Branch and Bound
Edmonds, Chapter 25

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Branch and Bound

Keep value of best solution so far
- As a global, perhaps. No need to consider solutions which will give a worse value.

Compute bound as we visit each node
- Will be a bound on the value of solution found in subtree.
  - Bound will be upper bound if we have a maximization problem or lower bound if we have a minimization problem.

Extend idea of promising
- If bound of a node is better than best solution, node is promising, otherwise it isn't.

Bounding allows us to prune more nodes

Integer Knapsack with bounding

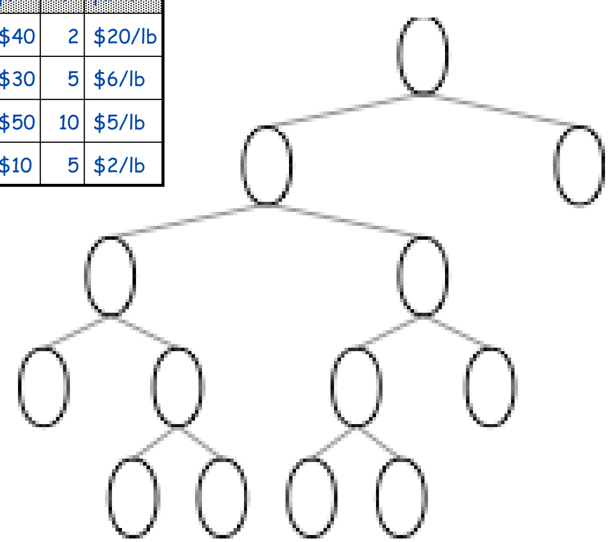
Example: W = 16

i	p_i	w_i	p_i/w_i
1	\$40	2	\$20/lb.
2	\$30	5	\$6/lb.
3	\$50	10	\$5/lb.
4	\$10	5	\$2/lb.

At a given node, we have a weight so-far and a profit so-far. A bound on the total profit would be if we greedily took remaining items with the best profit/weight (until an item won't fit), and then take a fraction of that item that won't fit.
- Example:
  - We've decide to not include item 1 and to include item 2. Weight-so-far = 5, profit-so-far = \$30. A greedy upper bound on the total profit is to take all of item 3 and 1/5 of item 4. Weight = 16 and bound on profit is \$30 + \$50 + 1/5(\$10) = \$82.

W=16 Knapsack Branch and Bound with Depth-First Search

i	p_i	w_i	p_i/w_i
1	\$40	2	\$20/lb
2	\$30	5	\$6/lb
3	\$50	10	\$5/lb
4	\$10	5	\$2/lb



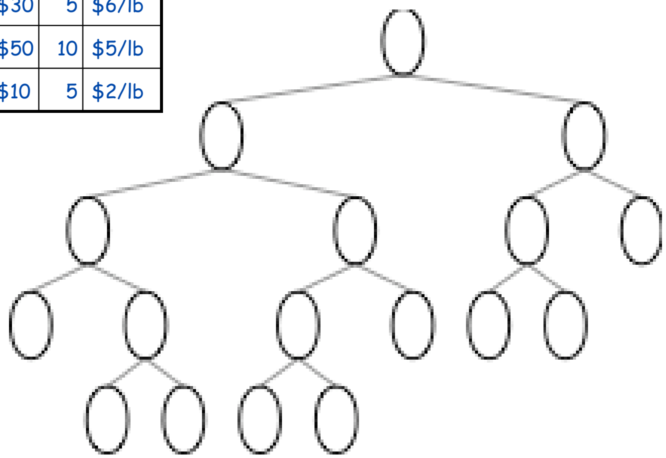
Example from Foundations of Algorithms by Neapolitan and Naimipour

W=16

Knapsack Branch and Bound with Breadth-First Search

Weight$_{sofar}$
\$Profit$_{sofar}$
\$bound

i	p_i	w_i	p_i/w_i
1	\$40	2	\$20/lb
2	\$30	5	\$6/lb
3	\$50	10	\$5/lb
4	\$10	5	\$2/lb



Example from Foundations of Algorithms by Neapolitan and Naimipour

Breadth-First Search

Speedup

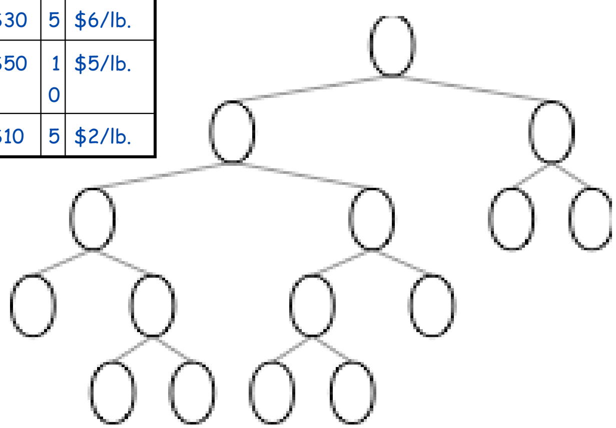
- When we remove a node from the queue, its bound may be lower than current best value. If so, skip it.

W=16

Knapsack Branch and Bound with Breadth-First Search (re-evaluate when removing from queue)

Weight$_{sofar}$
\$Profit$_{sofar}$
\$bound

i	p_i	w_i	p_i/w_i
1	\$40	2	\$20/lb.
2	\$30	5	\$6/lb.
3	\$50	10	\$5/lb.
4	\$10	5	\$2/lb.



Example from Foundations of Algorithms by Neapolitan and Naimipour

Best-First Search

Idea

- Of all nodes in the queue to be visited, pick the one with the best bounds. It might give better results than others.

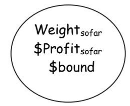
Implementation

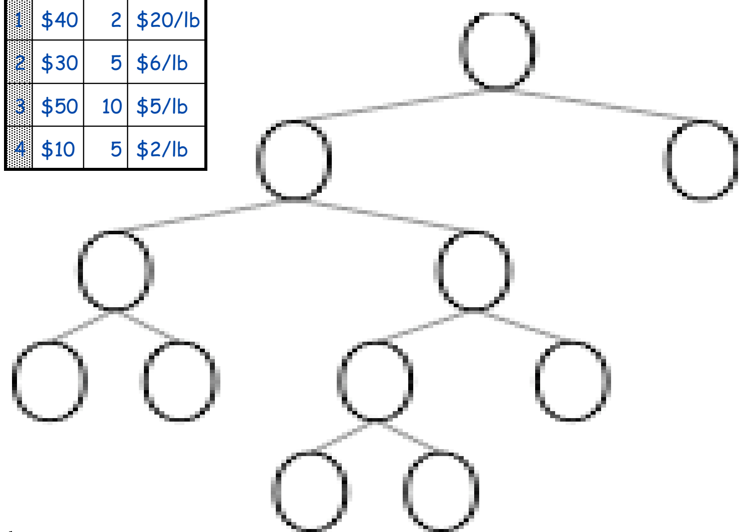
- Rather than the simple queue used for breadth-first search, use a prioirity queue.

Knapsack Branch and Bound with Best-First Search

W=16

i	p_i	w_i	p_i/w_i
1	$40	2	$20/lb
2	$30	5	$6/lb
3	$50	10	$5/lb
4	$10	5	$2/lb





Example from Foundations of Algorithms by Neapolitan and Naimipour

Master Algorithm for Branch and Bound with Best-First Search

```
BestFirstBranchAndBound(T)
Precondition: T is decision tree for problem. Nodes have bound and value.
Postcondition; returns best value of a solution

v = root of T
best = value(v)
Add v to priority queue (keyed on bound(v))
while priority queue is not empty
  remove element v from priority queue
  if bound(v) is better than best then
    foreach child u of v
      if value(u) is better than best then best = value(u)
      if bound(u) is better than best then insert u into priority queue
    end foreach
  end if
end while
return best
```


Knapsack Backtracking Branch and Bound (Best-First Search)

```
Knapsack(w, profit, W)
root.bound = bound(root)        bound returns upper bound on profit
maxprofit = root.profit = root.weight = root.level = 0
insert root into priority queue
while priority queue is not empty
  remove minimum element, v, from priority queue
  if v.bound > maxprofit then
    u.level = u.level + 1
    u.weight = v.weight + w_u.level
    u.profit = v.profit + profit_u.level
    if u.weight <= W and u.profit > maxprofit then
      maxprofit = u.profit
    u.bound = bound(u)
    if u.bound > maxprofit then insert u into priority queue
    u.weight = v.weight; u.profit = v.profit; u.bound = bound(u)
    if u.bound > maxprofit then insert u into priority queue
  end if
end while   Postcondition: maxprofit is value of most profitable solution
```


Branch and Bound Summary

Don't branch from a node whose bound is worse than the best solution found so far

- An extension of the idea of non-promising: impossible to find a valid solution from this node

A Best-first search searches along a frontier of nodes, choosing the one with the best bound first

- This is a greedy approach which will often find an optimal solution more quickly than using a predetermined order (like DFS or BFS). It will not always be quicker, but it is a reasonable heuristic.