

# BFS

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0. Label the starting vertex  $V_0$  as 0.
1. Let  $\{1, \dots, k\}$  be the set of assigned labels.
2. Find the vertex with the smallest label such that it has an unlabeled neighbor. Label the neighbor  $k + 1$ .

# DFS

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0. Label the starting vertex  $V_0$  as 0.
1. Let  $\{1, \dots, k\}$  be the set of assigned labels.
2. Find the vertex with the *largest* label such that it has an unlabeled neighbor. Label the neighbor  $k + 1$ .

# Dijkstra's Algorithm 1

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Given a directed graph  $G = (V, E)$  where there is a length  $d_{ij}$  on each arc (directed edge) from vertex  $i$  to vertex  $j$ , the assumptions are:

1.  $d_{ij} > 0$  for all  $i, j$
2.  $d_{ij} \neq d_{ji}$  for some  $i, j$
3.  $d_{ij} + d_{jk} \leq d_{ik}$  for all  $i, j, k$

## Dijkstra's Algorithm 2

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Properties of paths:

1. The length of a path is at least the length of its subpaths:

$$|\text{Path}| \geq |\text{Subpath}|.$$

2. Given a shortest path between two vertices, any subpath is also a shortest path.
3. Let  $P_k$  be the number of arcs (directed edges) from the starting vertex  $V_0$  to its  $k^{\text{th}}$  nearest vertex. Then  $P_k$  is at most  $k$ .

## Dijkstra's Algorithm 3

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The algorithm:

0. Vertex  $V_0$  is initialized with label  $l_0^* = 0$ .

For all other vertices  $V_i$ :

$l_i = d_{0,i}$  if  $V_i$  is a neighbor of  $V_0$

$l_i = \infty$  otherwise

1. Pick  $l_k = \min_i l_i$ , then update  $l_k \leftarrow l_k^*$ .

2. Relax vertex  $k$ 's neighbors:

$l_i \leftarrow \min[l_i, l_k^* + d_{k,i}]$ .