CSE 200
Computability and Complexity
Homework 3
NP, Completeness, and Reductions
Part 1
Due Wed, May 17

May 2, 2006

Give proofs for each problem. Proofs can be high-level, but be precise. You may use without giving a proof any result proved in class or in the textbook. In particular, to prove NP-completeness, it suffices to give a reduction from any of the NP-complete problems from the text or from class. However, you must show your reduction is valid, by showing the equivalence of the constructed instance and the original.

1. Let F be a positive integer valued function of two inputs, and $p(n)$ a polynomial. The $Opt(F,p)$ optimization problem is, given x, find $max_{y,|y|=p(|x|)}F(x,y)$. Let $OptP$ be the class of all problems $Opt(F,p)$ where F is computable in polynomial time and p is any polynomial. Prove that $P = NP$ if and only if $OptP \subseteq PF$.

2. Consider the problem of deciding whether a graph on n vertices has an independent set of size at least $n/2$. Prove that this problem is NP-complete.

3. Problem 7.28 from the Sipser text, page 298.

Sodoku The *sudoku* problem of size n is as follows. The input is an $n^2 \times n^2$ matrix M whose entries are either "blank" or an integer between 1 and n^2. A solution fills in the blank spaces with integers between 1 and n^2. The following constraints must be met: Each integer from 1 to n^2 appears exactly once in each row, in each column, and in each $n \times n$ sub-matrix of the form $M[jn+1...(j+1)n][in+1..(i+1)n]$ for each $0 \leq i,j \leq n-1$. The problem is to find any solution meeting the constraints, or return "no solution possible" if there is no such solution.

Give at least two ways of reducing sudoku to $CNFSAT$. For each, specify how many variables and clauses are created as a function of n, and also the size of the different categories of clauses (e.g., clauses of n variables vs. clauses with n^2 variables.) Give a guess for which reduction methods you would expect to work well with SAT solvers and why (see next assignment).