

CSE 105—Theory of Computability

Lecture 16—May 25, 2006

Reductions

Post Correspondence Problem

Instructor: Neil Rhodes

$\text{EMPTY}_{\text{TM}} = \{M \mid M \text{ is a TM and } M \text{ accepts the empty language}\}$

Is EMPTY_{TM} decidable?

Is $\text{EMPTY}_{\text{TM}}^c$ recursively enumerable?

$\text{REGULAR}_{\text{TM}} = \{M \mid M \text{ is a TM and } M \text{ accepts a regular language}\}$

Is $\text{REGULAR}_{\text{TM}}$ decidable?

$CF_{TM} = \{M \mid M \text{ is a TM and } M \text{ accepts a context-free language}\}$

Is CF_{TM} decidable?

$EQ_{TM} = \{(M_1, M_2) \mid M_1, M_2 \text{ are TM and } M_1, M_2 \text{ accept the same language}\}$

Is EQ_{TM} decidable?

Rice's Theorem

Any problem that takes a Turing machine as input and asks any non-trivial question about the language it accepts is undecidable.

- Trivial question:
 - Satisfied by all recursively enumerable languages
 - Satisfied by no languages
- Questions about the *machine* are decidable
 - How many states?
 - Ever move tape head to the left?

Linear Bounded Automaton

Turing machines that can't read/write outside the input

- $A_{LBA} = \{(M, w) \mid M \text{ is an LBA such that } M(w) \text{ accepts}\}$
- A_{LBA} is decidable

Post Correspondence Problem

Given two sets A and B of numbered strings A_i , B_i :

i	A_i	B_i
1	00	000
2	110	0
3	000	11

Is there a way to pick a sequence of numbers (i 's), such that:

- If you concatenate the A_i s using that sequence
- And concatenate the B_i s using that sequence
- The two concatenations are equal

Example:

Example

i	A_i	B_i
1	00	010
2	10	11
3	11	1

i	A_i	B_i
1	00	001
2	10	11
3	011	0110

Post Correspondence Problem

The Post Correspondence Problem

- Given sequence of A_i and sequence of B_i , is there a solution?
- Undecidable problem!
- Technical reduction that simulates the computation of a TM

Reducing PCP to $\cap_{CFG}$

$\cap_{CFG} = \{(G_1, G_2) \mid G_1 \text{ and } G_2 \text{ are CFGs and } L(G_1) \cap L(G_2) = \emptyset\}$

- Reducing PCP to $\cap_{CFG}$ ($PCP \leq \cap_{CFG}$)
- Make A into one grammar, B into another

- $A \rightarrow 00A1 \mid 110A2 \mid 000A3 \mid$
 $00\#1 \mid 110\#2 \mid 000\#3$
- $B \rightarrow 000B1 \mid 0B2 \mid 11B3 \mid$
 $000\#1 \mid 0\#2 \mid 11\#3$

i	A_i	B_i
1	00	000
2	110	0
3	000	11

Reducing $\cap_{CFG}$ to ALL_{CFG}

$ALL_{CFG} = \{G \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$

- Create $G' = (G_1 \cap G_2)^c$
- Check whether $G' \in ALL_{CFG}$. If yes, G_1 and G_2 don't intersect. If no, they do intersect.
- Is this valid? Is $G_1 \cap G_2$ a CFG? Is the complement of a CFL a CFL?
 - No
- However, look at $PCP \leq \cap_{CFG} \leq ALL_{CFG}$
 - The grammars used in the reduction of PCP were deterministic!
 - So, actually, the reduction is:
- $G' = G_1^c \cup G_2^c = (G_1 \cap G_2)^c$
 - G_1^c is DCFG
 - Union is CFG

Reducing PCP to AMBIG_{CFG}

AMBIG_{CFG} = {G | G is an ambiguous CFG}

- Reducing PCP to AMBIG_{CFG} (PCP ≤ AMBIG_{CFG})
- Make a grammar

- $A \rightarrow 00A1 \mid 110A2 \mid 000A3 \mid 00\#1 \mid 110\#2 \mid 000\#3$
- $B \rightarrow 000B1 \mid 0B2 \mid 11B3 \mid 000\#1 \mid 0\#2 \mid 11\#3$
- $S \rightarrow A \mid B$

i	A _i	B _i
1	00	000
2	110	0
3	000	11

Reducing ALL_{CFG} to $EQ_{CFG/REG}$

$EQ_{CFG/REG} = \{(G, R) \mid G \text{ is a CFG, } R \text{ is an FSA, } L(G)=L(R)\}$

- How to solve $ALL(G)$:
 - Check $EQ(G, \Sigma^*)$

Reducing $EQ_{CFG/REG}$ to EQ_{CFG}

$$EQ_{CFG} = \{(G_1, G_2) \mid G_1 \text{ and } G_2 \text{ are CFGs and } L(G_1) = L(G_2)\}$$

Reductions

