

CSE 105—Theory of Computability

Lecture 20—June 6, 2006
Time Complexity

Instructor: Neil Rhodes

NP-Complete

There exist problems which are as hard as any in NP (**NP-hard**).

- There is a polynomial time reduction from any problem in NP to an NP-Hard problem

If such a problem is also in NP, then it is **NP-complete**

- If you find a polynomial time algorithm for *any* NP-complete problem, all NP-complete problems have a polynomial-time algorithm

The first NP-complete problem

3-SAT: Given a boolean formula in CNF:

- n variables, m clauses, each clause is a disjunct of 3 variables (or negated variable)

$$- (x_1 \vee x_2 \vee x_3) \wedge (\sim x_1 \vee x_4 \vee \sim x_3) \wedge \dots (x_3 \vee x_2 \vee x_1)$$

Is there an assignment of truth values that satisfies the formula?

- $(x_1 \vee x_2 \vee x_3) \wedge (\sim x_1 \vee x_3 \vee \sim x_2) \wedge (x_1 \vee \sim x_2 \vee \sim x_3)?$

- $(x_1 \vee x_2 \vee x_3) \wedge (\sim x_1 \vee x_2 \vee \sim x_3) \wedge (\sim x_1 \vee \sim x_2 \vee x_3)?$

Showing a problem L is NP-Complete

Show polynomial-time reduction

- mapping reduction (Karp reduction)
- Turing reduction (Cook reduction)

Reduce from known NP-complete to L

If L has polynomial-time TM, then:

Example NP-Complete Problem

Vertex-cover is NP-complete

- Vertex-cover = $\{ \langle G, k \rangle \mid G \text{ is an undirected graph that has a } k\text{-node vertex cover} \}$
- Vertex-cover: subset of vertices V' such that each edge has an endpoint in V'

Example NP-Complete Problem

To show:

- Vertex-cover $\in$ NP
- 3-SAT $\leq_M$ Vertex-cover
- Example 3-SAT formula:
 - $I = (x_1 \vee x_2 \vee x_3) \wedge (\sim x_1 \vee x_3 \vee \sim x_2) \wedge (x_1 \vee \sim x_2 \vee \sim x_3)$
- Describe f , that maps from _____ to _____
- To show:
 - f is polynomial-time
 - _____ $\in$ _____ iff _____ $\in$ _____

EXPTIME

Exponential time:

- $\text{EXPTIME} = \{L \mid L \text{ is decidable by a deterministic TM in } \text{TIME}(2^{n^k}) \text{ for some } k\}$
- Relationship between:
 - LOGSPACE
 - P
 - NP
 - PSPACE
 - EXPTIME