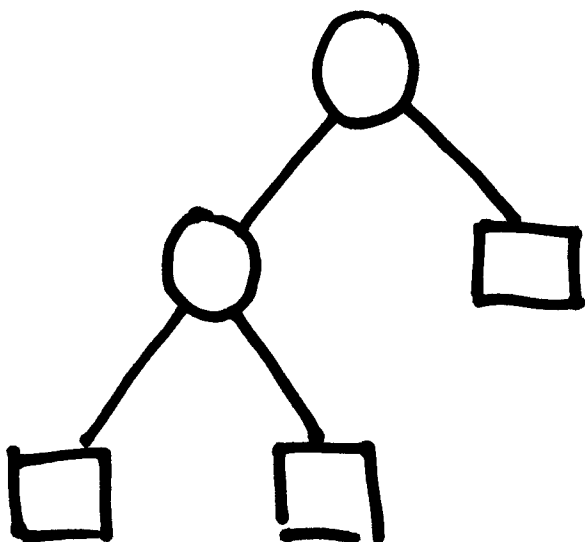
PreOrder O L R

InOrder L O R

PostOrder L R O

P399

Path length



BFS

DFS

Max Adjacency for

Global Min Cut

Almost BFS

Almost DFS

Math Models

Applications

Results ? a tree, a matrix?

correct?

restriction on Input data

Complexity on Time & Space

Extensions of algorithms

Numerical example

Optimal path algorithm

Shortest paths from one node

V_0 to all other nodes

Dijkstra Algorithm

l_i temp label

l_i^* permanent label

Step 0 $l_i = d_{oi}$
 $= \infty$ if no connection

Step 1 $l_k = \min_i l_i$

$$l_k \leftarrow l_k^*$$

Step 2 $l_i \leftarrow \min [l_i, l_k^* + d_{ki}]$

a node has n neighbors

update from one node in step 2

$|\text{shortest path}| \leq (n-1) \text{ Arcs}$

Longest Paths in Acyclic Network

P23 HU

Index nodes such that



Topological sorting

$$l_0 = 0$$

$$l_1 = d_{01}$$

$$l_2 = \text{Max} [(l_0 + d_{02}), (l_1 + d_{12})]$$

$$l_3 = \text{Max} [(l_0 + d_{03}), (l_1 + d_{13}), (l_2 + d_{23})]$$

Multi-terminal Shortest Paths

Floyd-Warshall

$$d_{ij} \geq d_{ji}$$

$$d_{ij} + d_{jk} \geq d_{ik}$$

$$d_{ij} \geq 0 \quad \text{but no negative cycle}$$

Algorithm

$$d_{ik} \leftarrow \min (d_{ik}, d_{ij} + d_{jk})$$

for $j = 1, 2, \dots, n$ (all $i, k \neq j$)

Start $p_{ik} = k$

$$p_{ik} = \begin{cases} p_{ij} & \text{if } d_{ij} + d_{jk} \leq d_{ik} \\ \text{same} & \text{if } d_{ij} + d_{jk} \geq d_{ik} \end{cases}$$

Concepts of basic arcs

vs Brown Arcs in Dijkstra

$$j=1 \quad \text{all } i, k \quad \binom{n}{2} = O(n^2)$$

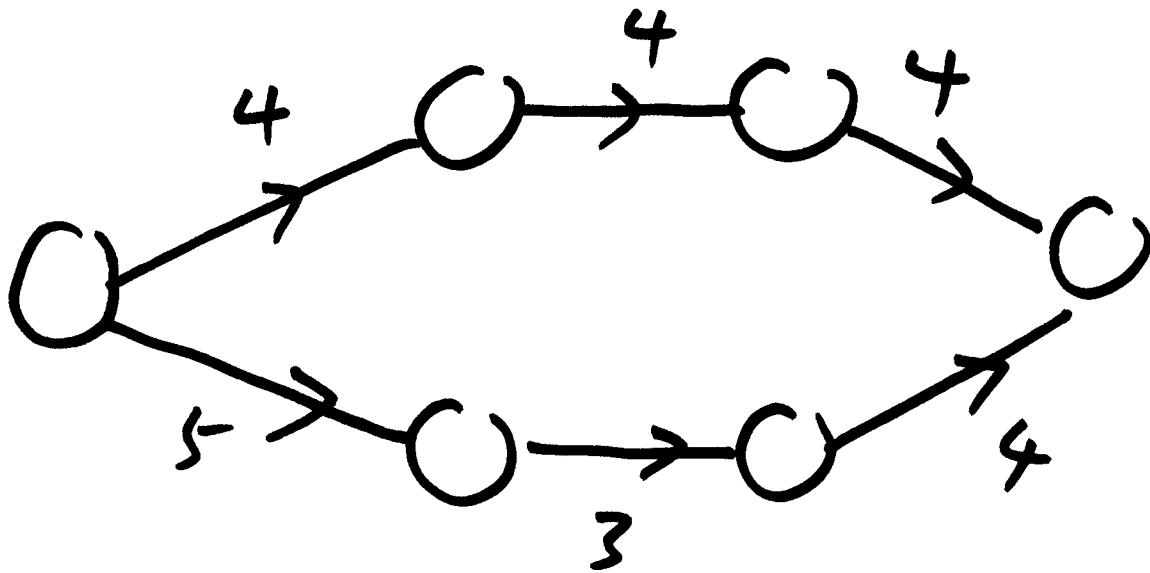
$$j=1 \sim n \quad \text{Time} = O(n^3)$$

Cost of (e_1, e_2, e_3)

$$= e_1 + e_2 + e_3$$

or $\text{Max min}(e_1, e_2, e_3)$

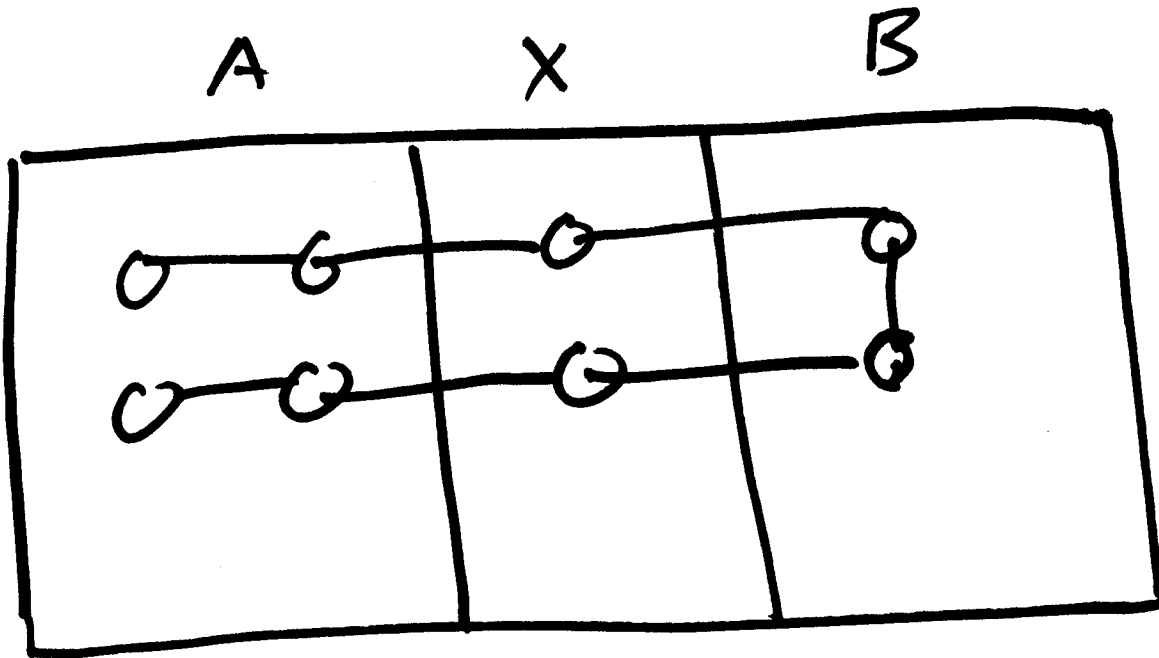
Max Capacity Path



$$d_{ik} \leftarrow \text{Max} [d_{ik}, \min(d_{ij}, d_{jk})]$$

Decomposition Algorithm

to save space



Optimal Tree Algorithms

edges $d_{ij} = d_{ji} = \text{cost}$

Want a Free Tree

with total cost min

Prim Algorithm from one node

Step 0

step 1 $l_k \leftarrow l_k^*$

step 2 $l_i \leftarrow \min[l_i, d_{ki}^*]$

MAX FLOW

Bottleneck in a system

Flow from (s) to (t)

MAX FLOW = Min Cut

S-t planar network

0. $v_s \in X$

1 IF $v_i \in X$, $x_{ij} < b_{ij}$, $v_j \in X$

2 IF $v_i \in X$, $x_{ji} > 0$, $v_j \in X$.

Multi-terminal Flows

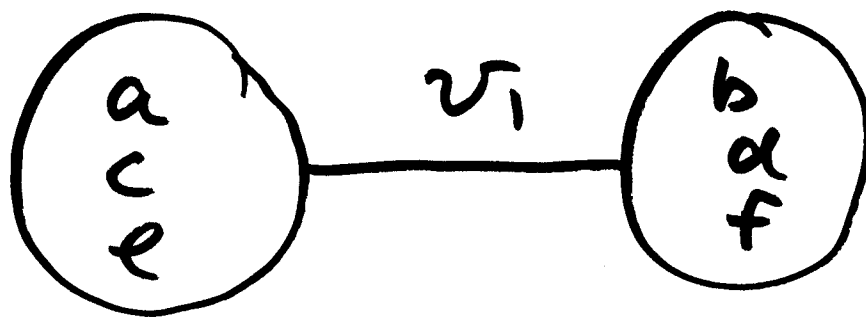
$$b_{ij} \equiv b_{ji}$$

$$F(i, k) \geq \min [F(i, j), F(j, k)]$$

Corollary:

There are at most $n-1$
distinct Max Flow values

Find all MAX Flows
doing $n-1$ max flow algorithms



until a Tree-network
is obtained

Flow-equivalent
cut-equivalent

Synthesis

Given R_{ij}

Raise link values

Form the Max R_{ij} spanning tree

Decompose into subtrees

Subtree $\leftarrow$ loop

Add all loops

Global Min Cut.

by labelling

Go through $n-2$ phases

$v_1, v_2, \dots, v_{n-1}, v_n$

The cut $[v_n]$ is recorded

The edge $v_{n-1} - v_n$ is shrinked.

123	678
45	9

7,	12345678
	9

Greedy Algorithm

EX. COIN-changing problem

$w_1, w_2, \dots, w_n$ are wts of coins

$v_1, v_2, \dots, v_n$ are values of coins

$$\min \sum_{i=1}^n w_i x_i$$

$$\text{sub } \sum v_i x_i = Y$$

$$x_i \geq 0, \text{ integers}$$

w_i, v_i are positive
integers

P204

$$1 = v_1 < v_2 < \dots < v_n$$

$$\frac{w_1}{v_1} \geq \frac{w_2}{v_2} \geq \dots \geq \frac{w_n}{v_n}$$

Define $F_k(y) = \min \sum_1^k w_i x_i$

sub to $\sum_1^k v_i x_i = y$

$x_i \geq 0$ integers

$$F_1(y) = w_1 \left\lfloor \frac{y}{v_1} \right\rfloor = w_1 v_1$$

$$F_2(y) =$$

$$\min_{0 \leq x_2 \leq \left\lfloor \frac{y}{w_2} \right\rfloor} \left[F_1(y - v_2 x_2) + w_2 x_2 \right]$$

Define $G_k(y)$ the total wt of coins
from the first k kinds of coins
by Greedy algorithms

$$G_1(y) = w_1 \left\lfloor \frac{y}{v_1} \right\rfloor = w_1 y$$

$$G_2(y) = w_2 \left\lfloor \frac{y}{v_2} \right\rfloor + w_1 [y \pmod{v_2}]$$

So $G_1(y) = F_1(y)$ for all y

$G_2(y) = F_2(y)$ for all y

If we introduce new coin
one kind at a time

$G_k(y) = F_k(y)$ for all y ?

TH 1. Given $G_K(y) = F_K(y)$

for all y & some K

Let $V_{K+1} > V_K$

$$V_{K+1} = P_K V_K - \delta_K$$

$$0 \leq \delta < V_K.$$

The following are equivalent

(a) $G_{K+1}(y) \leq G_K(y)$ for all y

(b) $G_{K+1}(y) = F_{K+1}(y)$ for all y

(c) $G_{K+1}(P V_K) = F_{K+1}(P V_K)$

(d) $W_{K+1} + G_K(\delta) \leq P W_K$

EX

$$w_1 = w_2 = w_3 = w_4 = 1$$

$$v_1 = 1, \quad v_2 = 5, \quad v_3 = 14 \quad v_4 = 18$$

$$G_1(y) = F_1(y) \text{ for all } y.$$

$$G_2(y) = F_2(y) \text{ for all } y.$$

$$v_3 = 14 = p v_2 - \delta = 15 - 1$$

$$p = 3, \quad \delta = 1$$

$$(d) \quad w_3 + G_2(1) \leq 3 w_2$$

$$1 + 1 \leq 3 \cdot 1$$

so Greedy works for 3 kinds

$$V_4 = 18 = p V_3 - \delta = 2 \cdot 14 - 10$$

$$p = 2, \quad \delta = 10$$

$$W_4 + G_3(\delta) \leq 2 \cdot W_3$$

$$1 + 2 \leq 2 \quad ? \quad \text{No}$$

So Greedy fails for $y = 28$

$$\left\lfloor \frac{28}{18} \right\rfloor + \left\lfloor \frac{10}{5} \right\rfloor = 3$$

$$\left\lfloor \frac{28}{14} \right\rfloor = 2$$