

CSE 105—Theory of Computability

Lecture 18—June 1, 2006
Reducibility

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Prove undecidable

Show that the following language is undecidable:

- $T = \{ \langle M \rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w \}$
- Step one: decide on undecidable language to reduce to T
 - $\leq_M T$
- Step two: Decide on two languages for a new TM, M' to accept:
 - $(\langle M \rangle, w) \in T \rightarrow M' \in T$. So, if M accepts w , M' accepts L_1 , where L_1 contains w^R whenever it contains w . Let $L_1 = \{ \quad \}$
 - $(\langle M \rangle, w) \notin T \rightarrow M' \notin T$. So, if M doesn't accept w , M' accepts L_2 , where L_2 contains some string whose reverse is not in L_2 . Let $L_2 = \{ \quad \}$
- Step three: write the description for the TM M' that takes input w'
 - step a) If $w' = \quad$, accept
 - step b) If $w' = \quad$, run M on w . If it accepts, accept
 - step c) Otherwise, reject
- Step four: describe the mapping function f :
 - f will, given M and w , create the TM M' as described above
- Step five: summary
 - We've reduced $\quad$ to T because:
 - If $(\langle M \rangle, w) \in T$, $f(\langle M \rangle, w) = M'$, and M' accepts $L_1 = \{ \quad \}$, so $\langle M' \rangle \in T$
 - If $(\langle M \rangle, w) \notin T$, $f(\langle M \rangle, w) = M'$, and M' accepts $L_2 = \{ \quad \}$, so $\langle M' \rangle \notin T$

Decidability example

Consider the problem of determining whether a two-tape Turing machine ever writes a nonblank symbol on its second tape when it is run on input w . Formulate this problem as a language and show that it is undecidable.

- $L = \{ \langle M, w \rangle \mid M \text{ is a TM that } \quad \quad \quad \}$
- Reduce what to L ?
- What will M' do?

Decidability example

Consider the problem of determining whether a two-tape Turing machine ever writes a nonblank symbol on its second tape when it is run on any input. Formulate this problem as a language and show that it is undecidable.

- $L = \{ \langle M \rangle \mid M \text{ is a TM that } \}$
- Reduce what to L ?
- What will M' do?

Decidability example

Consider the problem of determining whether a single-tape Turing machine ever writes a blank symbol over a nonblank symbol during the course of its computation on any input string. Formulate this problem as a language and show that it is undecidable.

- $L = \{ \langle M \rangle \mid M \text{ is a TM that} \quad \quad \quad \}$
- Reduce what to L?
- What will M' do?

Decidability example

Consider the problem of determining whether a single-tape Turing machine on any input ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.

- $L = \{ \langle M \rangle \mid M \text{ is a TM that } \}$
- Reduce what to L ?
- What will M' do?

Decidability example

Consider the problem of determining whether a single-tape Turing machine on any input ever writes a blank on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.

- $L = \{ \langle M \rangle \mid M \text{ is a TM that } \}$
- Reduce what to L ?
- What will M' do?

Decidability example

A *useless state* is a state that is never entered on any input string.

Consider the problem of determining whether a single-tape Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

- $L = \{ \langle M \rangle \mid M \text{ is a TM that } \}$
- Reduce what to L?
- What will M' do?

Decidability example

Consider the problem of determining whether a single-tape Turing machine on any input halts leaving the tape with the contents “011111”. Formulate this problem as a language and show that it is undecidable.

- $L = \{ \langle M \rangle \mid M \text{ is a TM that } \}$
- Reduce what to L ?
- What will M' do?

Decidability example

Consider the problem of determining whether a single-tape Turing machine on input w enters all the states of the machine. Formulate this problem as a language and show that it is undecidable.

- $L = \{ \langle M \rangle \mid M \text{ is a TM that } \}$
- Reduce what to L ?
- What will M' do?

Decidability example

Consider the problem of determining whether a single-tape Turing machine on input w ever moves the tape head right then immediately left then immediately right then immediately left. Formulate this problem as a language and show that it is undecidable.

- $L = \{ \langle M, w \rangle \mid M \text{ is a TM that } \quad \quad \quad \}$
- Reduce what to L ?
- What will M' do?

Reducing $\cap_{CFG}$ to ALL_{CFG}

$ALL_{CFG} = \{G \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$

- Create $G' = (G_1 \cap G_2)^c$
- Check whether $G' \in ALL_{CFG}$. If yes, G_1 and G_2 don't intersect. If no, they do intersect.
- Is this valid? Is $G_1 \cap G_2$ a CFG? Is the complement of a CFL a CFL?
 - No
- However, look at $PCP \leq \cap_{CFG} \leq ALL_{CFG}$
 - The grammars used in the reduction of PCP were deterministic!
 - So, actually, the reduction is:
- $G' = G_1^c \cup G_2^c = (G_1 \cap G_2)^c$
 - G_1^c is DCFG
 - Union is CFG