CSE105 Closed Book Quiz, Thursday, Dec. 2, 1999

1. (3 pts) Is f a one-to-one correspondence? Why or why not?

   $f(n) = 3 \times n$ for n in N, $n \geq 1$

   Yes, f is a one-to-one correspondence, because for each $a, b \geq 1$ where $a \neq b$, $f(a) \neq f(b)$.

2. (4 pts) Show that if $A \leq_m B$ and B is decidable, then A is decidable.

   If B is decidable, then some Turing machine M_B exists which decides it.

   By the definition of mapping reducibility, we know that some computable function f exists such that $w \in A \Leftrightarrow f(w) \in B$.

   To show that A is decidable, we will design some Turing machine M_A which decides A. Below is its description. Note that we use M_B as a subroutine, and we also make use of the function f.

   $M_A(x)$

   - **Compute $y = f(x)$.**
   - **Run machine M_B on input y. If $M_B(y)$ accepts, accept. If $M_B(y)$ rejects, reject.**

   We know that f is a computable function, so it is possible to accomplish our first step on a Turing machine. We know that M_B always halts and gives an answer (because M_B is a decider for language B). So our machine M_A will clearly halt on every input.

   The only remaining issue is whether the answer that M_B outputs given input y is the same output that our machine M_A should give on input x. Because of the fact that our function f must exist and $w \in A \Leftrightarrow f(w) \in B$, we know that $M_A(x)$ should always give exactly the same answer as $M_B(y)$, since $y = f(x)$.

3. (3 pts) State the differences between a TM which is a decider and a TM which is a recognizer.

   A TM which is a decider ALWAYS halts and make a decision to accept or reject the input. A TM which is a recognizer is only guaranteed to halt if the input is accepted. If the input is not in the language, the recognizer may either halt and reject, or it may loop forever.