CSE105 Closed Book Quiz, Thursday, Oct. 21, 1999

1. Write a regular expression over the alphabet of lower-case letters { **a-z** } to describe the language that consists of all strings containing exactly one occurrence of each vowel, such that the vowels appear in alphabetical order **a e i o u**. (For example the string **xyaxyewhyiwhyou** is in the language, but **eioubxyz** and **aaeiouxyz** are not.)

   Let $\Sigma = \{a - z, *, /\} - \{a, e, i, o, u\}$. Then the regular expression for the given language is

   $$\Sigma^* a \Sigma^* b \Sigma^* c \Sigma^* d \Sigma^* e \Sigma^*$$

   .

   Let $\Sigma = \{a - z\}$. Then the regular expression for the given language is

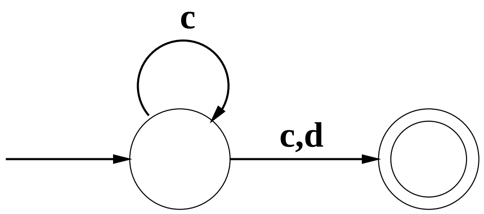
   $$/ * \Sigma^* * /$$

   .

2. Construct an NFA that recognizes the language described by the regular expression

   $(c^* \circ d) \cup (c \circ c^*)$

   where $\circ$ is the concatenation operator.



3. Show the following language is non-regular:
   L $= \{a^n b^n \mid n \geq 0\}$

   The proof is by contradiction. Assume L is regular. Then we know we can apply the pumping lemma. Let p be the pumping length given by the pumping lemma. Let the target string s be $a^p b^p$. Since $s \in L$ and has length greater than p, We know that s can be partioned into strings x, y, and z such that:

   (a) $|xy| \leq p$;
   (b) $|y| > 0$; and
   (c) $\forall i \geq 0, xy^i z \in L$.

   By conditions a and b, $y = a^k$, where $0 < k \leq p$. However, $xy^2z = a^{p+k}b^p$ is not in L since there are more a's than b's, contradicting condition c. Therefore, L cannot be regular.

4. True or False: The concatenation of two regular languages is regular.

   True.