

CSE 105—Theory of Computability

Lecture 9—May 2, 2006
Pushdown Automata/CFG
Pumping Lemma for CFL

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Example

Language = w *not* a palindrome (over $\{0, 1\}$)

Example

Language: $L\{xy \mid x \neq y, x, y \in \{0, 1\}^*\}$

CFG can be converted to NPDA

Easiest to use Chomsky Normal Form

- Book uses any grammar without conversion; idea is the same

Let's look at leftmost derivation with CNF

- Example grammar
 - $S \rightarrow TT \mid RT$
 - $T \rightarrow 0 \mid TT \mid 1$
 - $R \rightarrow 0 \mid RR$
- Example string
 - 01101

CFG can be converted to NPDA

Without Chomsky Normal Form

- Can have arbitrary mix of terminals and non-terminals in sentential form
- Store everything except leading terminals on stack
- Match input symbols to stack terminal symbols
- Example:
 - $S \rightarrow 0T1 \mid 1$
 - $T \rightarrow T0 \mid \epsilon$
- Example string:
 - 0001

CFG equivalent to PDA

Still need to show can convert PDA to CFG

- Believe me:) (In book, if you desire)

Pumping Lemma

A CFL can be pumped up (any string over a certain length can be used as a pattern for infinitely many more strings)

Pumping Lemma

For every Context-Free language, L

- There exists p
- such that, for all strings $s \in L$, with $|s| \geq p$
- s can be rewritten as $uvwxy$ with $|vwx| \leq p$, $|vx| \geq 0$
- where, for all $i \geq 0$, $uv^iwx^iy \in L$

Example

$$L = \{ww \mid w \in 0^*\}$$

Example

$$L = \{w1w \mid w \in 0^*\}$$

Palindromes

Example

$$L = \{0^n 1^n 2^n \mid n \geq 0\}$$

Example

$$L = \{ww \mid w \in \{0,1\}^*\}$$

Example

CFL not closed under intersection