

CSE 105—Theory of Computability

Lecture 4—April 13, 2006
Pumping Lemma

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Example converting RE to FSA

Want RE for binary strings not divisible by 3

No FSA that recognizes 0^n1^n

Imagine there were

- How many states?
 - p
- What states are visited when recognizing 0^p1^p ?
- This implies:

Pumping Lemma

For every regular language L :

- There exists a positive integer p such that:
 - For all strings $s \in L$ where $|s| \geq p$, s can be rewritten as xyz such that:
 - $|y| > 0$
 - $xy^iz \in L$ for all non-negative integer i
 - $|xy| \leq p$

Using the pumping lemma to prove 0^n1^n not regular

Example

Prove $L = \{w \mid w \text{ has an equal number of 0s and 1s}\}$ is irregular

- Using Pumping Lemma
- Using closed under complement

Example

$$L = \{ww^R \mid w \in \{0, 1\}^*\}$$

Example

Show $L = \{w \mid w \text{ is a palindrome over } \{0,1\}^*\}$ is not regular

Example

Let L (over $\{0, 1, +, =\}$) = $\{x=y+z \mid x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}$

- Is L regular?

Contradiction?

All finite sets are regular

All regular sets are pumpable

Finite sets are not regular

Example (1.57)

Example (1.58)

If A is any language, let A'' be the set of all strings in A with their middle thirds removed so that:

- $A'' = \{xz \mid \text{for some } y, |x| = |y| = |z| \text{ and } xyz \in A\}$

Show that if A is regular, A'' is not necessarily regular