CSE 202 Homework 1
Divide and conquer
Spring, 2004
Due Tuesday, April 27

For each of the algorithm problems, design as asymptotically efficient an algorithm as possible. Give a correctness argument (explanation, if it is relatively simple, or proof if not) and time analysis. You may use any well-known algorithm or data structure, or algorithm from the text or from class, as a subroutine without needing to provide details.

Base Conversion . Give an algorithm that inputs an array of n base base 10 digits representing a positive integer in base and outputs an array of bits representing the same integer in base 2. Your algorithm should be $o(n^2)$. You will probably need to use a divide-and-conquer strategy, and use a fast Multiplication sub-routine (from class).

Triangles Use the Strassen matrix multiply algorithm to give an $o(n^3)$ algorithm for the problem of deciding whether a graph has a triangle, a set of three nodes every two of which are connected. Assume the graph is given in adjacency matrix format.

Binary Tree Isomporphism Two rooted trees T_1 and T_2 are *isomorphic* if there is a 1-1 onto map $f : T_1 \rightarrow T_2$ so that $f(root_1) = root_2$ and $p_2(f(x)) = f(p_1(x))$, for every $x \in T_1$ except $root_1$. (Here, $root_1$ is the root of T_1, $root_2$ is the root of T_2, and p_1, p_2, represent the parents in the respective trees.) Give an efficient algorithm to determine whether two n node rooted binary trees are isomorphic. (Note that the f is what your algorithm is looking for, not an input.)

Weighted Median, Problem 9-2, part c., p. 194

Implementation: Integer Multiplication Implement the $O(n^{\log 3})$ divide-and-conquer algorithm for integer multiplication from class, but with a threshold, below which naive "gradeschool" multiplication is used. Experimentally determine the optimal threshold. For what values of n do you see an improvement in the time using divide-and-conquer, both using no threshold and using the optimal threshold?