CSE105 Closed Book Quiz, Thursday, Dec. 2, 1999

1. (3 pts) Is f a one-to-one correspondence? Why or why not?

   $f(n) = n \bmod 3$ for n in N, $n \geq 1$

   No, because it is not the case that for every pair $a, b \geq 1$ where $a \neq b$, $f(a) \neq f(b)$. For example, if $a = 4$ and $b = 7$, then $f(4) = f(7) = 1$.

2. (4 pts) Show that if $A \leq_m B$ and B is Turing-recognizable, then A is Turing-recognizable.

   If B is Turing-recognizable, then some Turing machine M_B exists which recognizes it.

   By the definition of mapping reducibility, we know that some computable function f exists such that $w \in A \Leftrightarrow f(w) \in B$.

   To show that A is Turing-recognizable, we will design some Turing machine M_A which recognizes A. Below is its description. Note that we use M_B as a subroutine, and we also make use of the function f.

   $M_A(x)$

   - **Compute $y = f(x)$.**
   - **Run machine M_B on input y. If $M_B(y)$ halts and accepts, accept. If $M_B(y)$ halts and rejects or loops, loop.**

   We know that f is a computable function, so it is possible to accomplish our first step on a Turing machine. We know that M_B always halts if the input is in B (because M_B is a recognizer for language B). So our machine M_A will definitely halt on every input x where $y = f(x)$ and $y \in B$.

   Note that M_B could loop for any input $y \notin B$, and therefore M_A will also loop sometimes. But this might be okay, since we're only building a recognizer, not a decider. The next step is to check whether M_A halts on and accepts everything in A, and nothing not in A. Because of the fact that our function f must exist and $w \in A \Leftrightarrow f(w) \in B$, we know that $M_A(x)$ should always give exactly the same answer as $M_B(y)$, since $y = f(x)$.

3. (3 pts) What does it mean for a language to be undecidable?

   If a language L is undecidable, then no TM exists which decides it. There may be some TM which recognizes it, but no TM exists that will always halt and give the correct answer for every possible input.