CSE 200 Final Exam
Due Friday, June 16 at noon

Answer four out of five questions with a complete proof. You may be informal, if you are precise. Each question has equal weight. You may cite without proof any result from the text or proved in class.

Computability Let $L = \{(M_1, M_2) | M_1, M_2$ are Turing Machine programs and $L_{M_1} \cap L_{M_2} = \emptyset\}$. Is L recursive? Is L recursively enumerable? Is L co-r.e.?

NP-Completeness Prove that Independent Set remains NP-complete when restricted to graphs of degree at most 3.

You may use the NP-Completeness of any of the following well-known problems: 3-SAT, 3-Colorability, Clique, Independent Set, Vertex Cover, Hamiltonian Circuit, Hamiltonian Path, Travelling Salesperson (decision version).

NP-Completeness Prove that the following "Bus Route" problem is NP-Complete:

- Instance:
  1. A set L of "locations"
  2. For each pair of locations, a non-negative integer called the "travel time" (to reach the second location from the first).
  3. A list of "riders", each consisting of a pair of locations the first being the "origin", and the second the "destination"
  4. An integer bound B on total travel time
- Solution Space: A list of locations, possibly with repetitions
- Constraints: First, for each rider, its destination must appear in the list after its origin. Second, the total travel time between consecutive locations on the list must be at most B, i.e., the total time the bus is on the road must be at most B.
- Problem: Decide whether a feasible solution exists

You may use the NP-Completeness of any of the following well-known problems: 3-SAT, 3-Colorability, Clique, Independent Set, Vertex Cover, Hamiltonian Circuit, Hamiltonian Path, Travelling Salesperson (decision version).

Hint: Consider the case when all customers have a common destination which is far away from all the origins, relative to the other distances.

Complexity Classes Remember, for a complexity class C, P^C is the class of all decision problems that are polynomial time Turing reducible to some language $L \in C$. Prove that $P^{NP \cap Co-NP} = NP \cap Co - NP$.

Space Complexity Let F be a function so that $|F(x)| = |x|$ for every x. Let F^k be F composed with itself k times, so $F^1(x) = F(x)$, and $F^{k+1}(x) = F(F^k(x))$. A *fixed point* for F on x is a value of k so that $F^{k+1}(x) = F^k(x)$. Let $F - FIX = \{x|$ there is some fixed point k of F on $x\}$. Prove that $P = PSPACE$ if and only if $F - FIX \in P$ for every such $F \in P$.