Homework 7

CSE 105, Fall 2025
Due: Friday, December 5, 11:59 pm

Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning, where applicable. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

Instructions:

- Read each question carefully.
- Selection of questions
    - You are required to answer one question from the first topic and **three** questions from the second topic in this homework.
    - You may choose any three questions from the second topic.
- Each question is worth 5 points (20 points total).
- Submission guidelines:
    - Upload a single file to Gradescope for each group.
    - All group members' names and PIDs should be on each page of the submission.
    - Include the text of each question you are answering, followed by your solution.

Topics:

1. *Undecidability of* E_{TM}
2. *Undecidability of Other Languages*

*For this **section**, you may use, without proof, the following languages:*

$$A_{TM} = \{< M, w > \mid M \text{ is a TM and } M \text{ accepts } w\}$$

$$HALT_{TM} = \{< M, w > \mid M \text{ is a TM and } M \text{ halts on } w\}$$

are undecidable.

Problem 1:

Given:

$$E_{TM} = \{< M > \mid M \text{ is a TM and } L(M) = \emptyset\}$$

Prove that E_{TM} is undecidable.

*For this **section**, you may use, without proof, the following languages:*

$$A_{TM} = \{< M, w > \mid M \text{ is a TM and } M \text{ accepts } w\}$$

$$HALT_{TM} = \{< M, w > \mid M \text{ is a TM and } M \text{ halts on } w\}$$

$$E_{TM} = \{< M > \mid M \text{ is a TM and } L(M) = \emptyset\}$$

are undecidable.

Problem 2:

Given:

$$EQ_{TM} = \{< M_1, M_2 > \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) = L(M_2)\}$$

Prove that EQ_{TM} is undecidable. *Hint: use* E_{TM}

Problem 3:

Given:

$$REGULAR_{TM} = \{< M > \mid M \text{ is a TM and } L(M) \text{ is a regular language}\}$$

Prove that $REGULAR_{TM}$ is undecidable.

Problem 4:

Given:

$$FIVE_{TM} = \{< M > \mid M \text{ is a TM and } |L(M)| = 5\}$$

Prove that $FIVE_{TM}$ is undecidable.

Problem 5:

Given:

$$010_{TM} = \{< M > \mid M \text{ is a TM and } 010 \in L(M)\}$$

Prove that 010_{TM} is undecidable.

Problem 6:

Given:

$$DISJOINT_{TM} = \{< M_1, M_2 > \mid M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) \cap L(M_2) = \emptyset\}$$

Prove that $DISJOINT_{TM}$ is undecidable.

Problem 7:

Given:

$$ALL_{TM} = \{< M > | M \text{ is a TM and } L(M) = \Sigma^*\}$$

Prove that ALL_{TM} is undecidable.

Problem 8:

Given:

$$CONTAIN_{TM} = \{< M_1, M_2 > | M_1 \text{ and } M_2 \text{ are TMs and } L(M_1) \subseteq L(M_2)\}$$

Prove that $CONTAIN_{TM}$ is undecidable.

Problem 9:

Given:

$$TOTAL_{TM} = \{< M > | M \text{ is a TM that halts on all inputs}\}$$

Prove that $TOTAL_{TM}$ is undecidable.

Problem 10:

A state q in a Turing machine M is called “unreachable” if there is no input w to M such that when running M(w), we reach the state q at some point in the computation.

Given:

$$UNREACHABLE_{TM} = \{< M, q > | M \text{ is a TM and } q \text{ is an unreachable state of } M\}$$

Prove that $UNREACHABLE_{TM}$ is undecidable.