Homework 4

CSE 105, Fall 2025
Due: Monday November 3, 11:59pm

Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning, where applicable. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

Instructions:

- Read each question carefully.
- Selection of questions
  - You are required to answer <u>one question from each topic</u> in this homework.
  - You may choose any one question from the set of questions within each topic.
- Each question is worth 5 points (25 points total).
- Submission guidelines:
  - Upload a single file to Gradescope for each group.
  - All group members' names and PIDs should be on each page of the submission.
  - Include the text of each question you are answering, followed by your solution.

Topics:

1. Pumping lemma
2. Context-free grammars
3. Push-down Automata
4. Closure property for CFGs
5. Pumping lemma for CFGs

Problem 1:

Prove using the pumping lemma that the language L is not regular.

$$L = \{uvu \mid u, v \in \{0, 1\}^*,\ u \neq \epsilon\}$$

Problem 2:

Consider the language $L = \{a^p \mid p \text{ is prime}\}$
Use the Pumping Lemma to show that L is not regular.

Problem 3:

Determine whether L is regular. Justify your answer using the pumping lemma.

$$L = \{ w \in \{0, 1\} \mid w \text{ has equal number of } 0s \text{ and } 1s\}$$

Problem 4:

Let, $L = \{a^m b^n \mid m \neq n;\ m, n \geq 0\}$ Is L regular? Prove your claim.

Problem 5:

Prove or disprove: L is regular. Use the Pumping Lemma in your reasoning.

$$L = \{ww \mid w \in \{0, 1\}^*\}$$

Problem 6:

Construct a CFG for the language $L = \{ 0^i 1^j 2^k \mid i + j = k\}$

Problem 7:

Construct a CFG for the language $L = \{a^n b^m \mid n \leq m \}$

Problem 8:

Design a CFG for the set of strings over $\{a, b\}$ with equal numbers of a's and b's.

Problem 9:

Construct a CFG for the language $L = \{a^n b^m c^m d^n \mid m, n \geq 0 \}$

Problem 10:

Construct a CFG for the language $L = \{a^m b^n \mid m \neq n;\ m, n \geq 0 \}$

Problem 11:

Construct a PDA (formal definition or diagram) for the language $L = \{a^n b^n \mid n \geq 0\}$

Problem 12:

Give a PDA that accepts $L = \{ a^n b^m c^n \mid n, m \geq 0\}$

Problem 13:

Design a PDA that accepts the language $L = \{w \in \{a, b\}^* \mid w \text{ is a palindrome}\}$

Problem 14:

Design a PDA that accepts the language $L = \{ 0^i 1^j 2^k \mid i + j = k\}$

Problem 15:

Consider the language $L = \{ a^i b^j c^k \mid i, j, k \geq 0;\ i = j \text{ or } j = k\}$.
Prove that L is context-free by describing a PDA that accepts it.

Problem 16:

Let $L_1 = \{a^n b^n \mid n \geq 0\}$, $L_2 = \{a^i b^j \mid i, j \geq 0 \text{ and } i \text{ is even}\}$
Use closure properties to determine whether $L_1 \cup L_2$ is context-free.

Problem 17:

Consider two CFGs

G_1 generating $L(G_1) = \{a^m b^m \mid m \geq 0\}$

G_2 generating $L(G_2) = \{b^n c^n \mid n \geq 0\}$

Construct a CFG that generates $L(G_1)L(G_2)$ – their concatenation.

Explain briefly why the class of CFLs is closed under concatenation.

Problem 18:

Let L be a context-free language and R a regular language. Prove that $L \cap R$ is context-free.

Give a brief explanation or sketch of how a PDA for L and a DFA for R can be combined to recognize this intersection.

Problem 19:

Let $L = \{a^i b^j c^k \mid i, j, k \geq 0,\ i = j \text{ or } j = k\}$.
Show that L can be expressed as the union of two simpler CFLs and conclude that L itself is context-free.

Problem 20:

We have seen earlier that the pumping lemma can be used to show that a language is not regular. Interestingly, with some modifications, the pumping lemma can also be used to prove whether a language is context-free.

The **pumping lemma for context-free languages** is as follows (see Theorem 2.34 in the book):

> For any context-free language *L*, there exists a number p, called the pumping length, such that for any string $w \in L$ which has length greater than *p*, can be written in the form *w=uvxyz*, so that either *v* or *y* is non-empty, and $uv^ixy^iz \in L$, for all $i \geq 0$. i.e.,
>
> 1. $|vxy| \leq p$
> 2. $|vy| > 0$
> 3. $uv^ixy^iz \in L, \ \forall \ i \geq 0$

Consider the language $L = \{a^i b^i c^i \mid i \geq 0\}$.
Use the pumping lemma for context free languages to prove that L is not context-free.