CSE252 – Computer Vision – Assignment #2
Instructor: Prof. Serge Belongie.
`http://www-cse.ucsd.edu/~sjb/classes/sp02/cse252`
Target Due Date: Tue. Apr. 30, 2002.

1. T&V, Exercise 7.6.

2. For this problem, download the script `make_scene.m` from the course web page and run it to produce the left and right camera images of a simple geometric figure.

   (a) Solve for the essential matrix E using Algorithm `EIGHT_POINT` from T&V with Hartley normalization.
   Matlab hints: `svd`, `reshape`.

   (b) Write a program that draws the epipolar line on the right image corresponding to an arbitrarily selected (clicked) coordinate in the left image. Demonstrate that corresponding points in the right image lie on epipolar lines for at least three points (e.g. 4, 5, and 6) in the left image.
   Matlab hints: `ginput`.

   (c) Recover R and $\mathbf{T}$ from E up to a scale factor using Algorithm `EUCLID_REC` from T&V. Use R and $\mathbf{T}$ to recover the depth of the 3D points relative to each camera and plot the two sets of reconstructed coordinates.
   Matlab hints: `trace`, `cross`, `plot3`.

3. This problem makes use of the script `get_coords.m` and the Star Wars stereo pair `x_sw21.jpg`, available on the course web page. (You are free to use a different stereo pair if you prefer.)

   (a) Use `imread` to load in the image and separate it into left and right images.

   (b) Click on at least 15 corresponding points using `get_coords.m`.

   (c) Repeat problems 2a and 2b from above.

4. Implement Algorithm `CORNERS` from T&V. Modify it to solve for the least-squares subpixel corner coordinates as described in class. Demonstrate your program on two grayscale images of your choice.

   Matlab hints: `gradient`, `rgb2gray/ind2gray`.

5. This problem deals with the *homography* between two images of coplanar points. Assume the two cameras are related by the rigid transformation $(R, \mathbf{T})$ from the left camera to the right camera, i.e. $\mathbf{P}_r = R\mathbf{P}_l + \mathbf{T}$.

   (a) Show that the relationship between $\mathbf{P}_r$ and $\mathbf{P}_l$ can be expressed as

   $$\mathbf{P}_r = H\mathbf{P}_l \quad \text{where} \quad H = R + \frac{1}{d_l}\mathbf{T}\mathbf{n}^T$$

   assuming the left camera is a distance of d_l from the plane and that the unit surface normal of the plane (relative to the left camera) is $\mathbf{n}$. The 3×3 matrix H is called the *homography matrix.*

   (b) Discuss what causes the Eight-Point algorithm to fail for coplanar points by considering $\mathbf{p}_r^T E \mathbf{p}_l = 0$ for a set of n correspondences.

   (c) Show that corresponding points satisfy the constraint $(\mathbf{p}_r)^\wedge H\mathbf{p}_l = \mathbf{0}$.

   (d) Explain how to recover the entries of H up to a scale factor using $n \geq 4$ corresponding points by solving an overdetermined linear system. Hint: the system has the form $A\mathbf{h} = \mathbf{0}$, where A is $2n \times 9$.