Geometric Image Formation

Introduction to Computer Vision I

CSE 152A

Lecture 2

Announcements

- Assignment 0 will be released today
  - Due Oct 7, 11:59 PM

Earliest Surviving Photograph

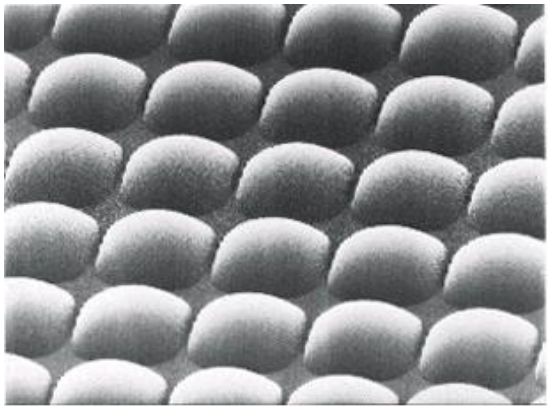


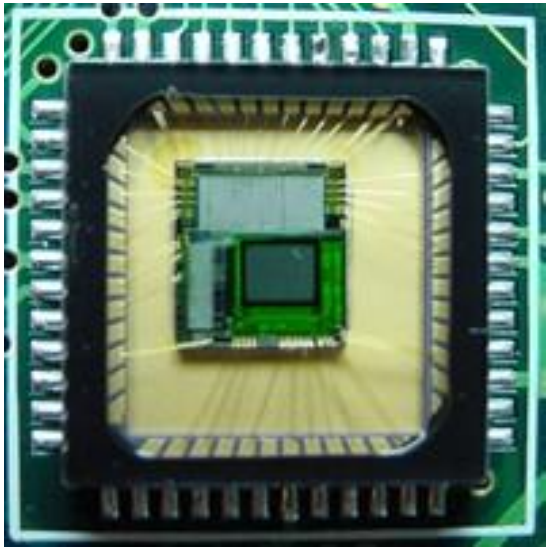
- First photograph on record, “la table service” by Nicephore Niepce in 1822
- Note: First photograph by Niepce was in 1816

How Cameras Produce Images

- Basic process:
  - photons hit a detector
  - the detector becomes charged
  - the charge is read out as brightness

- Sensor types:
  - CCD (charge-coupled device)
    - high sensitivity
    - high power
    - cannot be individually addressed
    - blooming
  - CMOS
    - simple to fabricate (cheap)
    - lower sensitivity, lower power
    - can be individually addressed





Images are two-dimensional patterns of brightness values.

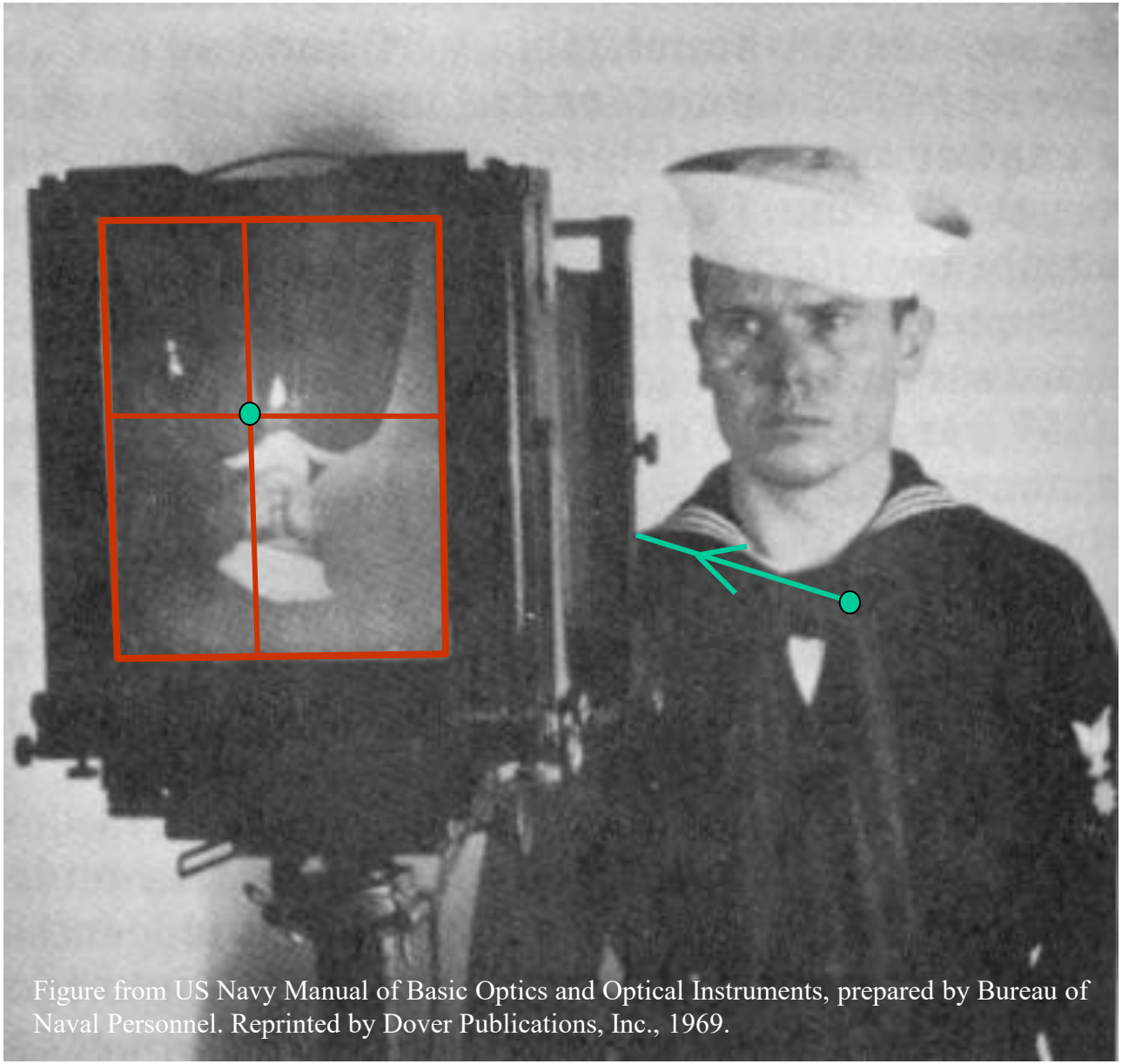
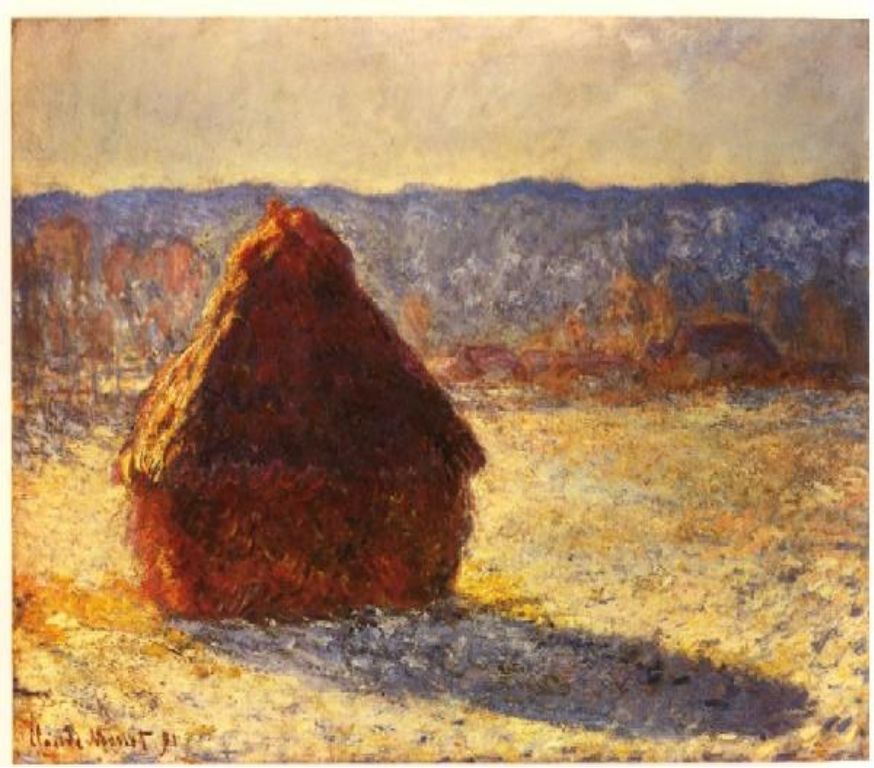


Figure from US Navy Manual of Basic Optics and Optical Instruments, prepared by Bureau of Naval Personnel. Reprinted by Dover Publications, Inc., 1969.

They are formed by the projection of 3D objects.

Lighting Affects Appearance: Monet





Viewpoint Affects Appearance: Monet



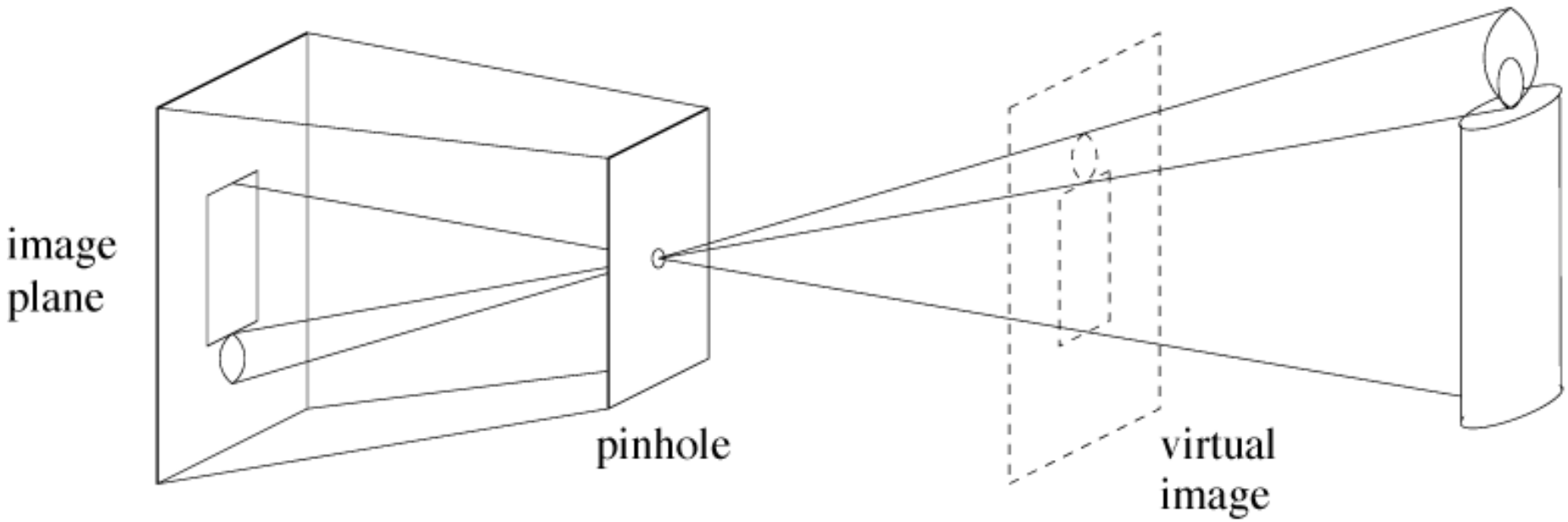
Haystack at Chailly at sunrise (1865)

Image Formation: Outline

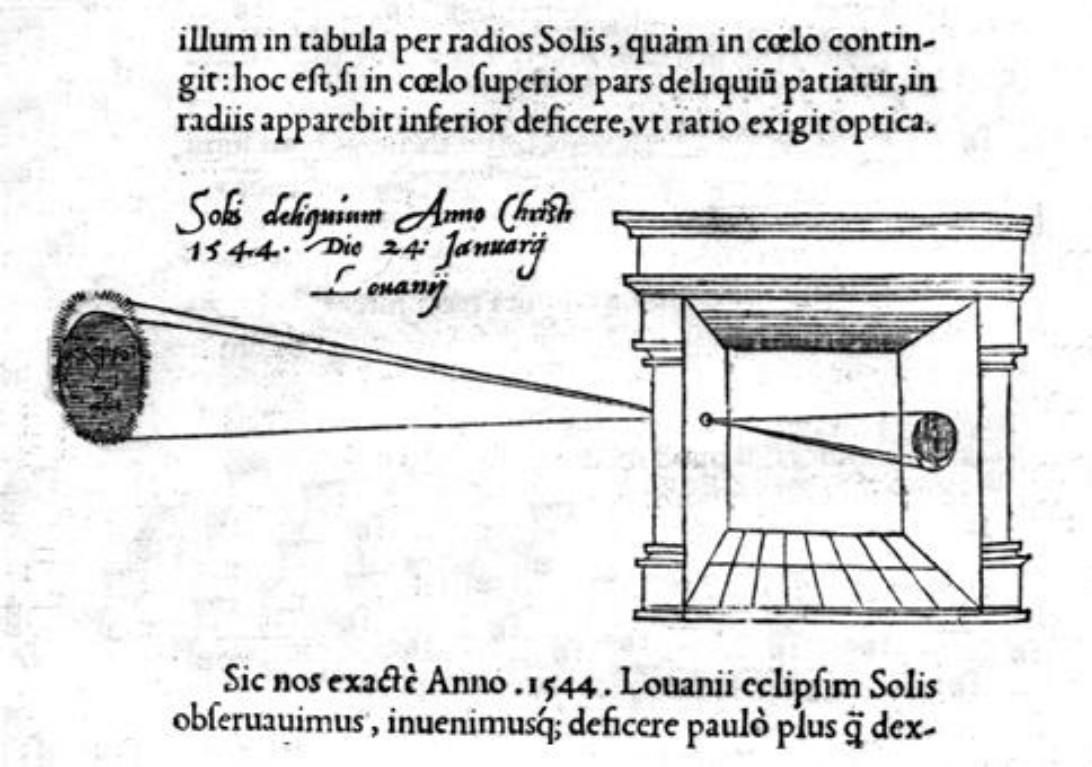
- Geometric image formation
  - How do 3D world points project to 2D image points?
- Photometric image formation
  - What color is the projected point?

Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it

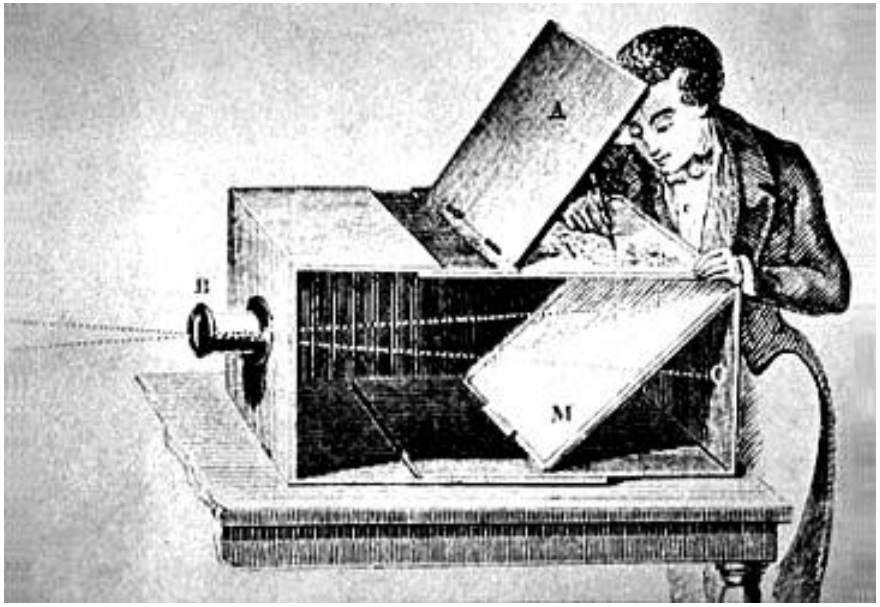


Camera Obscura



"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays". --- Leonardo *Da Vinci*

Camera Obscura



- Used to observe eclipses (e.g., Bacon, 1214-1294)
- By artists (e.g., Vermeer).

Camera Obscura

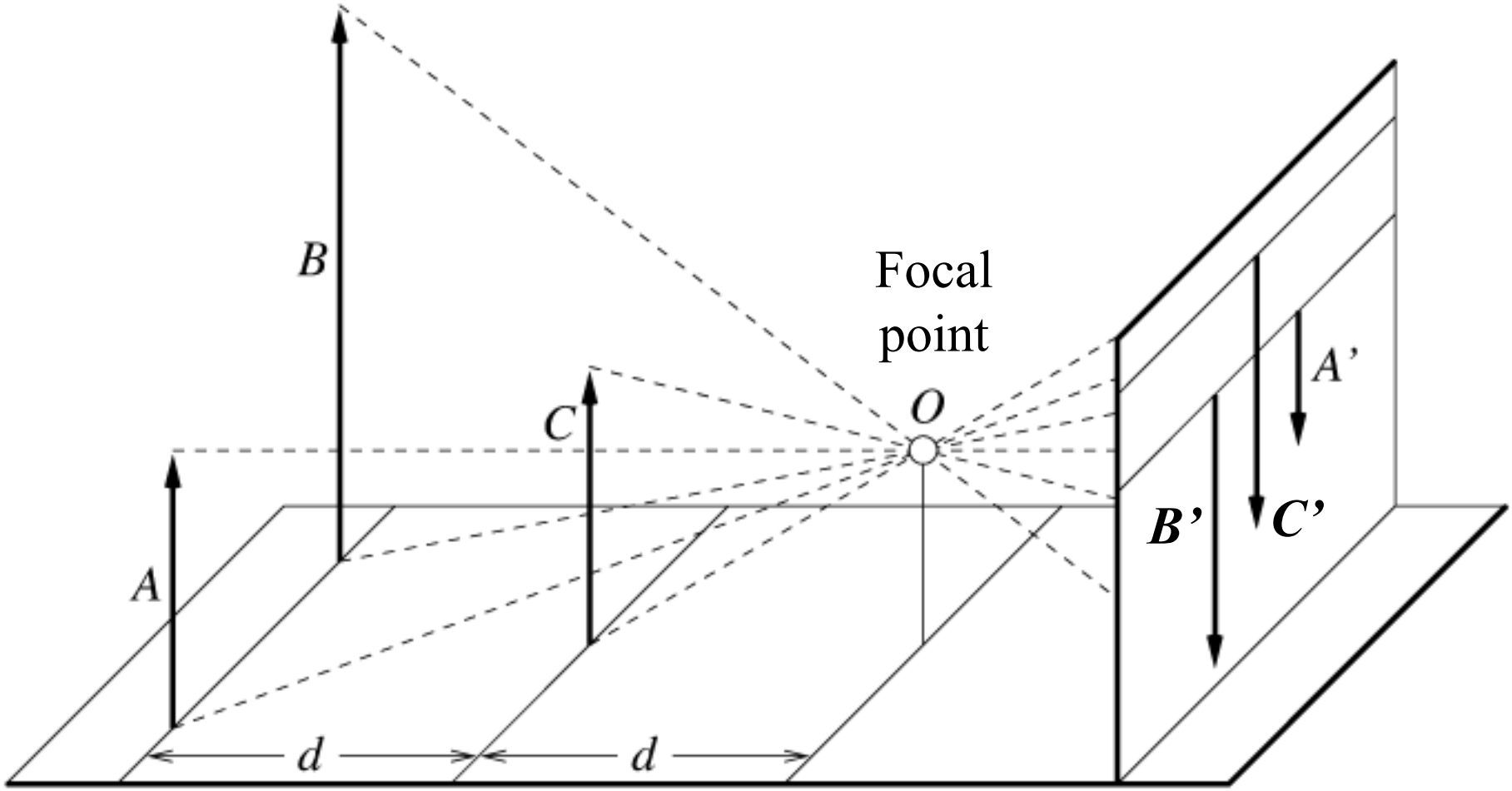


Jetty at Margate England, 1898.



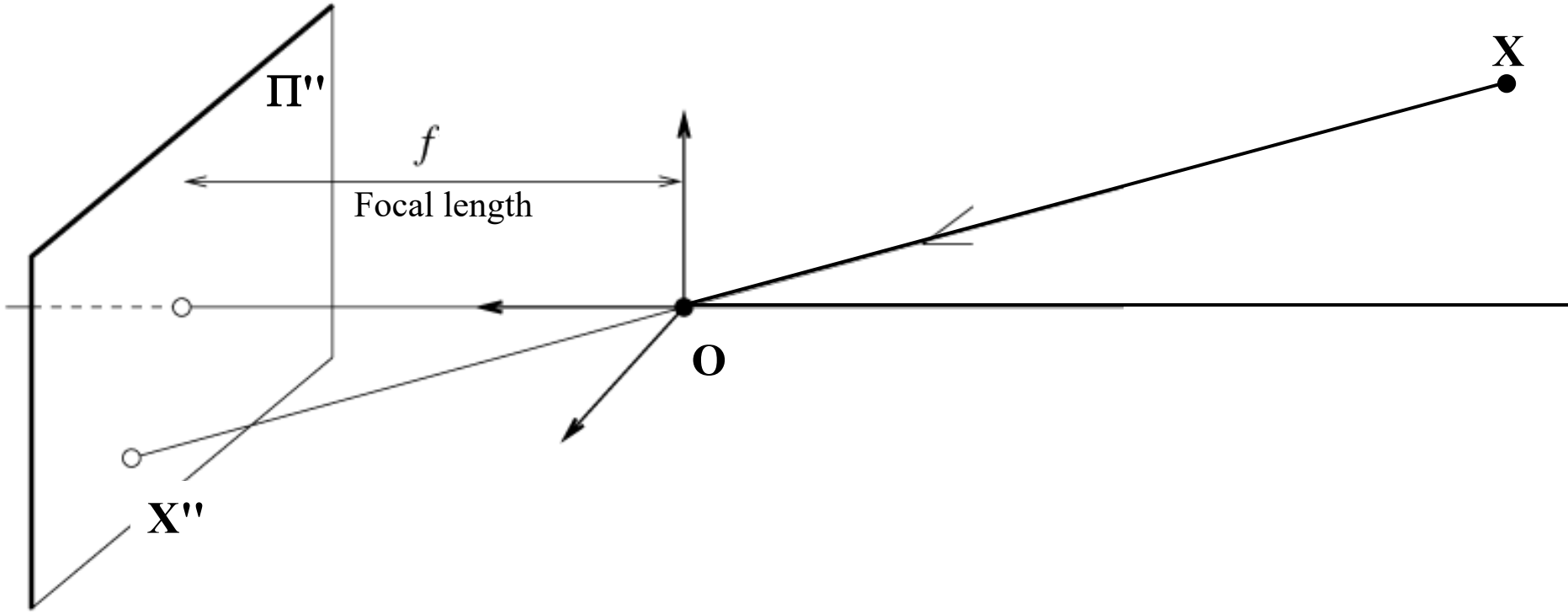
http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)

A and C are same size, but A is further from camera, so its image A' is smaller



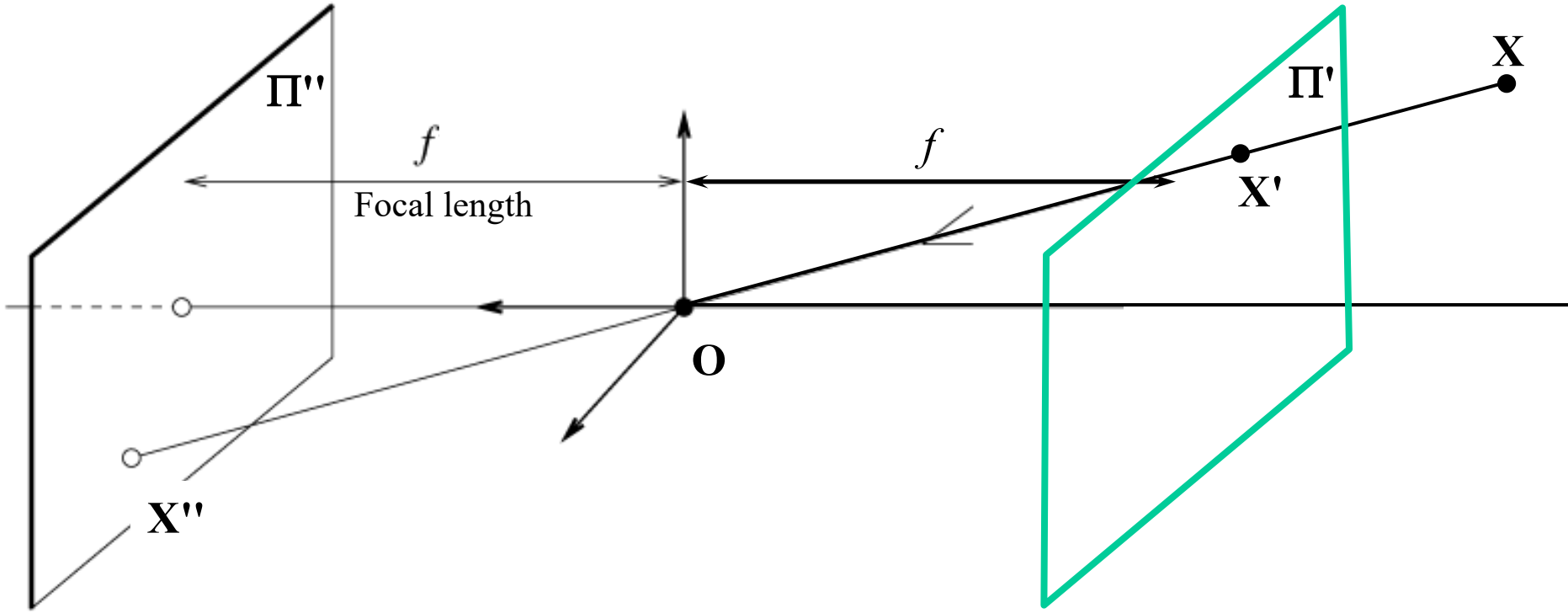
(Forsyth & Ponce)

Purely Geometric View of Perspective



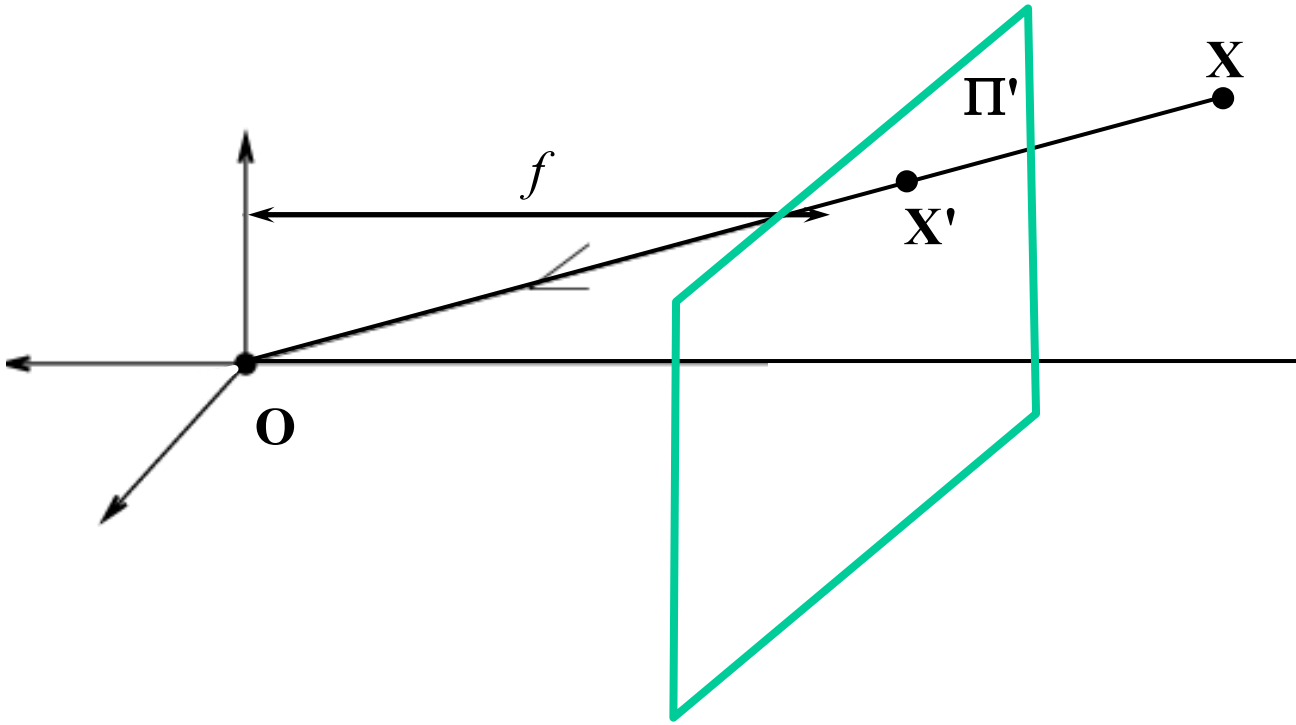
The projection of the point **X** on the image plane **Π''** is given by the point of intersection **X''** of the ray defined by **XO** with the plane **Π''**

Virtual image plane



- Virtual image plane in front of optical center
- Image is ‘upright’

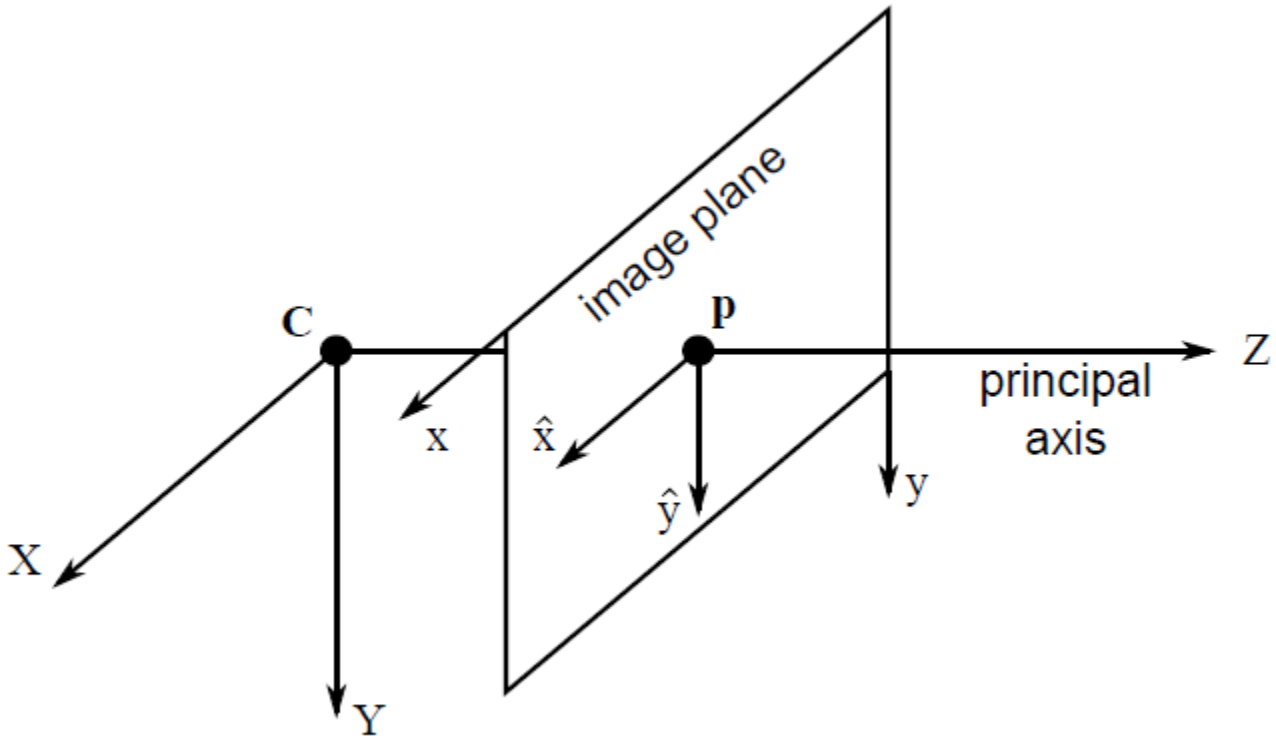
Virtual image plane



- Virtual image plane in front of optical center
- Image is ‘upright’

Geometric image formation

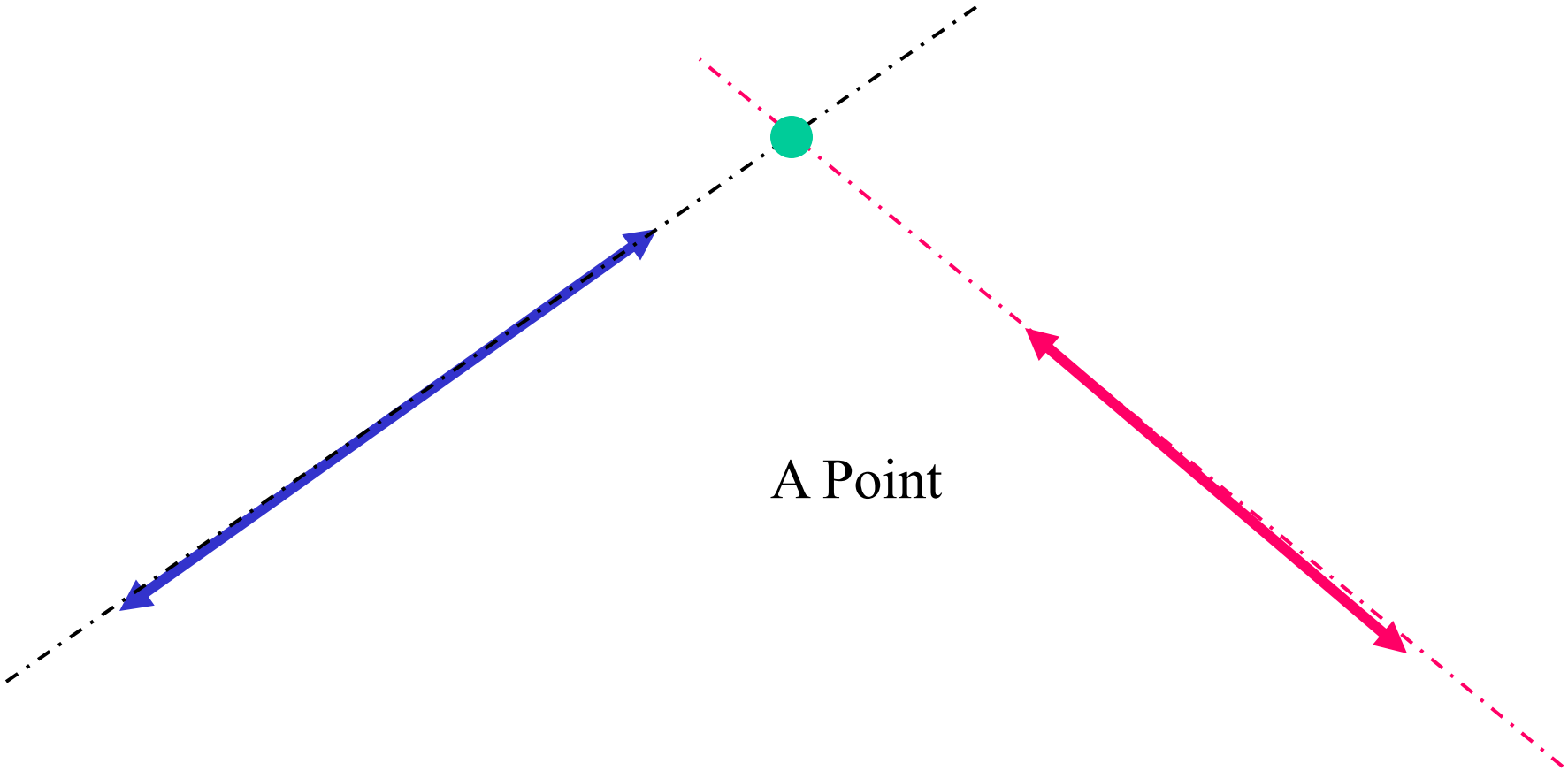
- How do 3D world points project to 2D image points?



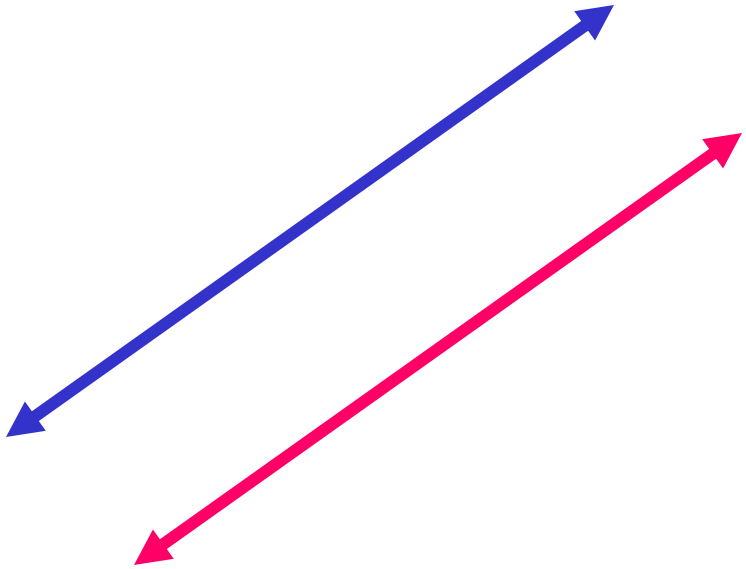
A Digression

Projective Geometry
and
Homogenous Coordinates

What is the intersection of two lines in a plane?



Do two lines in the plane always intersect at a point?



No, Parallel lines don't meet at a point.

Can the perspective projection of parallel lines in 3D meet at a point in an image?

YES



Projective geometry provides an elegant means for handling these different situations in a unified way

Homogenous coordinates are a way to represent entities (points & lines) in projective spaces

Geometric primitives

- 2D and 3D points
- 2D lines
- Joint and intersection of 2D points and lines

Points

- Inhomogeneous coordinates

2D

$$\tilde{\mathbf{x}} = (\tilde{x}, \tilde{y})^\top$$

3D

$$\tilde{\mathbf{X}} = (\tilde{X}, \tilde{Y}, \tilde{Z})^\top$$

- Homogeneous (i.e., defined up to nonzero scale) coordinates

2D

$$\mathbf{x} = (x, y, w)^\top$$

3D

$$\mathbf{X} = (X, Y, Z, T)^\top$$

Points

- Homogeneous coordinates to inhomogeneous coordinates

2D

$$\mathbf{x} = (x, y, w)^\top \mapsto \tilde{\mathbf{x}} = (x/w, y/w)^\top$$

3D

$$\mathbf{X} = (X, Y, Z, T)^\top \mapsto \tilde{\mathbf{X}} = (X/T, Y/T, Z/T)^\top$$

- Inhomogeneous coordinates to homogeneous coordinates

2D

$$\tilde{\mathbf{x}} = (\tilde{x}, \tilde{y})^\top \mapsto \mathbf{x} = (\tilde{x}, \tilde{y}, 1)^\top$$

3D

$$\tilde{\mathbf{X}} = (\tilde{X}, \tilde{Y}, \tilde{Z})^\top \mapsto \mathbf{X} = (\tilde{X}, \tilde{Y}, \tilde{Z}, 1)^\top$$

Points

- Homogeneous (i.e., defined up to nonzero scale) coordinates to inhomogeneous coordinates

2D

$$\mathbf{x} = \lambda\mathbf{x}, \text{ for nonzero } \lambda$$

$$\begin{aligned} \mathbf{x} = (x, y, w)^\top &\mapsto \tilde{\mathbf{x}} = (x/w, y/w)^\top \\ \lambda\mathbf{x} = (\lambda x, \lambda y, \lambda w)^\top &\mapsto \tilde{\mathbf{x}} = (\lambda x/\lambda w, \lambda y/\lambda w)^\top \\ &\tilde{\mathbf{x}} = (x/w, y/w)^\top \end{aligned}$$

3D

$$\mathbf{X} = \lambda\mathbf{X}, \text{ for nonzero } \lambda$$

$$\begin{aligned} \mathbf{X} = (X, Y, Z, T)^\top &\mapsto \tilde{\mathbf{X}} = (X/T, Y/T, Z/T)^\top \\ \lambda\mathbf{X} = (\lambda X, \lambda Y, \lambda Z, \lambda T)^\top &\mapsto \tilde{\mathbf{X}} = (\lambda X/\lambda T, \lambda Y/\lambda T, \lambda Z/\lambda T)^\top \\ &\tilde{\mathbf{X}} = (X/T, Y/T, Z/T)^\top \end{aligned}$$

Points

- Homogeneous coordinates allow for points at infinity to be treated like other points
  - Definition of infinity

2D $\infty = \lim_{w \to 0^+} \frac{1}{w}$ 3D $\infty = \lim_{T \to 0^+} \frac{1}{T}$

  - Set of points at infinity

2D $\mathbf{x}_\infty = (x, y, 0)^\top$ 3D $\mathbf{X}_\infty = (X, Y, Z, 0)^\top$

2D lines

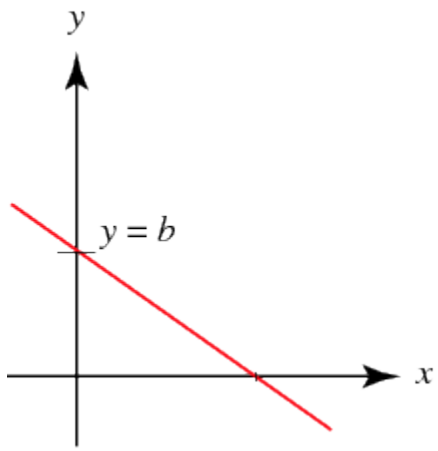
Equation of a 2D line, slope-intercept form

$$y = mx + b$$

with y-intercept b and slope

$$m = \frac{\Delta y}{\Delta x}$$

Slope of horizontal line is 0
Slope of vertical line is undefined (division by zero)



Do not use this

2D lines

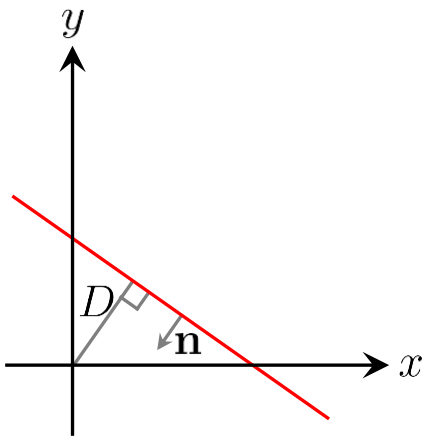
Equation of a 2D line, standard form

$$
\begin{aligned}
a\tilde{x} + b\tilde{y} + c &= 0 \\
a(x/w) + b(y/w) + c &= 0 \\
ax + by + cw &= 0 \\
\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} &= 0 \\
\boldsymbol{\ell}^\top \mathbf{x} &= 0
\end{aligned}
$$

2D lines

2D line in homogeneous coordinates

$$\boldsymbol{\ell} = (a, b, c)^\top$$
$$\boldsymbol{\ell} = (\mathbf{n}^\top, c)^\top$$



where normal vector

$$\mathbf{n} = (a, b)^\top$$

Signed distance from origin

$$D = \frac{c}{\sqrt{a^2 + b^2}} = \frac{c}{\|\mathbf{n}\|}$$

Set of 2D lines at infinity

$$\boldsymbol{\ell}_\infty = (0, 0, 1)^\top$$

2D points and lines

2D points and lines in homogeneous coordinates

$$\mathbf{x}^\top \boldsymbol{\ell} = 0$$
$$\boldsymbol{\ell}^\top \mathbf{x} = 0$$

if and only if $\mathbf{x}$ is on $\boldsymbol{\ell}$

2D points and lines

- Two 2D points define a 2D line

2D line joining two 2D points in general position is (right) null space of matrix

$$\begin{bmatrix} \mathbf{x}_1^\top \\ \mathbf{x}_2^\top \end{bmatrix} \boldsymbol{\ell} = \mathbf{0}$$

Two 2D points on 2D line is left null space of 2D line

$$[\boldsymbol{\ell}]^\perp = \begin{bmatrix} \mathbf{x}_1^\top \\ \mathbf{x}_2^\top \end{bmatrix}$$

2D points and lines

- Two 2D lines define a 2D point

2D point at intersection of two 2D lines in general position is (right) null space of matrix

$$\begin{bmatrix} \boldsymbol{\ell}_1^\top \\ \boldsymbol{\ell}_2^\top \end{bmatrix} \mathbf{x} = \mathbf{0}$$

Two 2D lines intersecting at a 2D point is the left null space of 2D point

$$[\mathbf{x}]^\perp = \begin{bmatrix} \boldsymbol{\ell}_1^\top \\ \boldsymbol{\ell}_2^\top \end{bmatrix}$$

2D points and lines

For 3-vectors, the cross product is the (right) null space. Note these assume points and lines are in general position.

Join

$$\boldsymbol{\ell} = \mathbf{x}_1 \times \mathbf{x}_2$$
$$\boldsymbol{\ell} = [\mathbf{x}_1]_\times \mathbf{x}_2 = (\mathbf{x}_1^\top [\mathbf{x}_2]_\times)^\top$$
$$[\boldsymbol{\ell}]_\times = \mathbf{x}_1\mathbf{x}_2^\top - \mathbf{x}_2\mathbf{x}_1^\top$$

$$\mathbf{a} = (a_1, a_2, a_3)^\top$$
$$[\mathbf{a}]_\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

Intersection

$$\mathbf{x} = \boldsymbol{\ell}_1 \times \boldsymbol{\ell}_2$$
$$\mathbf{x} = [\boldsymbol{\ell}_1]_\times \boldsymbol{\ell}_2 = (\boldsymbol{\ell}_1^\top [\boldsymbol{\ell}_2]_\times)^\top$$
$$[\mathbf{x}]_\times = \boldsymbol{\ell}_1\boldsymbol{\ell}_2^\top - \boldsymbol{\ell}_2\boldsymbol{\ell}_1^\top$$

Skew symmetric matrix

- Definition

$$\mathtt{A} = -\mathtt{A}^\top$$

- 3-vector in skew symmetric matrix form

$$\mathbf{a} = (a_1, a_2, a_3)^\top$$

$$[\mathbf{a}]_\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

  – 3x3 skew symmetric matrices are rank 2

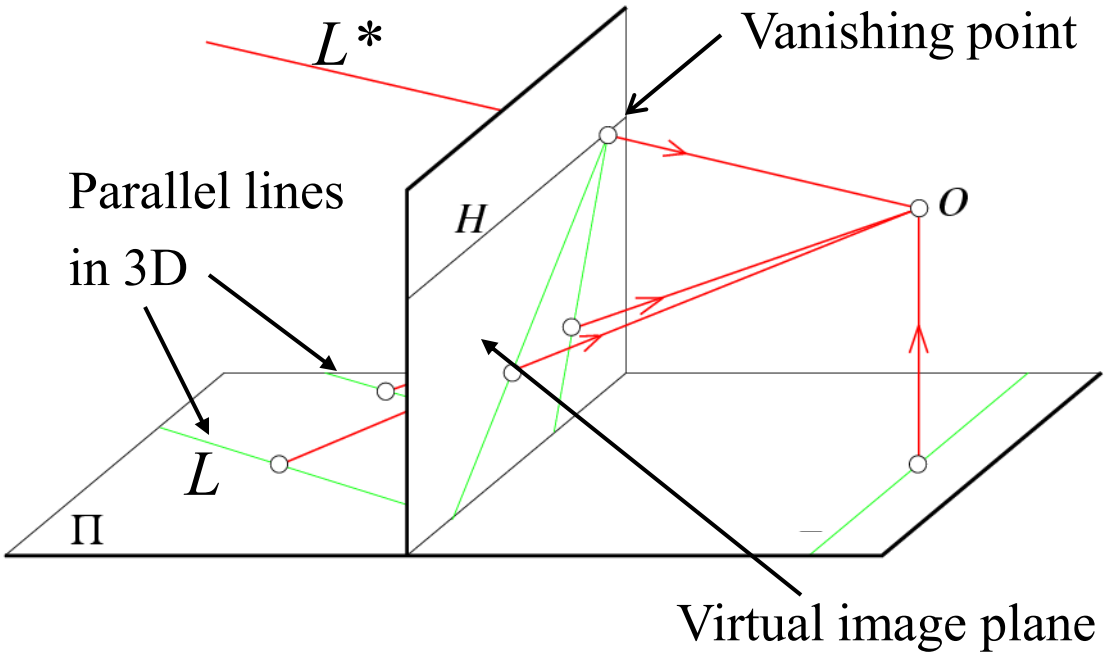
End of the Digression

In a perspective image, parallel lines meet at a point, called the vanishing point



Doesn’t need to be near the center of the image

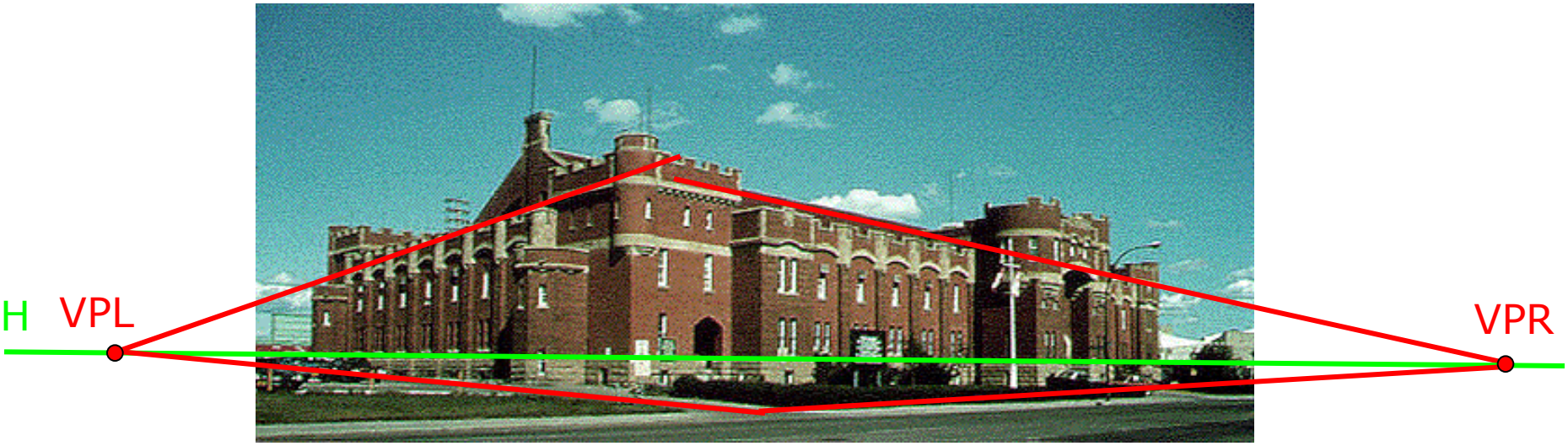
Parallel lines meet in the image



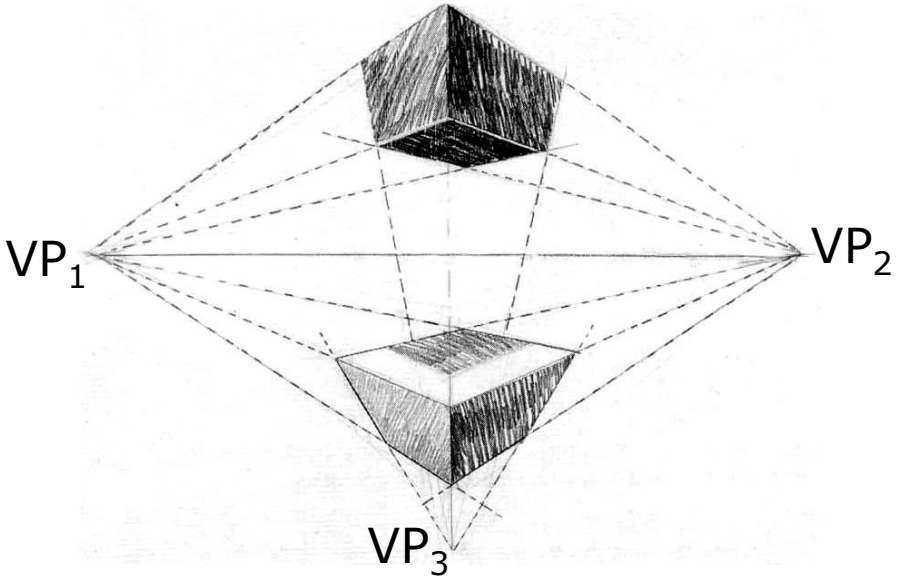


- A single line *L* can have a vanishing point.
- Vanishing point location: Intersection of image plane with a 3-D line *L** through optical center O parallel to *L*

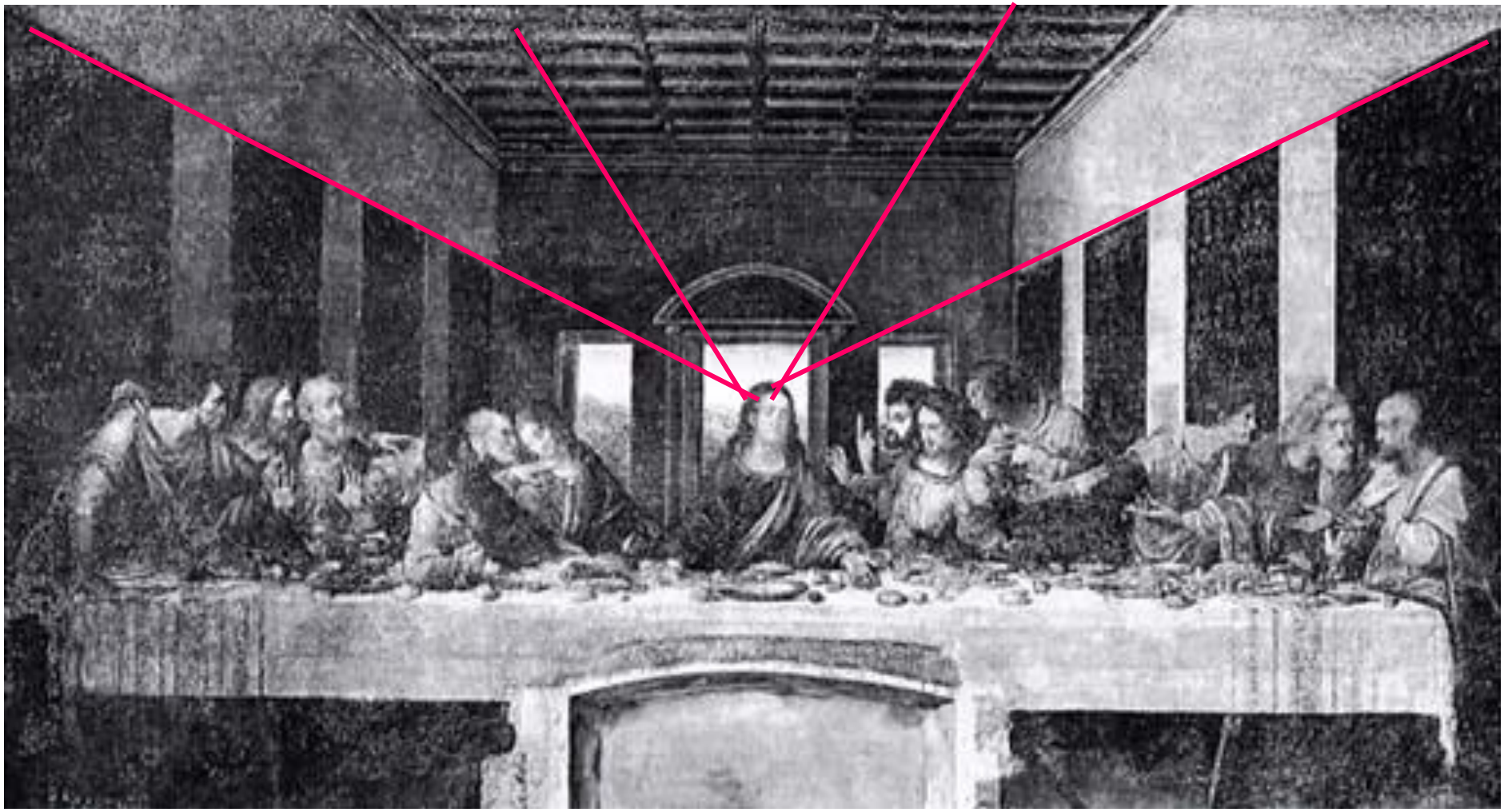
Vanishing Points



- A scene can have more than one vanishing point
- Different 3-D directions correspond different vanishing points



Vanishing Points



Vanishing Point

- In the **projective plane**, parallel lines meet at a point at infinity

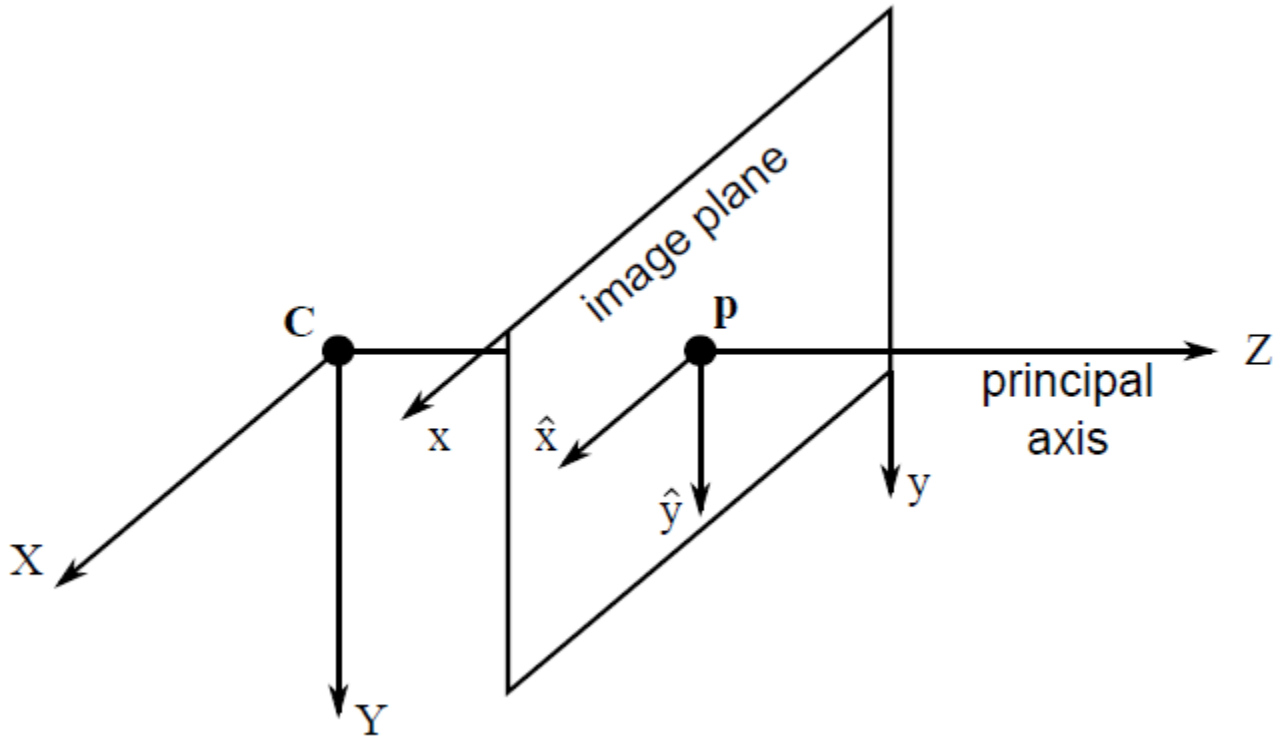
- The 2D vanishing point in the image is the perspective projection of this 3D point at infinity

What is a Camera?

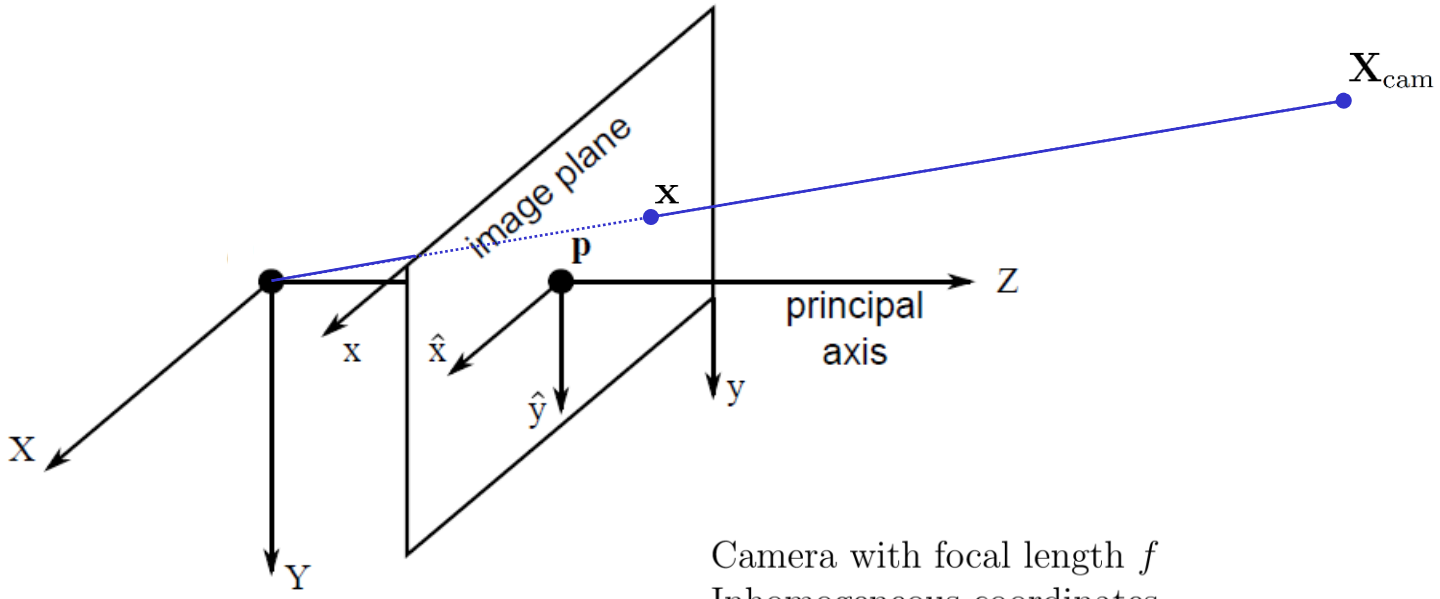
- A mathematical expression that relates points in 3D to points in an image for different types of physical cameras or imaging situations

Geometry

- How do 3D world points project to 2D image points?

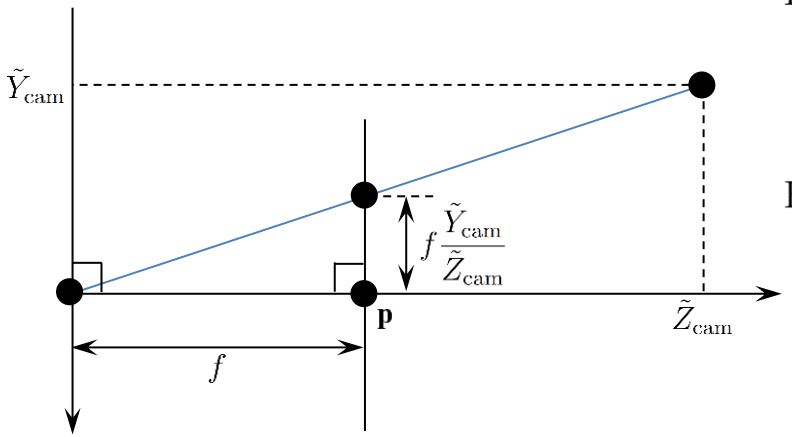


Camera projection



Camera with focal length f
Inhomogeneous coordinates

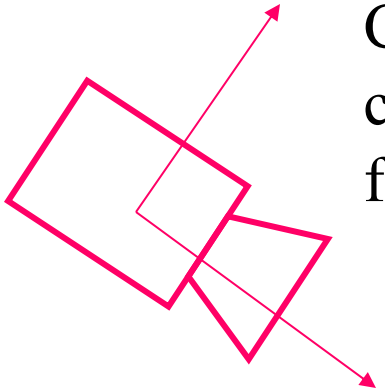
$$(\tilde{X}_{\text{cam}}, \tilde{Y}_{\text{cam}}, \tilde{Z}_{\text{cam}}) \mapsto \left(f\frac{\tilde{X}_{\text{cam}}}{\tilde{Z}_{\text{cam}}}, f\frac{\tilde{Y}_{\text{cam}}}{\tilde{Z}_{\text{cam}}} \right)$$



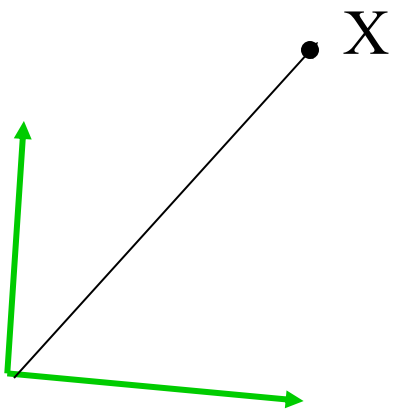
Homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_{\text{cam}} \\ Y_{\text{cam}} \\ Z_{\text{cam}} \\ T_{\text{cam}} \end{bmatrix}$$

What if camera coordinate system differs from world coordinate system?

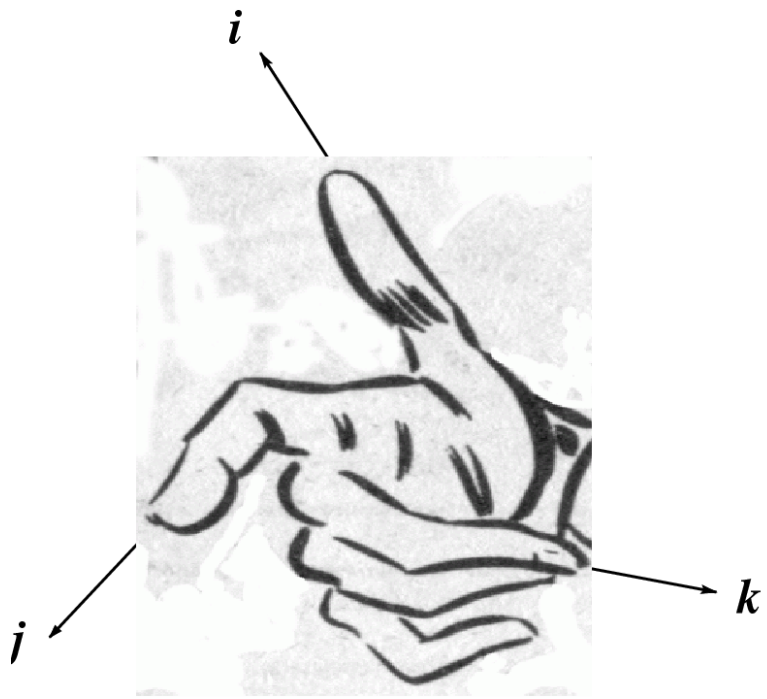


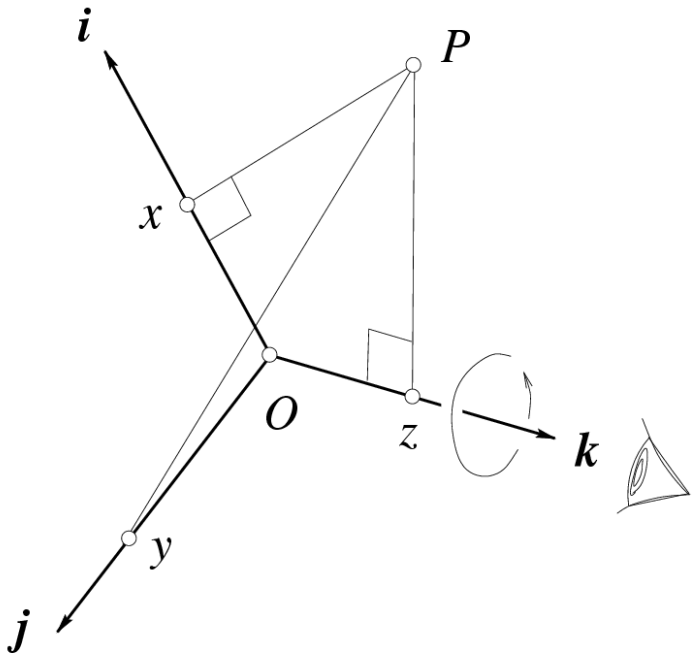
Camera coordinate frame



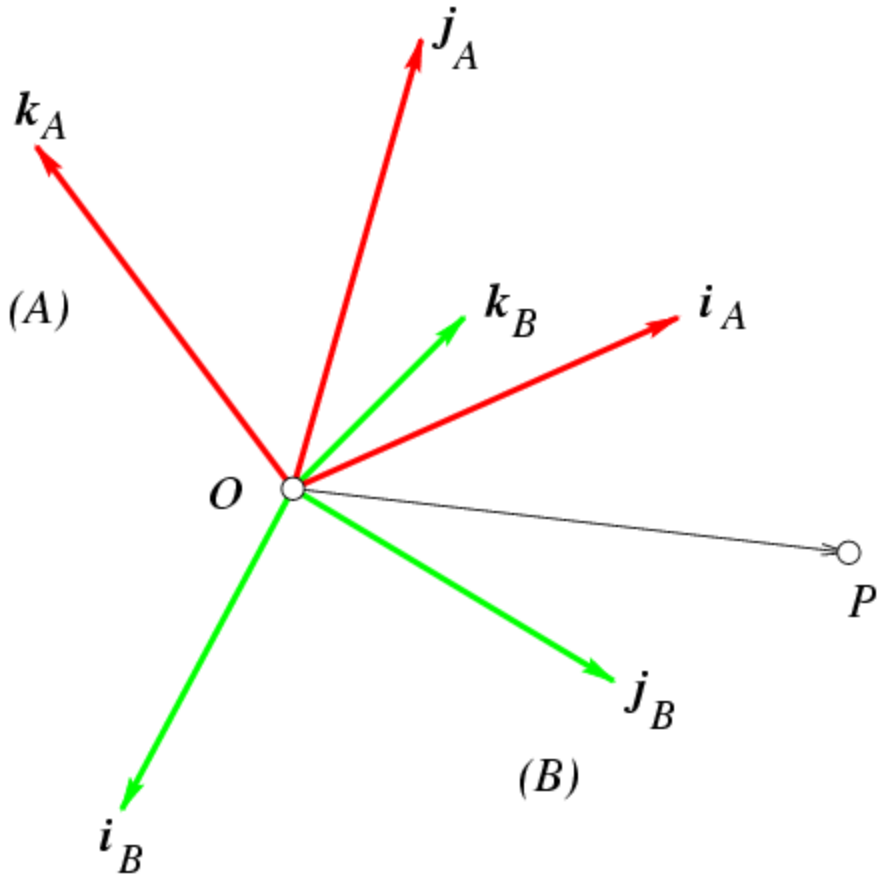
World coordinate frame

Euclidean Coordinate Systems





Coordinate Change: Rotation Only



$$\begin{bmatrix} \tilde{X}' \\ \tilde{Y}' \\ \tilde{Z}' \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix}$$

$$\tilde{\mathbf{X}}' = \mathtt{R}\tilde{\mathbf{X}}$$

$$\begin{bmatrix} \tilde{\mathbf{X}}' \\ 1 \end{bmatrix} = \begin{bmatrix} \mathtt{R} & \mathbf{0} \\ \mathbf{0}^\top & 1 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}} \\ 1 \end{bmatrix}$$

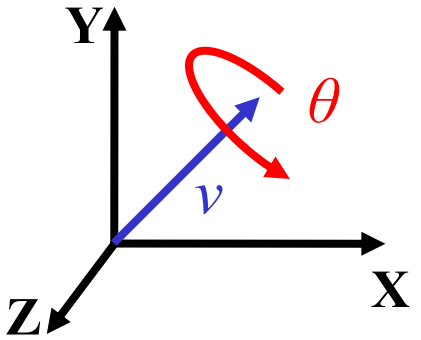
$$\mathbf{X}' = \begin{bmatrix} \mathtt{R} & \mathbf{0} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{X}$$

Some points about SO(n)

- $SO(n) = \{ R \in \Re^{nxn} : R^T R = I, \det(R) = 1 \}$
  - SO(2): rotation matrices in plane $\Re^2$
  - SO(3): rotation matrices in 3-space $\Re^3$
- Forms a Group under matrix product operation:
  - Identity
  - Inverse
  - Associative
  - Closure
- Closed (finite intersection of closed sets)
- Bounded $R_{i,j} \in [-1, +1]$
- Does not form a vector space.
- Manifold of dimension n(n-1)/2
  - Dim(SO(2)) = 1
  - Dim(SO(3)) = 3

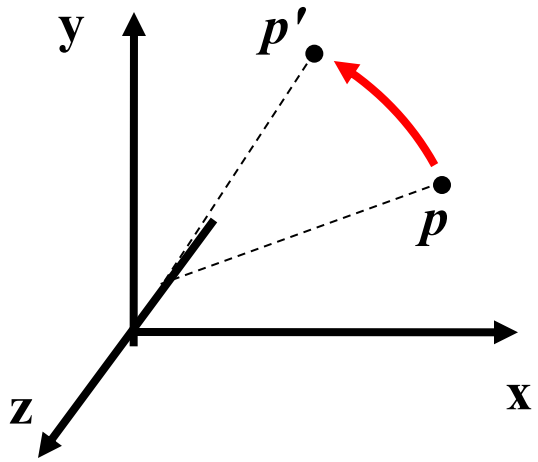
Parameterizations of SO(3)

- Even though a rotation matrix is 3x3 with nine elements, it only has three degrees of freedom. It can be parameterized with three elements. There are many parameterizations.
- Other common parameterizations
  - Euler Angles
  - Axis Angle
  - Quaternions
    - four parameters; homogeneous



Angle-axis representation

3D rotation about the Z axis



$$\begin{bmatrix} \tilde{X}' \\ \tilde{Y}' \\ \tilde{Z}' \end{bmatrix} = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix}$$

$$\tilde{\mathbf{X}}' = \mathtt{R}_Z(\gamma)\tilde{\mathbf{X}}$$

3D rotations about X and Y axes

- About X axis:

$$\begin{bmatrix} \tilde{X}' \\ \tilde{Y}' \\ \tilde{Z}' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix}$$

$$\tilde{\mathbf{X}}' = \mathtt{R}_X(\alpha)\tilde{\mathbf{X}}$$

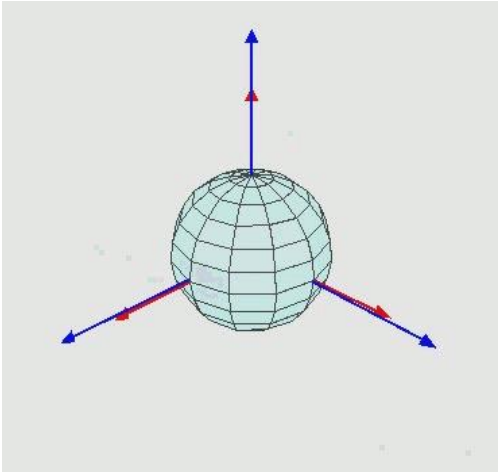
- About Y axis:

$$\begin{bmatrix} \tilde{X}' \\ \tilde{Y}' \\ \tilde{Z}' \end{bmatrix} = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix}$$

$$\tilde{\mathbf{X}}' = \mathtt{R}_Y(\beta)\tilde{\mathbf{X}}$$

3D rotation, Euler angles

- A sequence of 3 elemental rotations
  - Rotation about an axis, then a different axis, then a different axis
- 12 possible sequences



X-Y-X Y-X-Y Z-X-Y
X-Y-Z Y-X-Z Z-X-Z
X-Z-X Y-Z-X Z-Y-X
X-Z-Y Y-Z-Y Z-Y-Z

Tait-Bryan Angles

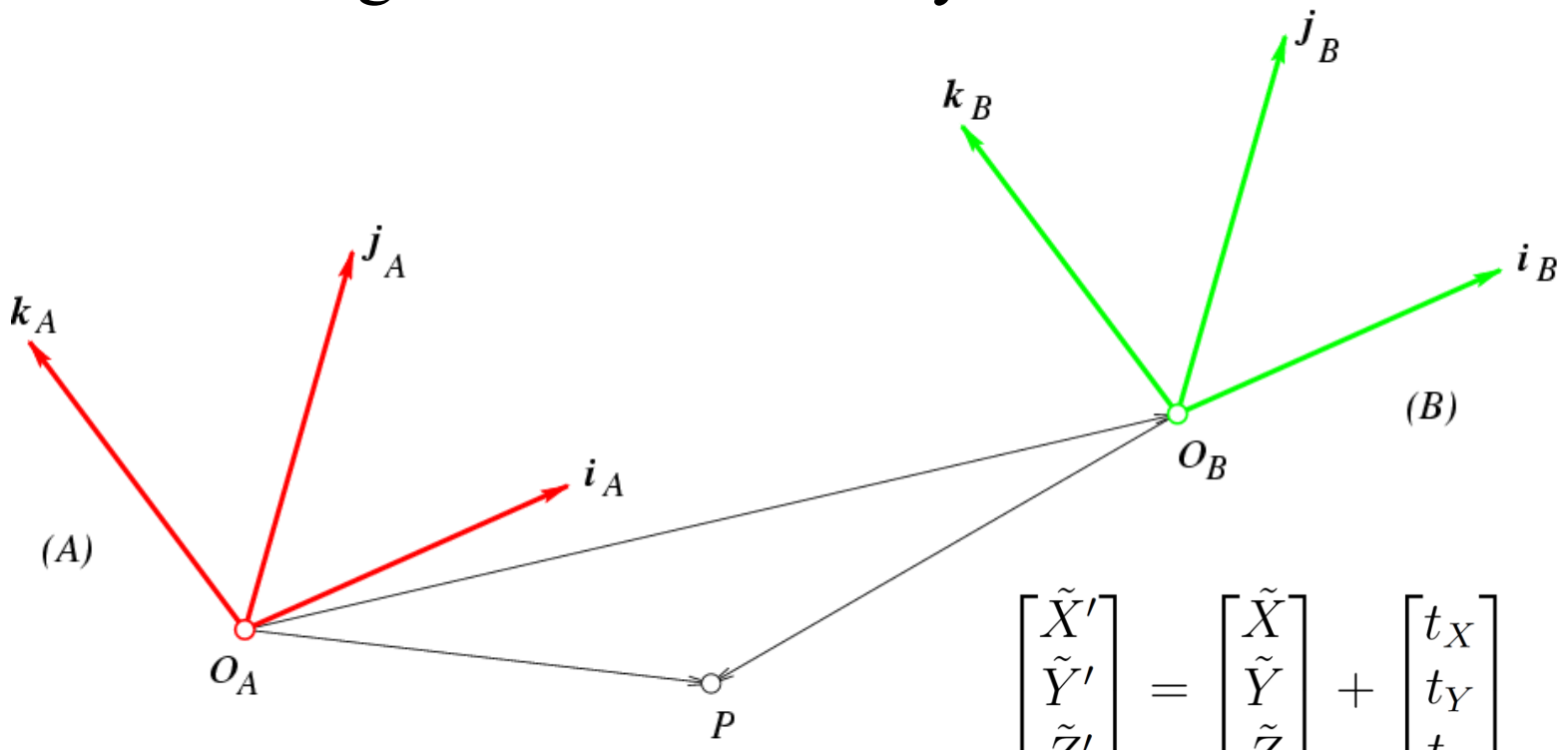
Euler Angles

- An example of 1 of the 12 possible sequences
  - Rotation about X-axis, followed by rotation about Y-axis, followed by rotation about Z-axis
  - Composition of rotations

$$R = R_Z(\gamma)\, R_Y(\beta)\, R_X(\alpha)$$

$$R = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix}$$

Coordinate Change: Translation Only



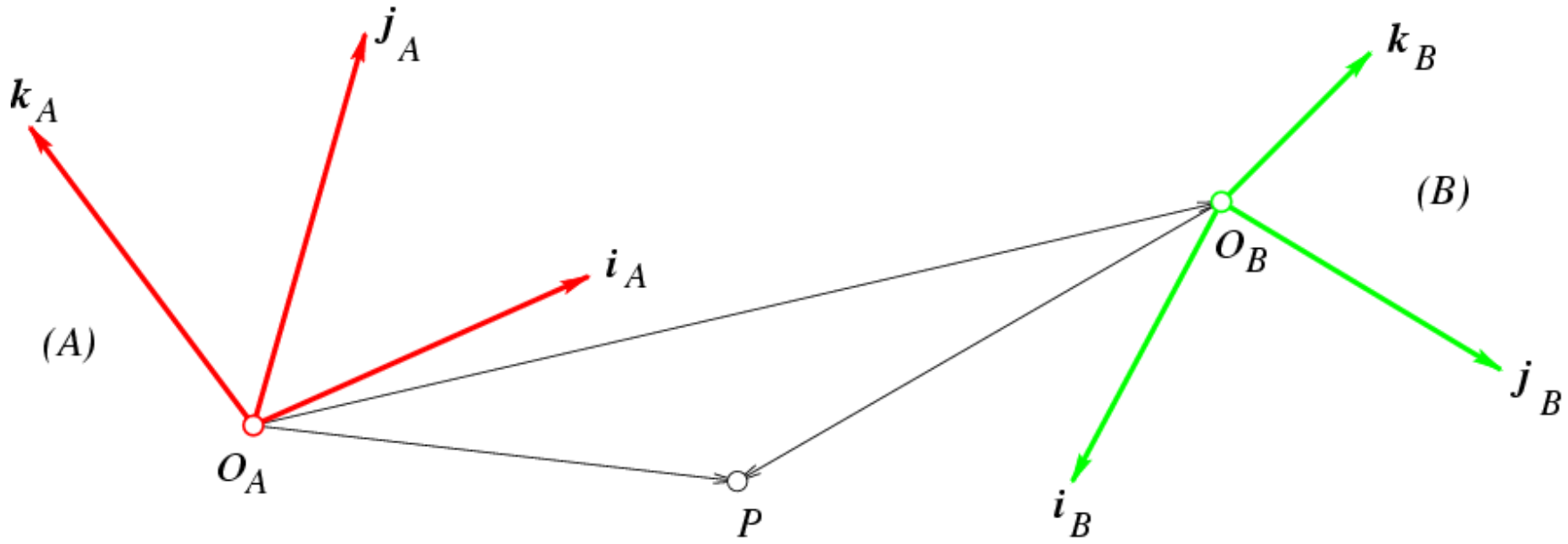
$$\begin{bmatrix} \tilde{X}' \\ \tilde{Y}' \\ \tilde{Z}' \end{bmatrix} = \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix} + \begin{bmatrix} t_X \\ t_Y \\ t_Z \end{bmatrix}$$

$$\tilde{\mathbf{X}}' = \tilde{\mathbf{X}} + \mathbf{t}$$

$$\begin{bmatrix} \tilde{\mathbf{X}}' \\ 1 \end{bmatrix} = \begin{bmatrix} \mathtt{I} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}} \\ 1 \end{bmatrix}$$

$$\mathbf{X}' = \begin{bmatrix} \mathtt{I} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{X}$$

Coordinate Changes: Rotation and Translation

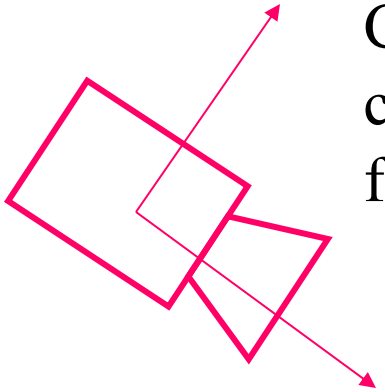


$$\tilde{\mathbf{X}}' = \mathtt{R}\tilde{\mathbf{X}} + \mathbf{t}$$

$$\begin{bmatrix} \tilde{\mathbf{X}}' \\ 1 \end{bmatrix} = \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}} \\ 1 \end{bmatrix}$$

$$\mathbf{X}' = \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{X}$$

What if camera coordinate system differs from world coordinate system?

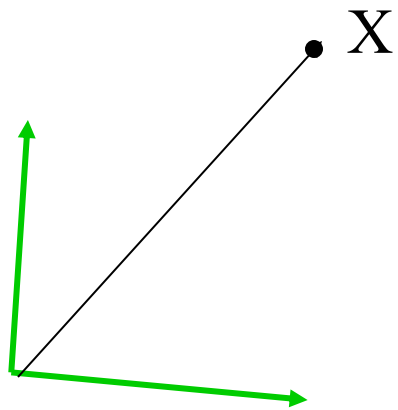


Camera coordinate frame

$$\tilde{\mathbf{X}}_{\text{camera}} = \mathtt{R}\tilde{\mathbf{X}}_{\text{world}} + \mathbf{t}$$

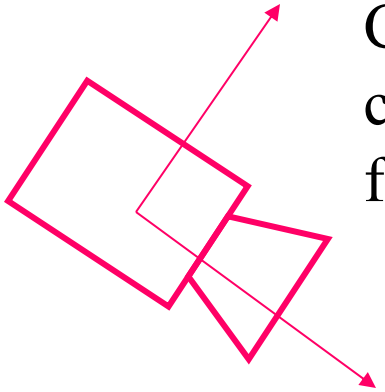
$$\begin{bmatrix} \tilde{\mathbf{X}}_{\text{camera}} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}}_{\text{world}} \\ 1 \end{bmatrix}$$

$$\mathbf{X}_{\text{camera}} = \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{X}_{\text{world}}$$



World coordinate frame

Where is the camera center in the world coordinate system?



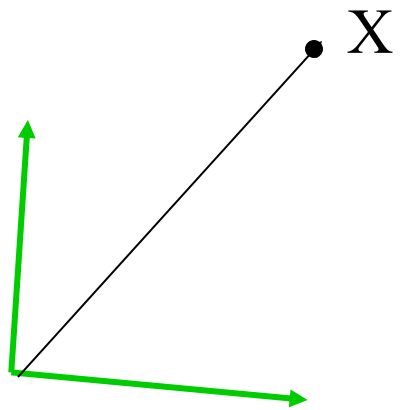
Camera coordinate frame

$$\tilde{\mathbf{0}}_{\text{cam}} = \mathtt{R}\tilde{\mathbf{C}} + \mathbf{t}$$
$$-\mathtt{R}\tilde{\mathbf{C}} = \mathbf{t}$$
$$\tilde{\mathbf{C}} = -\mathtt{R}^{\top}\mathbf{t}$$

$$\tilde{\mathbf{X}}_{\text{camera}} = \mathtt{R}\tilde{\mathbf{X}}_{\text{world}} + \mathbf{t}$$

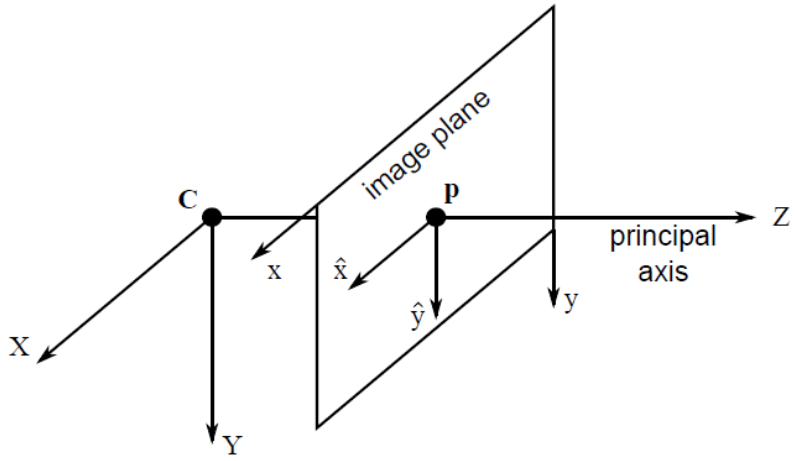
$$\begin{bmatrix} \tilde{\mathbf{X}}_{\text{camera}} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^{\top} & 1 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}}_{\text{world}} \\ 1 \end{bmatrix}$$

$$\mathbf{X}_{\text{camera}} = \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^{\top} & 1 \end{bmatrix} \mathbf{X}_{\text{world}}$$



World coordinate frame

Intrinsic parameters



$$\mathtt{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

where

$\alpha_x = f m_x$	Focal length in x direction in terms of pixel dimensions
$\alpha_y = f m_y$	Focal length in y direction in terms of pixel dimensions
s	Skew
$(x_0, y_0)^\top = (p_x m_x, p_y m_y)^\top$	Coordinates of principal point in terms of pixel dimensions

where m_x and m_y are number of pixels per unit disance in x and y directions, respectively

The projective camera

- Extrinsic parameters: Since the camera coordinate frame may not align with the world coordinate frame, there is a 3D Euclidean transformation from world coordinates to camera coordinates
- Intrinsic parameters: Since the scene units (e.g., cm) differ from the pixel coordinate frame units (i.e., pixels) and origin (i.e., upper left pixel), there is a 2D affine transformation comprised of focal length in x and y directions, skew (which is 0 in real cameras), and principal point, all in terms of pixel dimensions

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} \alpha_x & 0 & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_X \\ r_{21} & r_{22} & r_{23} & t_Y \\ r_{31} & r_{32} & r_{33} & t_Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ T \end{bmatrix}$$

Intrinsic parameters

Extrinsic parameters

The projective camera

- Projects 3D points in the world coordinates to 2D points in pixel coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} \alpha_x & 0 & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_X \\ r_{21} & r_{22} & r_{23} & t_Y \\ r_{31} & r_{32} & r_{33} & t_Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ T \end{bmatrix}$$

$$\mathbf{x} = \mathtt{K}[\mathtt{I} \,|\, \mathbf{0}] \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{X}$$

$$\mathbf{x} = \mathtt{K}[\mathtt{R} \,|\, \mathbf{t}]\mathbf{X}$$

$$\mathbf{x} = \mathtt{P}\mathbf{X}$$

Camera projection matrix

Linear projection of points in homogeneous coordinates

$$\mathbf{x} = \mathtt{P}\mathbf{X}$$

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ T \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} Xp_{11} + Yp_{12} + Zp_{13} + Tp_{14} \\ Xp_{21} + Yp_{22} + Zp_{23} + Tp_{24} \\ Xp_{31} + Yp_{32} + Zp_{33} + Tp_{34} \end{bmatrix}$$

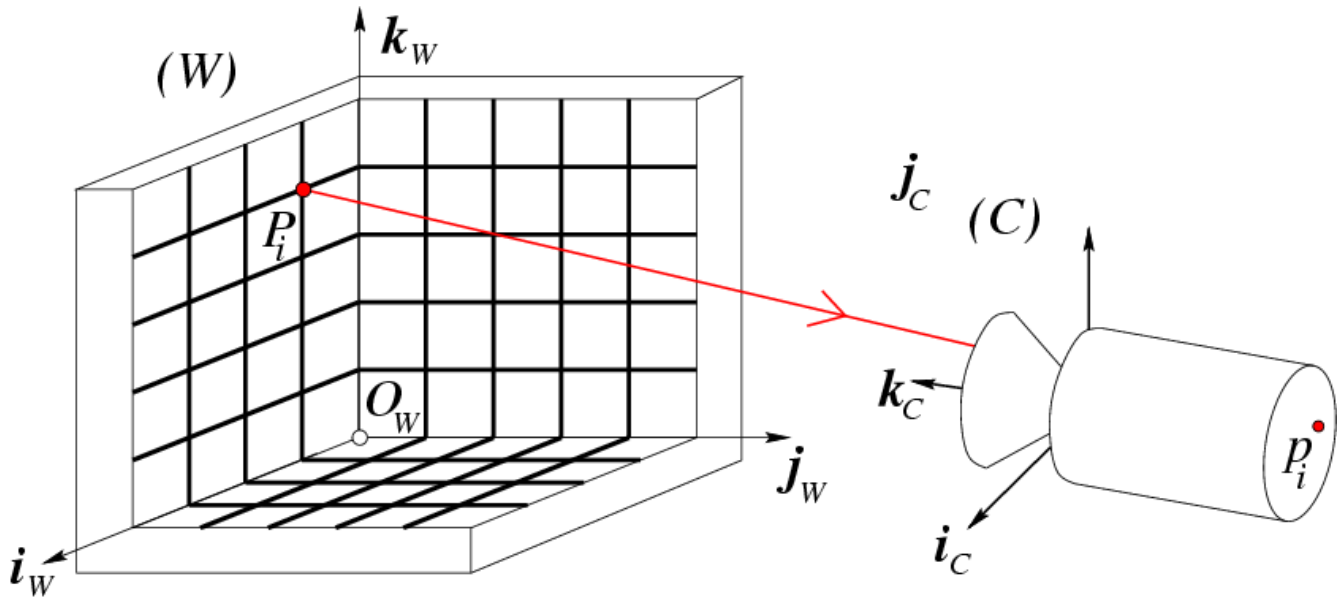
Nonlinear projection of points in inhomogeneous coordinates

$$\tilde{x} = \frac{x}{w} = \frac{Xp_{11} + Yp_{12} + Zp_{13} + Tp_{14}}{Xp_{31} + Yp_{32} + Zp_{33} + Tp_{34}} \text{ and}$$

$$\tilde{y} = \frac{y}{w} = \frac{Xp_{21} + Yp_{22} + Zp_{23} + Tp_{24}}{Xp_{31} + Yp_{32} + Zp_{33} + Tp_{34}}$$

Do not forget this is a nonlinear projection

Camera Calibration



- Given *n* points $\mathbf{P}_1,\ldots,\mathbf{P}_n$ with known 3-D position and their pixel coordinates $\mathbf{x}_1,\ldots,\mathbf{x}_n$, estimate intrinsic **K** and extrinsic camera parameters and lens distortion parameters
- See textbook for details
- Camera Calibration Toolbox for Matlab (Bouguet)

Archived

http://www.vision.caltech.edu/bouguetj/calib_doc/

Next Lecture

- Photometric image formation