

CSE105 - HOMEWORK 1 SOLUTIONS
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- 0.1**
- a. Odd natural numbers
 - b. Even integers
 - c. Even natural numbers
 - d. Even natural numbers that are multiples of 3
 - e. Symmetric binary numbers
 - f. The empty set
- 0.2**
- a. $\{1, 10, 100\}$
 - b. $\{x \in \mathbb{Z} \mid x > 5\}$
 - c. $\{x \in \mathbb{N} \mid x < 5\}$
 - d. $\{\text{aba}\}$
 - e. $\{\epsilon\}$
 - f. $\emptyset$
- 0.3**
- a. No: $A \not\subset B$
 - b. Yes: $B \subset A$
 - c. $A \cup B = \{x, y, z\}$
 - d. $A \cap B = \{x, y\}$
 - e. $A \times B = \{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)\}$
 - f. $\mathcal{P}(B) = \{\emptyset, \{x\}, \{y\}, \{x, y\}\}$
- 0.4** Each element of A is paired with each element of B , so $|A \times B| = ab$.
- 0.5** There are 2^c elements in the powerset of C . Suppose we label each element of C with a 1 or 0 and form the subset of elements of C that we labelled with a 1. Then, we can describe each element of the powerset – any subset of C – this way, so the size of the powerset is the number of 0-1 labellings of the elements of C .
- 0.6**
- a. 7
 - b. $\text{range}(f) = \{6, 7\}, \text{domain}(f) = \{1, 2, 3, 4, 5\}$
 - c. 6
 - d. $\text{range}(g) = \{6, 7, 8, 9, 10\}, \text{domain}(g) = \{(x, y) \in \mathbb{N} \times \mathbb{N} \mid 1 \leq x \leq 5, 6 \leq y \leq 10\}$
 - e. $g(4, f(4)) = g(4, 7) = 8$
- 0.7** Each of the following relations is defined over $A = \{a, b, c\}$
- a. $\{(a, a), (b, b), (c, c), (a, b), (b, a), (b, c), (c, b)\}$; since (a, c) is not in the set, the relation is not transitive.
 - b. $\{(a, a), (b, b), (c, c), (a, b), (b, c), (a, c)\}$
 - c. $\emptyset$.
- 0.8** $\deg(1) = 3; \deg(3) = 2$

0.9 $G = (V, E)$, where $V = \{1, 2, 3, 4, 5, 6\}$ and

$$\begin{aligned} E &= \{1, 2, 3\} \times \{4, 5, 6\} \\ &= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\} \end{aligned}$$

0.10 $(a - b) = 0$ if $a = b$, so there is a division by zero.

0.11 The proof breaks down at $k = 2$: H_1 and H_2 will be disjoint, so the induction hypothesis tells us only that each horse is the same color as itself. We cannot jump to the conclusion that the two horses are the same color.

0.12 Suppose towards contradiction that there is a graph G with n nodes ($n \geq 2$), all of which have different degree. The degree of any node must be at least 0 and at most $n - 1$, so there are only n possible degrees that a node can be. Since there are n different nodes, we conclude that for *each* integer i in the range $[0, n - 1]$, there is a node of degree i . In particular, there is a node of degree $n - 1$, meaning it is connected to all other nodes, and a node of degree 0, which is connected to no other nodes. However, the existence of each of these two nodes contradicts the existence of the other. Thus, all graphs of degree n must contain at least two nodes of the same degree.

- 1.1 a. $M_1: q_1; M_2: q_1$
 b. $M_1: \{q_2\}; M_2: \{q_1, q_4\}$
 c. $M_1: q_1, q_2, q_3, q_1, q_1; M_2: q_1, q_1, q_1, q_2, q_4$
 d. M_1 : no; M_2 : yes
 e. M_1 : no; M_2 : yes

1.2 answer in text

1.3 see other sheet

1.4g see other sheet

1.5g see other sheet

1.7d see other sheet

1.7g see other sheet

1.8b see other sheet

1.9a see other sheet

1.33 see other sheet. By the hint, we can read from right to left, so the given FSA actually processes the string from right to left (the reverse order). In the drawing, we wrote the transitions as $[x, y]$, which corresponds to $\begin{bmatrix} x \\ y \end{bmatrix}$ in the text's notation. For example, $[0, 1]$ means $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Recall that multiplying a binary number x by 3 is equivalent to bit-shifting x to the left, yielding $x0$, and adding $x0$ to x . We use the FSA to carry out this addition bit-by-bit. Let the top string of the input be $t_1 t_2 t_3 \cdots t_n$ and the bottom be $b_1 b_2 b_3 \cdots b_n$. Let c_i be the carry from adding $t_i + t_{i+1}$. Then, on reading a $\begin{bmatrix} t_i \\ b_i \end{bmatrix}$, we need to check if $t_i + t_{i+1} + c_{i+1} = b_i$. We read t_i and b_i from the input, and use the states to remember $(c_{i+1} t_{i+1})$.

Here is what the states represent:

state	$(\mathbf{c}_{i+1}, \mathbf{t}_{i+1})$
q_0	$(0, 0)$
q_1	$(0, 1)$
q_2	$(1, 1)$
q_3	$(0, 0)$
q_4	$(1, 0)$

Note that $(0, 0)$ is duplicated. This is done so that we don't accept the empty string. Note also that we are really only interested in $c_{i+1} + t_{i+1}$, so states q_1 and q_4 can be combined into one.

1.36 $(Q, \Sigma, \delta, q_0, A)$, where

$$\begin{aligned}
Q &= \{q_i \mid 0 \leq i \leq n-1\} \\
\Sigma &= \{a\} \\
\delta(q_i, a) &= q_{(i+1) \bmod n} \\
A &= \{q_0\}
\end{aligned}$$