CSE252 – Computer Vision – Assignment #1
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http://www-cse.ucsd.edu/classes/sp03/cse252
Target Due Date: Tue. Apr. 15, 2003.

1. What is the silhouette of a sphere of radius r with its center at coordinates $[X_o, 0, Z_o]^T$ under perspective projection on a planar surface? The perpendicular distance from the pinhole to the image plane is f.

2. F&P Problem 1.2.

3. F&P Problem 1.3. Hint: parameterize the line in 3D in the direction of the vector $\mathbf{D} = (D_x, D_y, D_z)^\top$ passing through the point $\mathbf{X}_o = (X_o, Y_o, Z_o)^\top$ as $\mathbf{X} = \lambda \mathbf{D} + \mathbf{X}_o$ for $\lambda \in \mathbb{R}$.

4. Consider the square viewed under perspective projection shown in Figure 1.

   (a) Solve for the vanishing point for each pair of parallel lines.

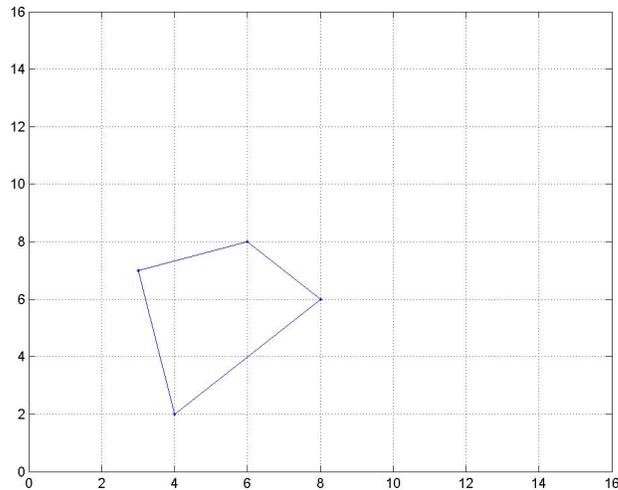   (b) Write down an expression for the line at the horizon.

Figure 1: A square viewed under perspective projection.

5. Write a Matlab function for computing the orthogonal matrix R corresponding to a rotation of ϕ about the unit-norm axis vector $\mathbf{n}$. Find the eigenvalues and eigenvectors of R and study their relationship to the axis vector. Verify the formula $\cos\phi = \frac{1}{2}\{\text{tr}(R) - 1\}$.

   Matlab hints: `trace`, `eig`, `angle`.

6. For small ϕ, we can approximate R by $I + \phi\hat{n}$. Investigate the range of ϕ over which this is a reasonable approximation.

7. Write a Matlab function for the converse of that in Problem 5, i.e. given R, compute the skew-symmetric matrix $\hat{n}$ and the associated $\mathbf{n}$ and ϕ. Do the skew-symmetric matrices have any non-zero real eigenvalues? Why not?

8. Write a function to display a planar grid of points under perspective projection. The inputs to your program should be the dimensions of the grid, its depth (the Z coordinates of the points), the focal length (f), and the camera pose (R and $\mathbf{T}$). Demonstrate your program for an 11×11 grid of points for three different non-trivial viewpoints. For example, pretend the grid is painted on the wall and your job is to display what it looks like to several observers of varying heights standing more or less in front of it.

   Matlab hints: `meshgrid`, `axis('equal')`.

9. Extend your function from Problem 8 to include texture mapping for an arbitrary image. Show the result applied to an image of your choice (e.g. `logo.tif`) from three different interesting viewpoints.

   Matlab hints: `imread`, `griddata`, `reshape`.