CSE252B – Computer Vision II – Assignment #3
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`http://www-cse.ucsd.edu/classes/sp05/cse252b`
Target Due Date: Tue. May. 10, 2005.

1. Show that an affine transformation can map a circle to an ellipse, but cannot map an ellipse to a hyperbola or parabola.

2. Consider the Lyapunov map from MaSKS Equation (6.42), p. 195:

$$L : \mathbb{C}^{3\times 3} \to \mathbb{C}^{3\times 3}; \quad X \mapsto X - CXC^\top$$

   Assume C has n independent eigenvectors $\{\boldsymbol{u}_i \in \mathbb{C}^n\}_{i=1}^n$, with eigenvalues given by $C\boldsymbol{u}_i = \lambda_i \boldsymbol{u}_i$.

   (a) Show that $X_{ij} = \boldsymbol{u}_i \boldsymbol{u}_j^* \in \mathbb{C}^{3\times 3}$ is an eigenvector of L. What is the corresponding eigenvalue?

   (b) Assuming $\det(C) = 1$, which eigenvectors are in $SRker(L)$? In other words, for which values of i and j does L map the symmetric real X to the zero matrix?

   (c) Explain the significance of $SRker(L)$ if we interpret X as the coefficient matrix for a conic.

3. 2D Upgrade from Affine to Euclidean via Orthogonal Lines.

   (a) Load in the affine-rectified image `affine_tile.gif`, identify two pairs of imaged orthogonal lines, and plot them on the raw image.

   (b) Implement the algorithm to solve for $K \in SL(2)/SO(2)$ from two imaged right angles on a plane as described in H&Z Example 2.26 (Metric Rectification I), p. 56.

   (c) Demonstrate your code on the tile image. Display the Euclidean-rectified image and plot the transformed line pairs on it.

4. Implement the algorithm described in H&Z Example 8.18 (A Simple Calibration Device), p. 211. Use it to estimate K from the image `squares.gif` depicting three metric planes.

5. Derive the solution for the homography $H \in GL(4)$ relating $n \geq 5$ corresponding 3D points $(\boldsymbol{X}_1^j, \boldsymbol{X}_2^j)$, $j = 1, 2, \ldots, n$, in general position.

6. Uncalibrated 3D Reconstruction.

   (a) Run the script `house_views.m` to produce two uncalibrated views of a wireframe house.

   (b) Recover F using the 8-pt. algorithm and compute the canonical camera matrices (Π_{1p}, Π_{2p}).

   (c) Triangulate to produce the projective structure $\boldsymbol{X}_p$.

   (d) Find the homography $H \in GL(4)$ to upgrade $\boldsymbol{X}_p$ directly to the Euclidean structure $\boldsymbol{X}_e$ using 5 ground truth points.

   (e) Solve for the image of the plane at infinity $(\boldsymbol{v}^\top, v_4)^\top$ from three vanishing points and use it to upgrade $\boldsymbol{X}_p$ to the affine structure $\boldsymbol{X}_a$.