Turing-Recognizability & Mapping Reducibility

Cor. If $A \leq_m B$ and A not Turing-recognizable, then B is not Turing-recognizable.

Proof: Suppose B were T. R. By the previous th., since $A \leq_m B$, then A would be T.R. Contradiction.

Th. EQ_{TM} is not Turing-recognizable.

Proof: We show that $A_{TM} \leq_m \overline{EQ}_{TM}$

F: on input<M,w> construct Rej and M2 so that $L(Rej) = \phi$

M2: on input x, run M on w. If M accepts, accept.

L(M2) is either Σ^* (if M accepts w) or ϕ (ow)

M accepts w $\leftrightarrow$ ϕ = L(Rej) $\neq$ L(M2)

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Turing-Recognizability & Mapping Reducibility

Th. $\overline{EQ}_{TM}$ is not Turing-recognizable.

Proof: We show that $A_{TM} \leq_m EQ_{TM}$

Want <M,w> $\rightarrow$ <M1,M2>

M accepts w $\leftrightarrow$ L(M1) = L(M2)

G: on input<M,w> construct Acc and M2 so that $L(Acc) = \Sigma^*$

M2: on input x, run M on w. If M accepts, accept.

L(M2) is either Σ^* (if M accepts w) or ϕ (ow)

M accepts w $\leftrightarrow$ Σ^* = L(Acc) = L(M2)

EQ_{TM} and $\overline{EQ}_{TM}$ are both Turing-unrecognizable

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Complexity

Decidability tells us answer (eventually)
but gives no idea of how long to compute!

Complexity: measure resources (time, space, power,...) to solve problem

We concentrate on TIME(n) = # of steps, as function of length n of input string

Decidability	**Complexity**
What can (conceptually) be solved, given unlimited resources?	*What can (practically) be solved, given limited resources?*

Example: Travelling Salesman Problem

Collection {C1, C2, ..., Cn} of cities, distance k

Distance matrix D(i,j) gives distance (in miles) between Ci and Cj.

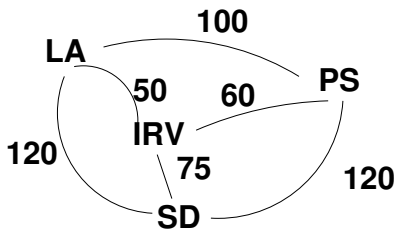
Problem: Is there a tour of all cities with distance less than k?

Solvable problem:

Can consider every possible tour (wo repet.)
For each tour, figure out total distance
Stop if find one of distance less than k.

Complexity:
n cities, (n-1)! choices for tours
n = 50, 49! ~ 10^{45}
not practically computable!

Example: Travelling Salesman Problem



SD

SD

Time Complexity

Def. Let M be a deterministic TM that always halts. The *running time or time complexity* of M is the function $f: N \longrightarrow N$ where f(n) is the maximum no. of steps of the TM M on any input of length n.

We say that M runs in time f(n).

Ex. $\{0^n 1^n \mid n \geq 0\}$

M on input w:

1. Scan the tape and reject if 0 to right of 1.
2. Repeat the following while both 0's and 1's:
   a. Scan across tape, crossing out one 0 and one 1.
3. If 0's remain and all 1's crossed out, reject.
   If 1's remain and all 0's crossed out, reject.
   OW accept.

How many steps required?

1. Scan the tape and reject if 0 to right of 1.

O(n) 000 .... 0 1111..11

1 scan of tape: n steps to determine right form; n steps to move head to left = 2n steps

2. Repeat the following while both 0's and 1's:

n

$O(n^2)$ a. Scan across tape, crossing out one 0 and one 1.

n/2 scans, n steps each: $n^2/2$ steps

3. If 0's remain and all 1's crossed out, reject.

O(n) If 1's remain and all 0's crossed out, reject. OW accept.

1 scan of tape: n steps.

Total Time: $O(n) + O(n^2) + O(n) = O(n^2)$

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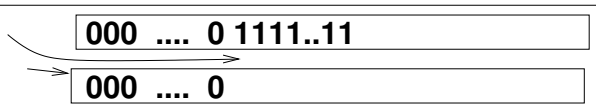
What about two tapes?



O(n) 1. scan across tape, reject if 0 to rt. of 1 at same time, write 0's on second tape.

2. Scan across the 1's on Tape 1 till end.

? For each 1 on Tape 1, cross off 0 on Tape 2. if all 0's are crossed off before all 1's read, reject!

3. If 0's remain and all 1's crossed out, reject.

O(n) If 1's remain and all 0's crossed out, reject. OW accept.

Total Time: $O(n) + O(n) + O(n) = O(n)$

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Complexity Models: Resources Matter

Linear time in single tape model

Quadratic time in two-tape model

Turns out time difference:

Det. single tape	Det. multitape
$O(t(n)^2)$	$O(t(n))$
Deterministic	**Nondeterministic**
$O(2^{t(n)})$	$O(t(n))$

All deterministic models differ by polynomial time

Exponential time difference in det. vs nondet.

Polynomial difference: small Exponential: large

The Class P

Def. P is the class of languages decidable on a determinstic, single tape TM in time bounded by some polynomial (in size of input)

P contains all problems that are realistically solvable

P turns out to be invariant over many models of computation

May still have problems in P we don't think are realistically solvable (e.g. $O(n^{\mathbf{v}})$ where **v** *is very large no.)*

Gives us fundamental way of grouping problems:

P	not in P

The Class NP

Many problems have as their only solution brute-force search

May not be solvable in polynomial time

Def. NP is the class of languages decidable on a nondeterminstic, single tape TM in time bounded by some polynomial (in size of input)

Ex. Travelling Salesman problem: **solution < k miles**

Only known solution is to try all tours- exponential time for deterministic TM

Nondeterminstic TM can guess the tour, then check if its length is less than k in polynomial time!