Redundancy Elimination

Aim: Eliminate redundant operations in dynamic execution

Why occur?

Loop-invariant code: Ex: constant assignment in loop

Same expression computed Ex: addressing

Value numbering is an example

Requires dataflow analysis

Other optimizations:

Constant subexpression elimination

Loop-invariant code motion

Partial redundancy elimination

Common Subexpression Elimination

Replace recomputation of expression by use of temp which holds value

Ex. (s1) y := a + b

(s2) z := a + b

Ex. (s1) temp := a + b
(s1') y := temp

(s2) z := temp

Illegal?

How different from value numbering?

Ex. (s1) read(i)
(s2) j := i + 1
(s3) k := i
(s4) l := k + 1

i + 1, k+1

no cse, same value number

Why need temp?

Local and Global

Local CSE (BB)

Ex. (s1) c := a + b
(s2) d := m&n
(s3) e := a + b
(s4) m := 5
(s5) if(m&n) ...

→

(s1) t1 := a + b
(s1') c := t1
(s2) d := m&n
(s3) e := t1
(s4) m := 5
(s5) if(m&n) ...

5 instr, 4 ops, 7 vars

6 instr, 3 ops, 8 vars

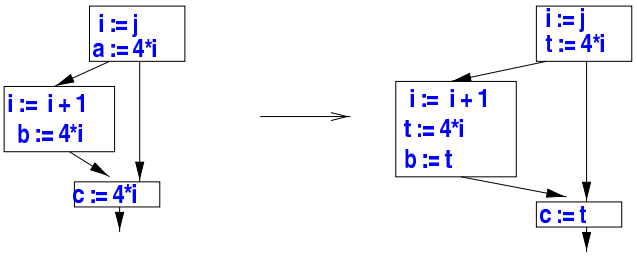
Always better?

Method: keep track of expressions computed in block whose operands have not changed value

CSE Hash Table

(+, a, b)
(&,m, n)

Global CSE example



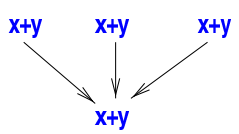
Assumes b is used later

Global CSE

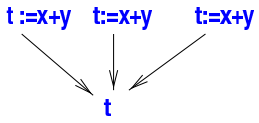
An expression e is ***available*** at entry to B if on every path p from Entry to B, there is an evaluation of e at B' on p whose values are not redefined between B' and B.

Solve by:

1. ***Find Available Expressions (Data flow problem)***
2. ***For each available expression e***



Do backward search from e in CFG to find the evaluations of e



Create new temp t to hold previous evaluations, and replace e by t

Solving Available Expressions

Gen(B) set of expressions evaluated in B available on exit from B

Kill(B) set of expressions killed by B

```
c := a + b
d := w & z
a := 5
e := e - 7
```

Gen(B) = {w&z}

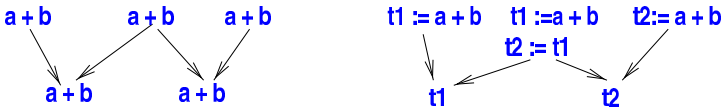
Kill(B) = {all expr. involving c,d,a,e}

Forward or Backward?

$$AEin(B) = \bigcap_{p \text{ pred of } B} AEout(p)$$

$$AEout(B) = Gen(B) \cup (AEin(B) - Kill(B))$$

Applying CSE



Create new temps for each occurrence of expression--necessary?

Is CSE always desirable?

When should CSE be applied?

Forward Substitution: Inverse of CSE

Replace copy by reevaluation of expression

Legal?

Loop-invariant Code Motion

Move computations that do not vary in loop outside of loop (pre-header)

```
Ex.  do i := 1, 100
         I := i + (c * 5)
         do j := 1, 100
             a(i.j) := 100*c  + 10*i +j
         enddo
     enddo
```

```
t1 :=  (100*c)
t2 :=  (c * 5)
do i := 1, 100
    I := i + t2
    t3 := t1 + 10 * i
    do j := 1, 100
        a(i.j) := t3 +j
    enddo
enddo
```

Almost always a good idea

Especially useful on addressing code

Need def-use information (dataflow analysis)

Finding Code that is Loop-invariant

Find loops

From inner to outer loop order, repeat till no change:

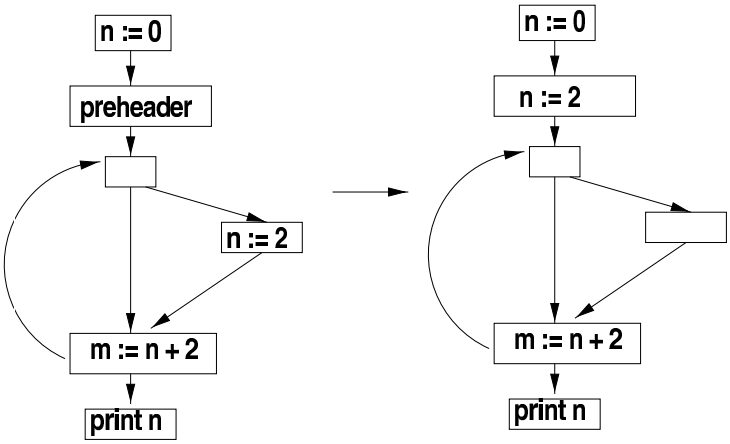
Mark all constant operands as loop-invar.

Mark all operands with all its defs from outside loop-inv.

Mark all expressions whose operands are loop-inv. as loop-inv.

Mark all instructions whose only defs are loop -inv. as loop-inv.

Problems



Safe to move loop invariant code?

Assignments: NO

If move executed ass't to preheader:

May change data flow

May raise exceptions that would not o.w. be raised

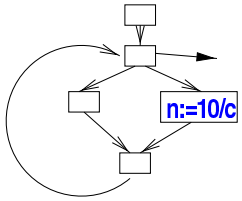
Conditional tests: YES, if

No side effects (ie no assignments)

Changes in C.D. are allowed

May be redundantly executed....

Code motion of assignments



1. ***Don't move asst unless***
   asst. dominates all uses, and
   asst dominates all exits of loop
2. ***Move with conditional*** (if possible)

Problems with Code Motion

Procedure Calls

```
y:= rand()
```

Value may be different each call, even if arguments the same

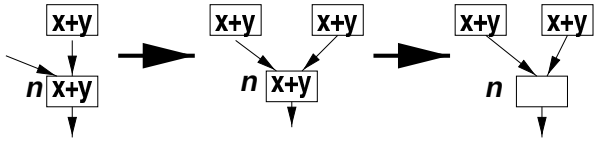
Fix:

1. Don't move, or

2. Interprocedural analysis

Partial Redundancy Elimination (PRE)

An expression is **partially redundant** at node n if it is evaluated on some path from the entry to n, and there are no redefinitions of any of its operands on the path after the last evaluation.



Elimination

Discovers partial redundancies
Converts to full redundancies
Removes redundancy

Code Hoisting

Application of Very Busy Expressions

Def. ***An expression e is*** very busy ***at B if on all paths from B to exit, e is evaluated before any of its operands are defined.***

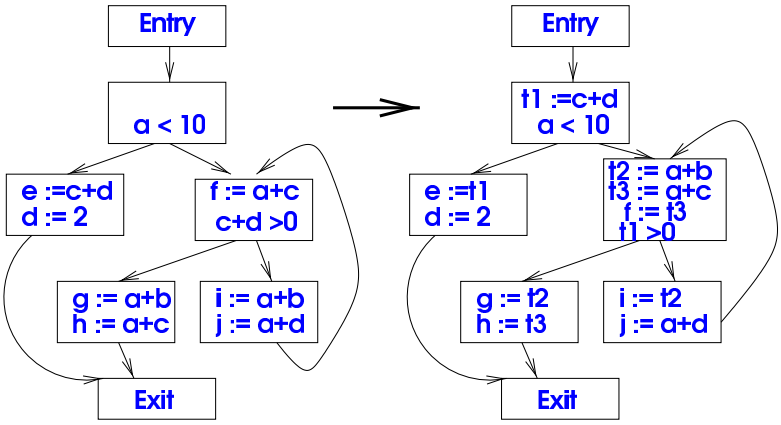
B is potential point to hoist code, may not be legal!

Want earliest such point

Beneficial?

May not always improve execution time

Example



Induction Variables & Strength Reduction

Induction variable: ***variable whose successive values form an arithmetic progression in loop***

```
Ex.  do i = 1,99                    t := -1
       a(i) :=2*i-1          →      do i = 1,99
     enddo                           t:=t+2
                                     a(i) := t
                                   enddo
```

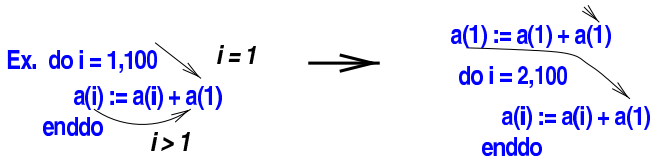
i is inductive
2*i-1 is inductive

Strength Reduction

Replaces expensive ops (like *) with less expensive ops (like +)

Loop Peeling

Remove 1 or more iterations from beginning/end of loop



Beneficial?

Always possible?

Index Set Splitting

Split index set 1,...,N into multiple sets

```
Ex.  do i = 1,100
        a(i) :=b(i) +c(i)
        if (i>50) then
            d(i) := a(i) + a(i-10)
     enddo
```

→

```
do i = 1,50
    a(i) :=b(i) +c(i)
enddo
do i = ,51,100
    a(i) :=b(i) +c(i)
    d(i) := a(i) + a(i-10)
enddo
```

Beneficial?

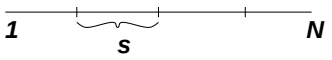
Always possible?

Strip Mining

Turn 1 loop into 2:

Outer: by strip size s

Inner: inside each strip




```
Ex.  do i = 1,100
        a(i+20) :=a(i) +b
     enddo
```

→

```
do is = 1,100,20
   do i = is, is+19
      a(i+20) :=a(i) +b
   enddo
   enddo
```

Always possible?

Beneficial?

Loop Unrolling

Copy body of loop k times, substituting successive indices and adjusting outer loop accordingly

```
Ex.  do i = 1,99
       a(i) := b(i) +c
     enddo
```



```
.  do i = 1,99,3
     a(i) := b(i) +c
     a(i+1) := b(i+1) +c
     a(i+2) := b(i+2) +c
   enddo
```

Always possible?

Beneficial?

Loop Reversal

Run the loop backwards

```
Ex.  for i=1,n
       a[i] := b[i] + 1
       c[i]  := a[i] -2
     endfor
     for i=1,n
       d[i] := 1/c[i+1]
     endfor
```

Can't fuse two loops

```
Ex.  for i= n,1
       a[i] := b[i] + 1
       c[i]  := a[i] -2
     endfor
     for i= n,1
       d[i] := 1/c[i+1]
     endfor
```

Can fuse two loops

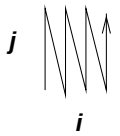
Illegal if loop-carried dependences

Reverses dependence direction

Loop Interchange

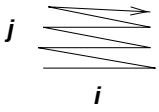
Interchange

Ex. for i=1,n
for j=1,n



→

for j=1,n
for i=1,n



Not always legal

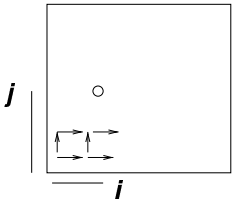
Can get any permutation of loops by composing interchanges

Can be used to implement tiling

Loop Skewing

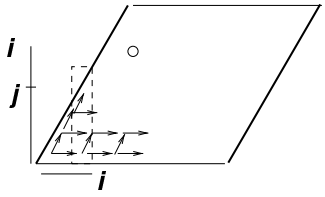
iteration (i,j) → *iteration (i, j+i)*

for i=1,n
for j=1,n
a[i,j] := a[i-1,j] + a[i, j-1]



→

for i=1,n
for j= i+1, i+n
a[i,j-i] := a[i-1,j-i] + a[i, j-i-1]



Unimodular Tranformations

Any number of interchanges, skews, reversals

Can be represented by square integer matrices with det. ± 1

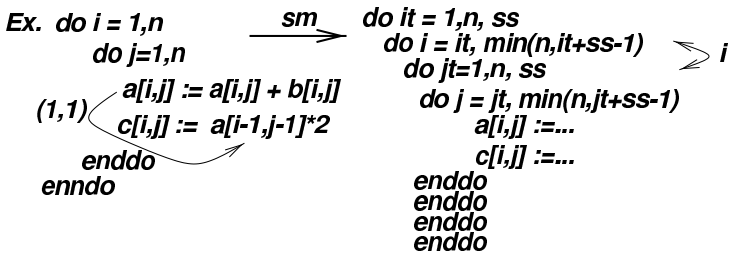
Interchange	*i,j*	*j,i*	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Reversal	*i,j*	*i,-j*	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Skew	*i,j*	*i, i+j*	$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

Easy test for legality, using matrix representation

Tiling

Partitioning of ISG into regular size and shape pieces, + schedule for executing each piece

Can be done by strip mine + interchange



Do ss by ss block at a time