8. NP and Computational Intractability

CSE 101

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Neil Rhodes

Algorithmic Design Patterns and Anti-Patterns

Algorithmic design patterns.

- Greed. $O(n \log n)$ interval scheduling.
- Divide-and-conquer. $O(n \log n)$ sorting.
- Dynamic programming. $O(n^2)$ edit distance.
- Network flow. $O(n^3)$ bipartite matching.

Algorithmic anti-design patterns.

- NP-completeness $O(n^k)$ algorithm unlikely.
- PSPACE-completeness. $O(n^k)$ certifying algorithm unlikely.
- Undecidability. No algorithm possible.

NP

We'll look only at **decision** problems

- Problems for which the answer is **yes** or **no**
- Optimization problem: what is the longest path?
- Decision problem: is there a path of size k or larger?

NP = Set of problems for which a solution can be **verified** in polynomial time

- Given a **certificate** for a solution (certificate is polynomial in the input size), certifier checks to see whether the solution is correct
- For example, a certificate for path of size k or larger is the path itself
  - Certifier: check that each edge is in the original graph, check that the path has at least k edges

NP problems can be solved in exponential time

- Run the certifier on each of the exponential number of certificates

Historical Note

NP stands for **N**on-deterministic **P**olynomial time

- Alternative definition of NP: guess a certificate (non-deterministically) and then verify it in polynomial time
- Our definition doesn't care where the certificate comes from: just concentrates on the existence of a polynomial-time certifier

P (Polynomial time) = Set of problems for which a solution can be found in polynomial time

Biggest question in Computer Science: Does P = NP?

NP Problems

NP-hard: as hard as any problem in NP

- But need not be in NP itself

NP-complete (NPC): the hardest of the NP problems

- NP-hard and in NP itself
- If any NP-complete problem can be solved in polynomial time, all problems in NP can be solved in polynomial time
- Alternatively: if some NP problem **can't** be solved in polynomial time, then no NP-complete problem an be solved in polynomial time

Relationship between P and NP

$P \subseteq NP$

Possibility 1: P = NP=NPC

- Every NP problem can be solved in polynomial time

Possibility 2: $P \neq NP$

- No NP-complete problem can be solved in polynomial time
- Believed by many to be true

8.1. Polynomial-Time Reductions

Classify Problems According to Computational Requirements

Q. Which problems will we be able to solve in practice?

A working definition. [Cobham 1960, Edmonds, 1962] Those with polynomial-time algorithms.

Yes	Probably No
Shortest path	Longest path
Euler cycle	Hamiltonian cycle
Min cut	Max cut
2-SAT	3-SAT
Planar 4-color	Planar 3-color
Bipartite vertex cover	Vertex cover
Matching	3D-matching
Primality testing	Factoring

Classify Problems

Desiderata. Classify problems according to those that can be solved in polynomial-time and those that cannot.

Provably requires exponential-time.

- Given a Turing machine, does it halt in at most k steps?
- Given a board position in an n-by-n generalization of chess, can black guarantee a win?

Unfortunately… huge number of fundamental problems have defied classification for decades.

Fortunately… they were shown to be "computationally equivalent" and intractable for all practical purposes.

Polynomial-Time Reduction

Desiderata'. Suppose we could solve X in polynomial-time. What else could we solve in polynomial time?

don't confuse with reduces from

Reduction. Problem X polynomial reduces to problem Y if arbitrary instances of problem X can be solved using:

- Polynomial number of standard computational steps, plus
- Polynomial number of calls to oracle that solves problem Y.

computational model supplemented by special piece of hardware that solves instances of Y in a single step

Remarks.

- We pay for time to write down instances sent to black box $\Rightarrow$ instances of Y must be of polynomial size.
- Note: Cook reducibility. ← as opposed to Karp reductions
- Notation: $X \leq_P Y$.

Polynomial-Time Reduction

Purpose. Classify problems according to relative difficulty.

Design algorithms. If $X \leq_P Y$ and Y can be solved in polynomial-time, then X can be solved in polynomial time.

Establish intractability. If $X \leq_P Y$ and X cannot be solved in polynomial-time, then X cannot be solved in polynomial time.

Establish equivalence. If $X \leq_P Y$ and $Y \leq_P X$, we use notation $X \equiv_P Y$.

up to cost of reduction

Showing a Problem Q is NP-Complete

Choose a known NP-complete problem, P

Reduce P to Q

- $P \leq_P Q$
  - Prove reduction is correct
  - Prove reduction works in polynomial time

Prove that Q is in NP

Polynomial-Time Reduction

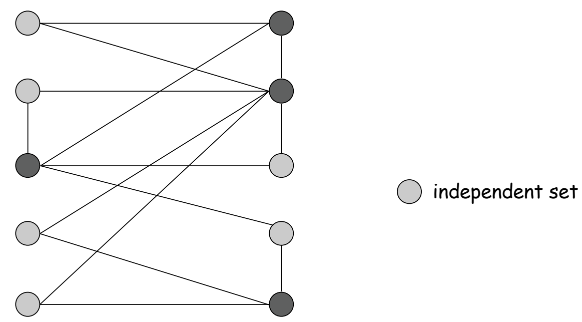
Basic strategies.

- Reduction by simple equivalence.
- Reduction from special case to general case.
- Reduction by encoding with gadgets.

Independent Set

INDEPENDENT SET: Given a graph $G = (V, E)$ and an integer k, is there a subset of vertices $S \subseteq V$ such that $|S| \geq k$, and for each edge at most one of its endpoints is in S?

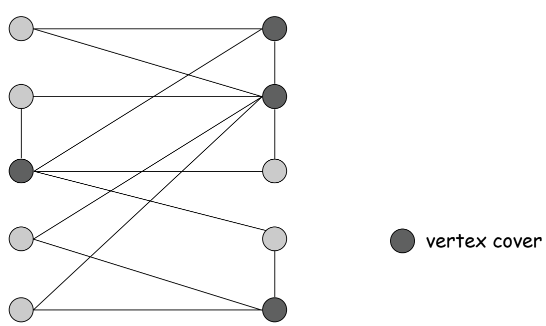
Ex. Is there an independent set of size ≥ 6? Yes.
Ex. Is there an independent set of size ≥ 7? No.



Vertex Cover

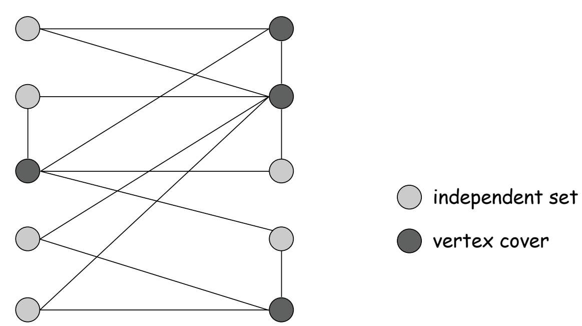
VERTEX COVER: Given a graph $G = (V, E)$ and an integer k, is there a subset of vertices $S \subseteq V$ such that $|S| \leq k$, and for each edge, at least one of its endpoints is in S?

Ex. Is there a vertex cover of size ≤ 4? Yes.
Ex. Is there a vertex cover of size ≤ 3? No.



Vertex Cover and Independent Set

Claim. VERTEX-COVER $\equiv_P$ INDEPENDENT-SET.
Pf. We show S is an independent set iff $V - S$ is a vertex cover.



Vertex Cover and Independent Set

Claim. VERTEX-COVER $\equiv_P$ INDEPENDENT-SET.

Pf. We show S is an independent set iff V – S is a vertex cover.

$\Rightarrow$

- Let S be any independent set.
- Consider an arbitrary edge (u, v).
- S independent $\Rightarrow u \notin S$ or $v \notin S \Rightarrow u \in V - S$ or $v \in V - S$.
- Thus, V – S covers (u, v).

$\Leftarrow$

- Let V – S be any vertex cover.
- Consider two nodes $u \in S$ and $v \in S$.
- Observe that $(u, v) \notin E$ since V – S is a vertex cover.
- Thus, no two nodes in S are joined by an edge $\Rightarrow$ S independent set.

Polynomial-Time Reduction

Basic strategies.

- Reduction by simple equivalence.
- Reduction from special case to general case.
- Reduction by encoding with gadgets.

Set Cover

SET COVER: Given a set U of elements, a collection $S_1, S_2, \ldots, S_m$ of subsets of U, and an integer k, does there exist a collection of $\le k$ of these sets whose union is equal to U?

Sample application.

- m available pieces of software.
- Set U of n capabilities that we would like our system to have.
- The ith piece of software provides the set $S_i \subseteq U$ of capabilities.
- Goal: achieve all n capabilities using fewest pieces of software.

Ex:

$U = \{1, 2, 3, \ldots, 12\}$, $k = 3$.

$S_1 = \{1, 2, 3, 4, 5, 6\}$	$S_2 = \{5, 6, 8, 9\}$
$S_3 = \{1, 4, 7, 10\}$	$S_4 = \{2, 5, 7, 8, 11\}$
$S_5 = \{3, 6, 9, 12\}$	$S_6 = \{10, 11\}$

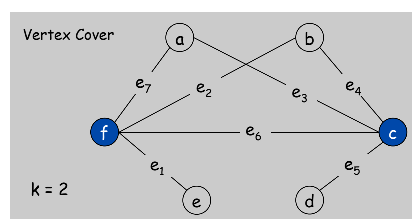
Vertex Cover Reduces to Set Cover

Claim. VERTEX-COVER $\le_P$ SET-COVER.

Pf. Given a VERTEX-COVER instance G = (V, E), k, we construct a set cover instance whose size equals the size of the vertex cover instance.

Construction.

- Create SET-COVER instance:
  - $k = k$, $U = E$, $S_v = \{e \in E : e \text{ incident to } v\}$
- Set-cover of size $\le k$ iff vertex cover of size $\le k$.



Set cover

$U = \{1, 2, 3, 4, 5, 6, 7\}$

$k = 2$

$S_a = \{3, 7\}$	$S_b = \{2, 4\}$
$S_c = \{3, 4, 5, 6\}$	$S_d = \{5\}$
$S_e = \{1\}$	$S_f = \{1, 2, 6, 7\}$

Integer Programming

INTEGER-PROGRAMMING: Given rational numbers a_{ij}, b find integers x_j that satisfy:

$$\begin{array}{rcll} \sum_{j=1}^{n} a_{ij} x_j & \geq & b_i & 1 \leq i \leq m \\ x_j & \geq & 0 & 1 \leq j \leq n \\ x_j & & \text{integral} & 1 \leq j \leq n \end{array}$$

Claim. VERTEX-COVER $\leq_P$ INTEGER-PROGRAMMING.

$$\begin{array}{rcll} \sum_{u \in V} x_u & \leq & k & \\ x_u + x_v & \geq & 1 & (u,v) \in E \\ x_u & \geq & 0 & u \in V \\ x_u & & \text{integral} & u \in V \end{array}$$

Polynomial-Time Reduction

Basic strategies.
- Reduction by simple equivalence.
- Reduction from special case to general case.
- Reduction by encoding with gadgets.

Satisfiability

Literal: A Boolean variable or its negation. x_i or $\overline{x_i}$

Clause: A disjunction of literals. $C_j = x_1 \vee \overline{x_2} \vee x_3$

Conjunctive normal form: A propositional formula Φ that is the conjunction of clauses. $\Phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

SAT: Given propositional formula in conjunctive normal form, does it have a satisfying truth assignment?

3-SAT: SAT where each clause is of length exactly 3.

Ex: $\left(\overline{x_1} \vee x_2 \vee x_3\right) \wedge \left(x_1 \vee \overline{x_2} \vee x_3\right) \wedge \left(x_2 \vee x_3\right) \wedge \left(\overline{x_1} \vee \overline{x_2} \vee \overline{x_3}\right)$

Yes: x_1 = true, x_2 = true x_3 = false.

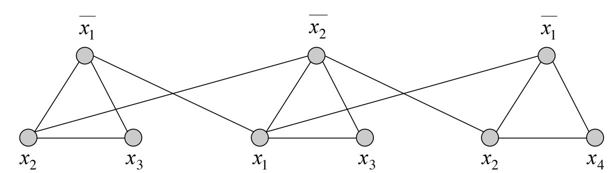
3 Satisfiability Reduces to Independent Set

Claim. 3-SAT $\leq_P$ INDEPENDENT-SET.

Pf. Given an instance Φ of 3-SAT (with k clauses), we construct an instance (G, k) of INDEPENDENT-SET such that Φ is satisfiable iff G has an independent set of size k.

Construction.
- G contains 3 vertices for each clause, one for each literal.
- Connect 3 literals in a clause in a triangle.
- Connect literal to each of its negations.

G



k = 3

$\Phi = \left(\overline{x_1} \vee x_2 \vee x_3\right) \wedge \left(x_1 \vee \overline{x_2} \vee x_3\right) \wedge \left(\overline{x_1} \vee x_2 \vee x_4\right)$

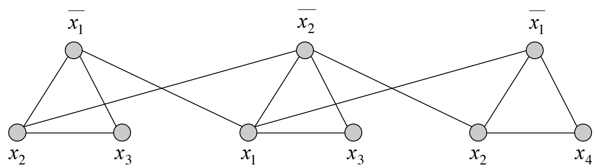
3 Satisfiability Reduces to Independent Set

Claim. G contains independent set of size $k = |\Phi|$ iff Φ is satisfiable.

Pf. $\Rightarrow$ Let S be independent set of size k.
- S must contain exactly one vertex in each triangle.
- Set these literals to true. ← and any other variables in a consistent way
- No conflicts.

Pf $\Leftarrow$ Given satisfying assignment, select one true literal from each triangle. This is an independent set of size k.

G



k = 3

$$\Phi = (\overline{x_1} \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee x_4)$$

Review

Basic reduction strategies.
- Simple equivalence: INDEPENDENT-SET $\equiv_P$ VERTEX-COVER.
- Special case to general case: VERTEX-COVER $\le_P$ SET-COVER.
- Encoding with gadgets: 3-SAT $\le_P$ INDEPENDENT-SET.

Transitivity. If $X \le_P Y$ and $Y \le_P Z$, then $X \le_P Z$.

Ex: 3-SAT $\le_P$ INDEPENDENT-SET $\le_P$ VERTEX-COVER $\le_P$ SET-COVER.

Self-Reducibility

Decision problem. Does G have a vertex cover of size $\le$ k?
Search problem. Find vertex cover of minimum cardinality.

Self-reducibility. Search problem $\le_P$ decision version.
- Applies to all problems in this chapter.
- Justifies our emphasis on decision problems.

Ex: to find min cardinality vertex cover.
- (Binary) search for cardinality k* of min vertex cover.
- Find a vertex v such that G – { v } has a vertex cover of size $\le$ k* - 1.
  - any vertex in any min vertex cover will have this property
- Include v in the vertex cover.
- Recursively find a min vertex cover in G – { v }.

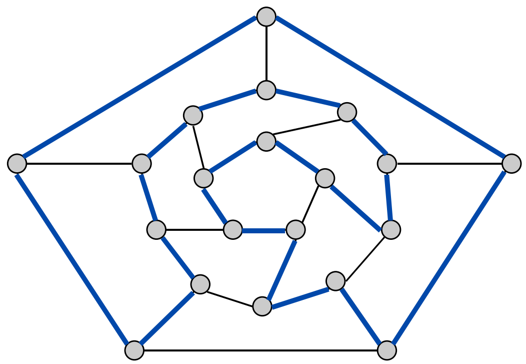
↑ delete v and all incident edges

8.4 Sequencing Problems

Basic genres.
- Packing problems: SET-PACKING, INDEPENDENT SET.
- Covering problems: SET-COVER, VERTEX-COVER.
- Constraint satisfaction problems: SAT, 3-SAT.
- Sequencing problems: HAMILTONIAN-CYCLE, TSP.
- Partitioning problems: 3-COLOR, 3D-MATCHING.
- Numerical problems: SUBSET-SUM, KNAPSACK.

Hamiltonian Cycle

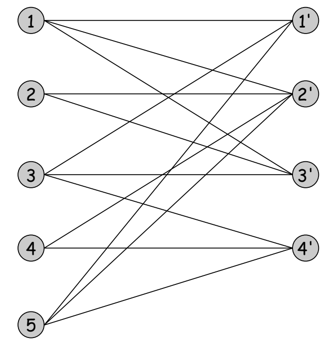
HAM-CYCLE: given an undirected graph $G = (V, E)$, does there exist a simple cycle C that contains every vertex in V.



YES: vertices and edges of a dodecahedron.

Hamiltonian Cycle

HAM-CYCLE: given an undirected graph $G = (V, E)$, does there exist a simple cycle C that contains every vertex in V.



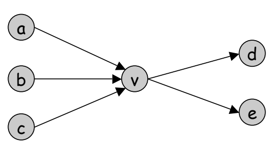
NO: bipartite graph with odd number of nodes.

Directed Hamiltonian Cycle

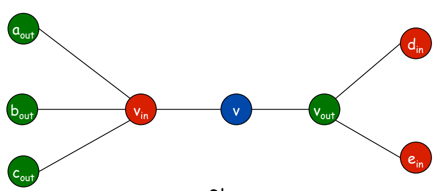
DIR-HAM-CYCLE: given a digraph $G = (V, E)$, does there exists a simple directed cycle C that contains every vertex in V?

Claim. DIR-HAM-CYCLE $\leq_P$ HAM-CYCLE.

Proof. Given a directed graph $G = (V, E)$, construct an undirected graph G' with $3n$ vertices.



G



G'

Directed Hamiltonian Cycle

Claim. G has a Hamiltonian cycle if and only if G' does.

Proof. $\Rightarrow$

- Suppose G has a directed Hamiltonian cycle C.
- Then G' has an undirected Hamiltonian cycle (same order).

Proof. $\Leftarrow$

- Suppose G' has an undirected Hamiltonian cycle C'.
- C' must visit nodes in G' using one of following two orders:
  - . . . , G, R, B, G, R, B, G, R, B, . . .
  - . . . , R, G, B, R, G, B, R, G, B, . . .
- Blue nodes in C' make up directed Hamiltonian cycle C in G, or reverse of one.

3-SAT Reduces to Directed Hamiltonian Cycle

Claim. 3-SAT $\le_P$ DIR-HAM-CYCLE.

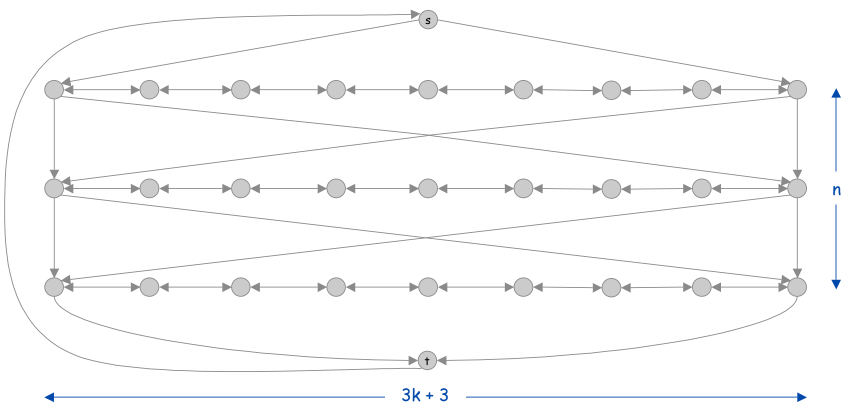
Proof. Given an instance Φ of 3-SAT, we construct an instance of DIR-HAM-CYCLE that has a Hamiltonian cycle iff Φ is satisfiable.

Construction. First, create graph that has 2^n Hamiltonian cycles which correspond in a natural way to 2^n possible truth assignments.

3-SAT Reduces to Directed Hamiltonian Cycle

Construction. Given 3-SAT instance Φ with n variables x_i and k clauses.

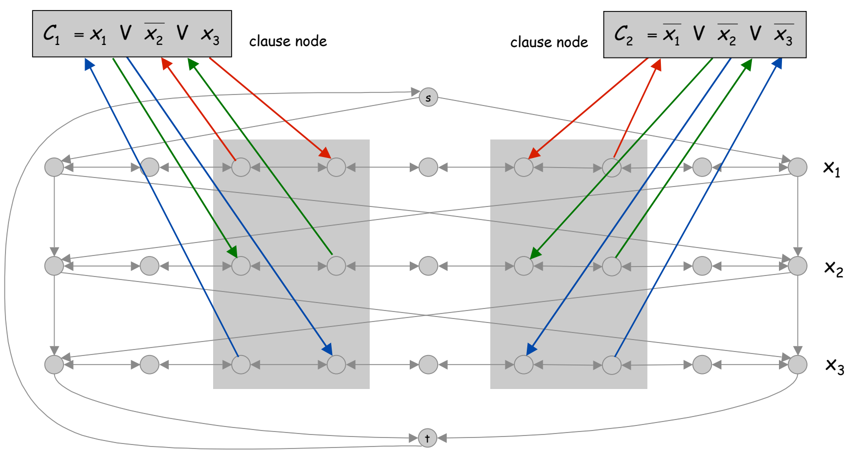
- Construct G to have 2^n Hamiltonian cycles.
- Intuition: traverse path i from left to right $\Leftrightarrow$ set variable $x_i = 1$.



3-SAT Reduces to Directed Hamiltonian Cycle

Construction. Given 3-SAT instance Φ with n variables x_i and k clauses.

- For each clause: add a node and 6 edges.



3-SAT Reduces to Directed Hamiltonian Cycle

Claim. Φ is satisfiable iff G has a Hamiltonian cycle.

Pf. $\Rightarrow$

- Suppose 3-SAT instance has satisfying assignment x*.
- Then, define Hamiltonian cycle in G as follows:
  - if $x^*_i = 1$, traverse row i from left to right
  - if $x^*_i = 0$, traverse row i from right to left
  - for each clause C_j, there will be at least one row i in which we are going in "correct" direction to splice node C_j into tour

3-SAT Reduces to Directed Hamiltonian Cycle

Claim. Φ is satisfiable iff G has a Hamiltonian cycle.

Pf. $\Leftarrow$

- Suppose G has a Hamiltonian cycle Γ.
- If Γ enters clause node C_j, it must depart on mate edge.
  - thus, nodes immediately before and after C_j are connected by an edge e in G
  - removing C_j from cycle, and replacing it with edge e yields Hamiltonian cycle on $G - \{C_j\}$
- Continuing in this way, we are left with Hamiltonian cycle Γ' in $G - \{C_1, C_2, \ldots, C_k\}$.
- Set $x^*_i = 1$ iff Γ' traverses row i left to right.
- Since Γ visits each clause node C_j, at least one of the paths is traversed in "correct" direction, and each clause is satisfied.

8.5 3-Dimensional Matching

Basic genres.

- Packing problems: SET-PACKING, INDEPENDENT SET.
- Covering problems: SET-COVER, VERTEX-COVER.
- Constraint satisfaction problems: SAT, 3-SAT.
- Sequencing problems: HAMILTONIAN-CYCLE, TSP.
- Partitioning problems: 3-COLOR, 3D-MATCHING.
- Numerical problems: SUBSET-SUM, KNAPSACK.

3-Dimensional Matching

3D-MATCHING. Given n instructors, n courses, and n times, and a list of the possible courses and times each instructor is willing to teach, is it possible to make an assignment so that all courses are taught at different times?

Instructor	Course	Time
Wayne	COS 423	MW 11-12:20
Wayne	COS 423	TTh 11-12:20
Wayne	COS 226	TTh 11-12:20
Wayne	COS 126	TTh 11-12:20
Tarjan	COS 523	TTh 3-4:20
Tarjan	COS 423	TTh 11-12:20
Tarjan	COS 423	TTh 3-4:20
Sedgewick	COS 226	TTh 3-4:20
Sedgewick	COS 226	MW 11-12:20
Sedgewick	COS 423	MW 11-12:20

3-Dimensional Matching

3D-MATCHING. Given disjoint sets X, Y, and Z, each of size n and a set $T \subseteq X \times Y \times Z$ of triples, does there exist a set of n triples in T such that each element of $X \cup Y \cup Z$ is in exactly one of these triples?

Claim. 3-SAT $\leq_P$ 3D-MATCHING.

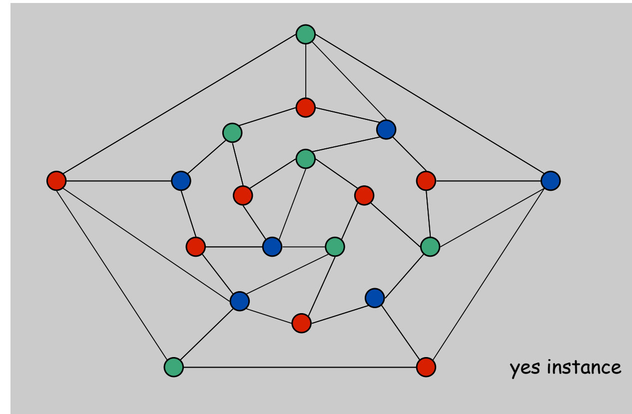
8.6 Graph Coloring

Basic genres.

- Packing problems: SET-PACKING, INDEPENDENT SET.
- Covering problems: SET-COVER, VERTEX-COVER.
- Constraint satisfaction problems: SAT, 3-SAT.
- Sequencing problems: HAMILTONIAN-CYCLE, TSP.
- Partitioning problems: 3-COLOR, 3D-MATCHING.
- Numerical problems: SUBSET-SUM, KNAPSACK.

3-Colorability

3-COLOR: Given an undirected graph does there exists a way to color the nodes R, G, and B such no adjacent nodes have the same color?



Register Allocation

Register allocation. Assign program variables to machine register so that no more than k registers are used and no two program variables that are needed at the same time are assigned to the same register.

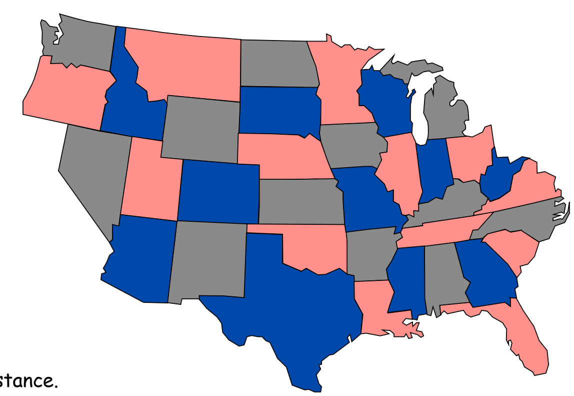
Interference graph. Nodes are program variables names, edge between u and v if there exists an operation where both u and v are "live" at the same time.

Observation. [Chaitin, 1982] Can solve register allocation problem iff interference graph is k-colorable.

Fact. 3-COLOR $\le_P$ k-REGISTER-ALLOCATION for any constant $k \ge 3$.

Planar 3-Colorability

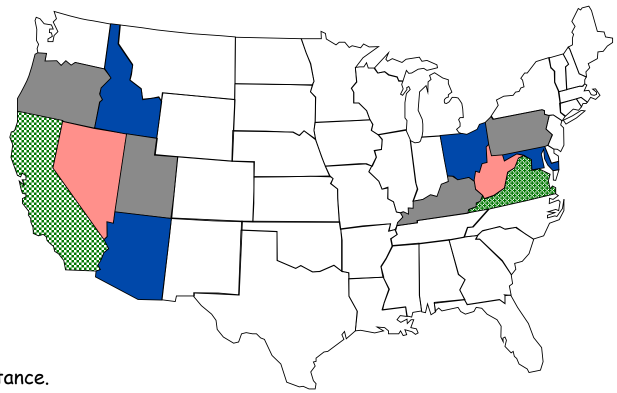
PLANAR-3-COLOR. Given a planar map, can it be colored using 3 colors so that no adjacent regions have the same color?



YES instance.

Planar 3-Colorability

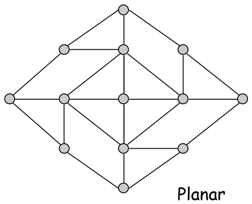
PLANAR-3-COLOR. Given a planar map, can it be colored using 3 colors so that no adjacent regions have the same color?



NO instance.

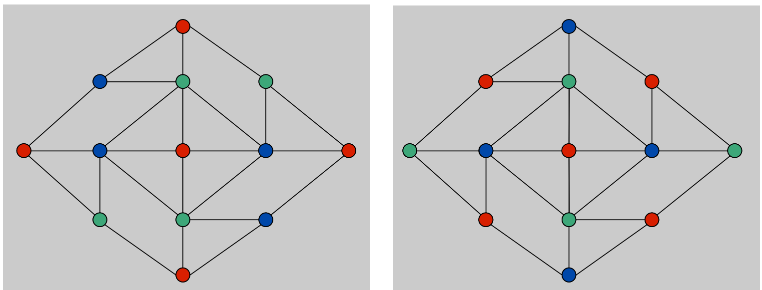
Planarity

Def. A graph is planar if it can be embedded on the plane (or sphere) in such a way that no two edges cross.
Applications: VLSI circuit design, computer graphics.



Planar 3-Colorability

Claim. 3-COLOR $\le_P$ PLANAR-3-COLOR.
Proof sketch: Given instance of 3-COLOR, draw graph in plane, letting edges cross if necessary.
- Replace each edge crossing with the following planar gadget W.
  - in any 3-coloring of W, opposite corners have the same color
  - any assignment of colors to the corners in which opposite corners have the same color extends to a 3-coloring of W

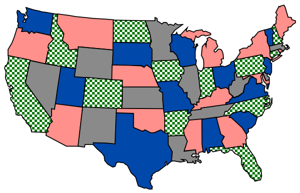


Planar k-Colorability

PLANAR-2-COLOR. Solvable in linear time.

PLANAR-3-COLOR. NP-complete.

PLANAR-4-COLOR. Solvable in O(1) time.



Theorem. [Appel-Haken, 1976] Every planar map is 4-colorable.
- Resolved century-old open problem.
- Used 50 days of computer time to deal with many special cases.
- First major theorem to be proved using computer.

False intuition. If PLANAR-3-COLOR is hard, then so is PLANAR-4-COLOR and PLANAR-5-COLOR.

8.7 Numerical Problems

Basic genres.

- Packing problems: SET-PACKING, INDEPENDENT SET.
- Covering problems: SET-COVER, VERTEX-COVER.
- Constraint satisfaction problems: SAT, 3-SAT.
- Sequencing problems: HAMILTONIAN-CYCLE, TSP.
- Partitioning problems: 3-COLOR, 3D-MATCHING.
- Numerical problems: SUBSET-SUM, KNAPSACK.

Subset Sum

SUBSET-SUM. Given natural numbers $w_1, \ldots, w_n$ and an integer W, is there a subset that adds up to exactly W?

Ex: { 1, 4, 16, 64, 256, 1040, 1041, 1093, 1284, 1344 }, W = 3754.
Yes. 1 + 16 + 64 + 256 + 1040 + 1093 + 1284 = 3754.

Remark. With arithmetic problems, input integers are encoded in binary. Polynomial reduction must be polynomial in binary encoding.

Claim. 3-SAT $\le_P$ SUBSET-SUM.
Pf. Given an instance Φ of 3-SAT, we construct an instance of SUBSET-SUM that has solution iff Φ is satisfiable.

Subset Sum

Construction. Given 3-SAT instance Φ with k clauses and n variables, form the following 2n + 2k integers (in base 10).

Claim. 3D-matching iff some subset sums to W.
Pf. No carries possible.

$$C_1 = \overline{x_1} \vee x_2 \vee x_3$$
$$C_2 = x_1 \vee \overline{x_2} \vee x_3$$
$$C_3 = \overline{x_1} \vee \overline{x_2} \vee \overline{x_3}$$

	x	y	z	C_1	C_2	C_3	
x1	1	0	0	0	1	0	100,010
¬ x1	1	0	0	1	0	1	100,101
x2	0	1	0	1	0	0	10,100
¬ x2	0	1	0	0	1	1	10,011
x3	0	0	1	1	1	0	1,110
¬ x3	0	0	1	0	0	1	1,001
	0	0	0	1	0	0	100
	0	0	0	2	0	0	100
	0	0	0	0	1	0	10
	0	0	0	0	2	0	10
	0	0	0	0	0	1	1
	0	0	0	0	0	2	1
W	1	1	1	3	3	3	111,333

$\underbrace{\text{last six rows}}$: 2k dummies to get clause columns to sum to 3

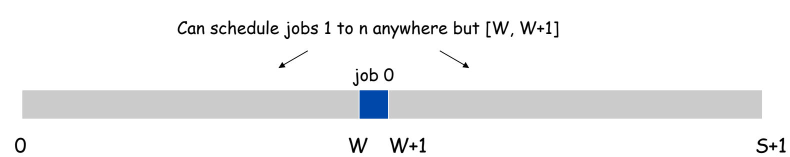
Scheduling With Release Times

SCHEDULE-RELEASE-TIMES. Given a set of n jobs with processing time t_i, release time r_i, and deadline d_i, is it possible to schedule all jobs on a single machine such that job i is processed with a contiguous slot of t_i time units in the interval $[r_i, d_i]$?

Claim. SUBSET-SUM $\le_P$ SCHEDULE-RELEASE-TIMES.
Pf. Given an instance of SUBSET-SUM $w_1, \ldots, w_n$, and target W,

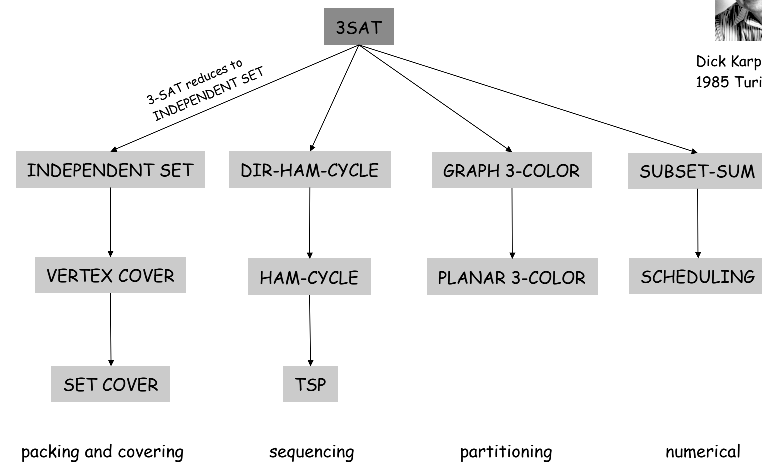
Let $S = 1 + \sum_j w_j$

- Create n jobs with processing time $t_i = w_i$, release time $r_i = 0$, and deadline $d_i = S+1$
- Create job 0 with $t_0 = 1$, release time $r_0 = W$, and deadline $d_0 = W+1$.



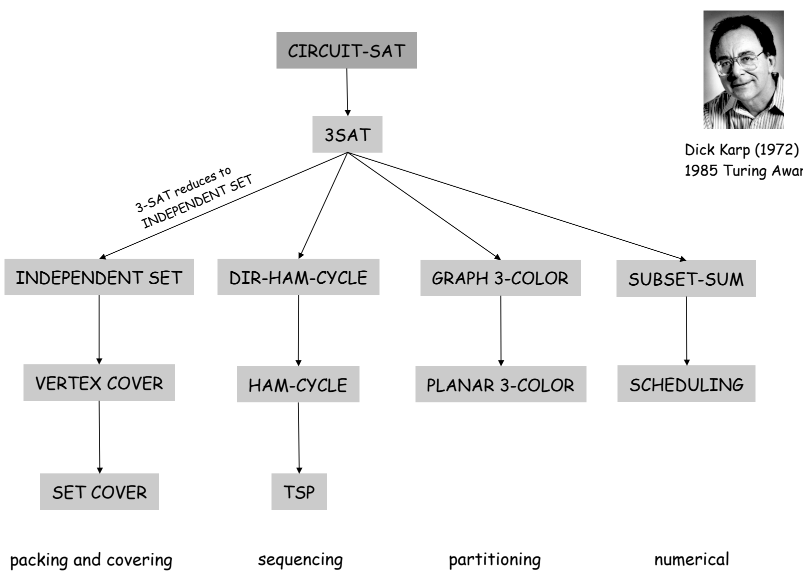
8.9 A Partial Taxonomy of Hard Problems

Polynomial-Time Reductions



Dick Karp (1972)
1985 Turing Award

Polynomial-Time Reductions



Dick Karp (1972)
1985 Turing Award

Undecidable Problems

Given a computer program, does it halt?

- Program1
  i = 10
  while i > 0
      total = total + i

- Program2
  i = 10
  while i > 0
      total = total + i
      i = i - 1

Halting Problem

Does it halt?

- ```
  i = j = k = n = 3
  loop
     exit when i^n + j^n = k^n
    increment minimum of i, j, k, n
  end loop
  ```
- I have a marvelous proof that this program never halts, but this slide is too small to contain it!

Halting Problem

Assume we have an algorithm H that can decide whether a given program A halts on input I

- H(A, I)
  - Returns true if the program A halts on input I
  - Returns false if the program A doesn't halt on input I
- Construct H':
  - H'(A)

    ```
    if H(A, A) then          A halts when fed A as input
      loop forever
    else
       return
    ```
  - What about H'(H')? If H' halts on itself then it doesn't halt on itself. If H' doesn't halt on itself, then it does halt on itself

By contradiction, no such algorithm H exists!

Can't decide much useful about a program

Summary

If you are given a problem:

- If it is undecidable, don't bother trying to solve it
- If it is NP-complete, don't bother trying to come up with a polynomial algorithm
  - Try an approximation algorithm
  - Try a simpler version of the problem
    ```