

CSE 207B: Applied Cryptography

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UCSD

Fall 2025 Lecture 1

Course Mechanics

- Course Web Page:
<https://cseweb.ucsd.edu/classes/fa25/cse207B-a/>
 - Schedule, assignments, etc.
- Lectures: Tu/Th 2-3:20pm APM 2301
- Canvas: <https://canvas.ucsd.edu/courses/68532>
 - Gradebook
- Piazza: <https://piazza.com/class/fall12025/cse207b>
 - Asynchronous Q&A
- My office hours: Tuesday 3:30-4:30pm EBU3B 3138 or outside CSE depending on weather
- TA: Laura Shea
 - Office hours Monday 12:30-1:30pm B250A or outside CSE
 - Zoom OH: Thursdays 5:30-6:30pm, link on Piazza

Assignments and grading

100% Homework assignments

- Approximately 8 assignments, one a week
- Both programming and written exercises
- Submit to Gradescope by deadline

Course policies

- Collaboration policy:
 - Talk to your classmates about approaches, write your own code and solutions.
 - This is a grad class: Cite your sources.
 - Any use of AI tools is disallowed.
- Late policy:
 - We're all in this together, and we're all adults.
 - Please turn in your work on time and try not to push boundaries.
 - That said, if you have *unforeseen* extenuating circumstances let me know.
- Lecture modality:
 - Lectures are in person only.
 - I'm not taking attendance, but you'll get more from the class if you attend.
 - Please don't come to class if you are sick!

Textbook and other resources

We will mostly be following *Applied Cryptography* by Boneh and Shoup.

- Draft available here: <https://toc.cryptobook.us/>
- *Introduction to Cryptography* by Katz and Lindell also covers a lot of the same material.

Links to other textbooks and lots of papers for additional reading on web site.

Topics Covered

- Cryptographic security notions
 - Security definitions and reductions between concepts
 - Attacks exemplifying security notions
 - Focus on cryptography in broad use
- Symmetric cryptography
 - Stream ciphers
 - Block ciphers, modes of operation, chosen plaintext attacks
 - Message integrity, chosen ciphertext attacks
 - Hash functions, constructions and collision attacks
- Public-key cryptography
 - Basic computational number theory
 - Diffie-Hellman key exchange, discrete log security and attacks
 - RSA encryption, factoring hardness and attacks
 - Digital signatures: RSA, (EC)DSA, practical attacks
- Advanced/fun topics
 - Lattices
 - Post-quantum cryptography

Topics *not* covered

- Cryptocurrencies
- Blockchains
- NFTs
- Zero-knowledge proofs
- Threshold encryption
- ...

Prerequisites

- Mathematical maturity, CS-style reductions and proofs. This should not be the first time you see a mathematical proof.
- You should have done well in your algorithms and complexity classes.
- You will be happier in the second half of the course if you know what a group and field are.
- Programming challenges will be in Python3/Sage. This should not be your introduction to programming.

My work: Cryptographic systems security





Crypto shocker: four of every 1,000 public keys provide no security (updated)

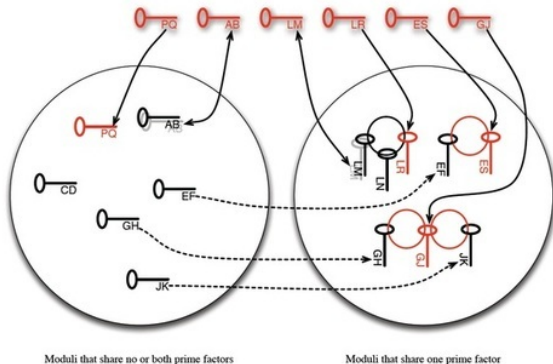
Almost 27,000 certificates used to protect webmail, e-commerce, and other ...

by Dan Goodin - Feb 15 2012, 7:00am EST

Share

Tweet

68



Keys that share one prime factor are vulnerable to cracking by anyone. Keys that share both prime factors can be broken by the other holder.

Researchers reveal a method the NSA may use to spy on Web traffic

By Sean Spósito | October 21, 2015 | Updated: October 21, 2015 5:05pm



RISK ASSESSMENT —

NSA could put undetectable “trapdoors” in millions of crypto keys

Technique allows attackers to passively decrypt Diffie-Hellman protected data.

DAN GOODIN - 10/11/2016, 7:30 AM

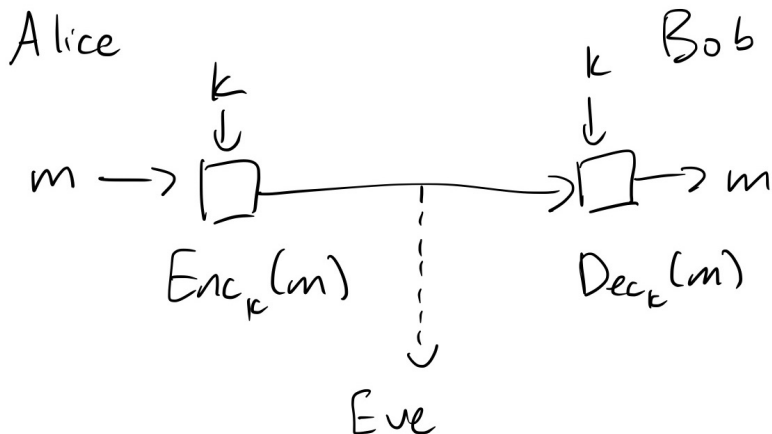


We'll see a lot of this stuff in this class.

Today's topics

- A little bit of classical cryptanalysis
- One-time pad

Cipher syntax



- Key generation: Generate key $k \in K$
- Encryption: $c = Enc_k(m)$
- Decryption: $m = Dec_k(c)$
- Correctness: $Dec_k(Enc_k(m)) = m$

Example: Substitution cipher

Alphabet $\Sigma = \{A, \dots, Z, \sqcup\}$

Key generation: Choose random permutation $k : \Sigma \rightarrow \Sigma$

Encryption: $\text{Enc}_k(m) = k[m[0]], k[m[1]], \dots, k[m[\ell - 1]] \quad m \in \Sigma^\ell$

Decryption: $\text{Dec}_k(c) = k^{-1}[c[0]], k^{-1}[c[1]], \dots, k^{-1}[c[\ell - 1]]$
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k^{-1} is inverse permutation of k

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- Correctness?
- Cryptanalysis?
 - Frequency analysis
 - Brute force?

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- Correctness?
- Cryptanalysis?
 - Frequency analysis
 - Brute force? No: $27! \approx 2^{93}$

Caesar Cipher in the wild

- 2006: Mafia boss Bernardo Provenzano used Caesar cipher encoded with numbers
 - <http://itre.cis.upenn.edu/~myl/languagelog/archives/003049.html>
- 2011: Rajib Karim plotted to blow up airplanes using Excel Caesar cipher
 - https://www.theregister.co.uk/2011/03/22/ba_jihadist_trial_sentencing/

Kerckhoff's Principle

Auguste Kerckhoff 1883

- Encryption scheme should not be a secret
- Only key needs to be a secret
- For modern cryptography: Algorithms should be public, standardized, and scrutinized in public

One-time pad

Alphabet: $\{0, 1\}$

Key generation: $k \in_R \{0, 1\}^\ell$

Encryption: $m \in \{0, 1\}^\ell \quad c = k \oplus m$

Decryption: $m = c \oplus k$

Correctness?

One-time pad

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Correctness? $\text{Dec}_k(\text{Enc}_k(m)) = k \oplus m \oplus k = m$

What does “secure” mean?

What does “secure” mean?

- No adversary can compute secret key from ciphertext?
- No adversary can compute plaintext from ciphertext?
- No adversary can determine a character of plaintext?
- No adversary can derive meaningful information from ciphertext?
- No adversary can compute any function of plaintext from ciphertext?

Perfect secrecy

Definition (Perfect secrecy)

Let (Enc, Dec) be a cipher over (K, M, C) .

Let k be a random variable uniformly distributed over K .

$\forall m_0, m_1 \in M, \forall c \in C$:

$$\Pr[\text{Enc}_k(m_0) = c] = \Pr[\text{Enc}_k(m_1) = c]$$

Theorem

The following are equivalent:

- (1) (Enc, Dec) is perfectly secure.*
- (2) $\forall c \in C, \exists N_c$ s.t. $\forall m \in M, |\{k \in K \mid \text{Enc}_k(m) = c\}| = N_c$*
- (3) Let k be a random variable uniformly distributed over K .
Then $\text{Enc}_k(m)$ has the same distribution for all m .*

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Proof.

$$(2) \implies (3) \Pr(\text{Enc}_k(m) = c) = N_c/|K|$$

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Proof.

$$(2) \implies (3) \Pr(\text{Enc}_k(m) = c) = N_c/|K|$$

$$(1) \implies (2) \text{ Choose } m_0. \text{ By (1),}$$

$$\Pr[\text{Enc}_k(m_0) = c] = \Pr[\text{Enc}_k(m) = c] \quad \text{for any } m \in M$$

$$\text{and } \Pr[\text{Enc}_k(m_0) = c] = N_c/|K|.$$

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- (1) (Enc, Dec) is perfectly secure.
- (2) $\forall c \in C, \exists N_c$ s.t. $\forall m \in M, |\{k \in K \mid Enc_k(m) = c\}| = N_c$
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Then $Enc_k(m)$ has the same distribution for all m .

Proof.

$$(2) \implies (3) \Pr(Enc_k(m) = c) = N_c/|K|$$

$$(1) \implies (2) \text{ Choose } m_0. \text{ By (1),}$$

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$$\text{and } \Pr[Enc_k(m_0) = c] = N_c/|K|.$$

$$(3) \implies (1) \text{ Fix } m_0, m_1 \in M, c \in C. \text{ By (3),}$$

$$\Pr[Enc_k(m_0) = c] = P_c = \Pr[Enc_k(m_1) = c]$$



Theorem

The one-time pad is perfectly secret.

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The one-time pad is perfectly secret.

Proof.

Fix $m \in \{0, 1\}^\ell$, $c \in \{0, 1\}^\ell$.

\exists unique $k \in \{0, 1\}^\ell$ satisfying $k \oplus m = c$, $k = m \oplus c$.

Thus $N_c = 1$ in above theorem, part (2).



Example: Variable-length one-time pad

$$k \in \{0, 1\}^L, m, c \in \{0, 1\}^{\ell \leq L}$$

$$\text{Enc}_k(m) = k[0, \dots, \ell - 1] \oplus m$$

$$\text{Dec}_k(c) = k[0, \dots, \ell - 1] \oplus c$$

Example: Variable-length one-time pad

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c has length 1.

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Let m_0 have length 1, m_1 have length 2.

c has length 1.

$$\Pr[\text{Enc}_k(m_0) = c] = 1/2 \quad \Pr[\text{Enc}_k(m_1) = c] = 0$$

\implies Not perfectly secret.

Theorem (Shannon)

(Enc, Dec) perfectly secret $\implies |K| \geq |M|$.

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Proof.

Show $|K| < |M| \implies$ not perfectly secret.

Choose arbitrary m_0, k_0 . Let $c = \text{Enc}_{k_0}(m_0)$.

Let $S = \{\text{Dec}_k(c) \mid k \in K\}$.

$|S| \leq |K| < |M|$.

$\implies \exists m_1 \in M \setminus S$.

$$\left. \begin{array}{l} \Pr[\text{Enc}_k(m_1) = c] = 0 \\ \Pr[\text{Enc}_k(m_0) = c] > 0 \end{array} \right\} \text{not perfectly secret}$$



One-time pad too difficult to use in practice: Basically only used by spies.

Numerous usability problems, including reusing one-time pad material.

Recent example: Russian spies using Cuban “numbers” station, but there was a bug and dummy “fill” traffic was missing the number 9!

According to Matt Blaze, this let the FBI tell when messages were transmitted to the spies.

<https://www.mattblaze.org/blog/neinnines/>

Your first assignment is already online! Break “one-time pad” where pad is reused multiple times.

We recommend you start early. Due in 1.5 weeks.

Next time: Relax security definitions to be more usable in the real world.