

CSE105 - HOMEWORK 7 SOLUTIONS

3.7 Since there are infinitely many settings of $x_1, \dots, x_k$ to integer values, any Turing machine implementing this psuedocode will never get to step 2. No Turing machine can behave like this psuedocode would: it would not be able to get out of step one and hence couldn't implement steps 2 and 3. Moreover, the description of the first step is not meaningful: what does it mean to "try all possible settings of $x_1, \dots, x_k$ to integer values"? Store all the settings in a table? It is not clear.

3.8 When we scan across the tape, we only look at the part of the tape that the input is on – *i.e.* we stop at the blank symbols.

a. Answer in text.

b. While there are 0s or 1s left on the tape:

- Scan across to a 0. If there is no 0, *reject*.
- Replace the 0 with a #.
- Scan across to a 0. If there is no 0, *reject*.
- Replace the 0 with a #.
- Scan across to a 1. If there is no 1, *reject*.
- Replace the 1 with a #.

Accept.

c. While there are 0s or 1s left on the tape:

- Scan across to a 1. If there is no 1, *accept*.
- Replace the 1 with a #.
- Scan across to a 0. If there is no 0, *accept*.
- Replace the 0 with a #.
- Scan across to a 0. If there is no 0, *accept*.
- Replace the 0 with a #.

Reject.

3.22 Answer in text.

4.6 Suppose towards contradiction that $\mathcal{B}$ is countable. Then, we could list the elements of $\mathcal{B}$:

b_1	0	0	1	0	...
b_2	1	0	0	1	...
b_3	1	1	1	0	...
b_4	0	0	0	1	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

We define $b = d_1 d_2 d_3 d_4 \dots$, where each d_i is a 0 or 1, as

$$d_i = \begin{cases} 0 & \text{if the } i\text{th bit of } b_i \text{ is 1,} \\ 1 & \text{if the } i\text{th bit of } b_i \text{ is 0.} \end{cases}$$

b is an infinite binary sequence, so is in $\mathcal{B}$. However, b differs from each element of the list b_i by at least one bit, so is not in the list. Thus such a list cannot contain all of $\mathcal{B}$, so $\mathcal{B}$ is not countable.

4.7 We prove that if R and S are countable sets, $R \times S$ is countable. Let $R = \{r_1, r_2, \dots\}$ and $S = \{s_1, s_2, \dots\}$. Consider the following table of $R \times S$.

	s_1	s_2	s_3	s_4	$\dots$
r_1	(r_1, s_1)	(r_1, s_2)	(r_1, s_3)	(r_1, s_4)	$\dots$
r_2	(r_2, s_1)	(r_2, s_2)	(r_2, s_3)	(r_2, s_4)	$\dots$
r_3	(r_3, s_1)	(r_3, s_2)	(r_3, s_3)	(r_3, s_4)	$\dots$
r_4	(r_4, s_1)	(r_4, s_2)	(r_4, s_3)	(r_4, s_4)	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

We can list all of the elements of $R \times S$ by following the diagonals that run right-to-left and bottom-to-top:

(r_1, s_1)
 (r_2, s_1)
 (r_1, s_2)
 (r_3, s_1)
 (r_2, s_2)
 (r_1, s_3)
 (r_4, s_1)
 (r_3, s_2)
 (r_2, s_3)
 (r_1, s_4)
 $\vdots$

This is essentially the same technique that was used to show that the rationals are countable on p. 176 of the text.

Since $\mathcal{N}$ is countable, our above proof demonstrates that $\mathcal{N} \times \mathcal{N}$ is countable. Then, since $\mathcal{N} \times \mathcal{N}$ is countable, $\mathcal{N} \times \mathcal{N} \times \mathcal{N}$ is countable.