

Homework 2

CSE 105, Fall 2025

Due: Monday October 13, 11:59pm

Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning, where applicable. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

Instructions:

- Read each question carefully.
- Selection of questions
 - You are required to answer one question from the first 3 topics and two questions from the last topic in this homework.
 - You may choose any question from the set of questions within each topic.
- Each question is worth 5 points (25 points total).
- Submission guidelines:
 - Upload a single file to Gradescope for each group.
 - All group members' names and PIDs should be on each page of the submission.
 - Include the text of each question you are answering, followed by your solution.

Topics:

1. Proof-Based Questions (Closure Properties & Regularity)
2. Direct NFA Constructions
3. NFAs with Operations (Union, Intersection, Concatenation)
4. Proving Regularity (Advanced/Reasoning)

Section 1 – Proof-Based Questions (Closure Properties & Regularity)

Problem 1:

Proof of Regularity (Finite Languages)

Prove that if L is a finite language over some alphabet Σ , then L is regular.
(Short-answer, rigorous proof required.)

Problem 2:

Closure under Union

Let L_1 and L_2 be regular languages. Prove formally that $L_1 \cup L_2$ is regular.

Problem 3:

Closure under Complementation

Suppose L is regular. Prove that its complement language \bar{L} is regular.

Problem 4:

Prove Regularity of Cofinite Sets

A language $L \subset \Sigma^*$ is cofinite if its complement $\Sigma^* \setminus L$ is finite. Prove that every cofinite language is regular.

Provide a state diagram and clearly show the start/accept states. No proof needed.

Problem 5:

Prefix/Suffix Recognition

Construct an NFA over $\{0,1\}$ that accepts strings that begin with 00 or end with 101.

Problem 6:

Substring Recognition

Build an NFA for $L = \{w \in \{0,1\}^* \mid w \text{ contains } 010 \text{ as a substring}\}$.

Problem 7:

Multiple Ends

Construct an NFA for $L = \{w \mid w \text{ ends in either } 00 \text{ or } 11\}$.

Problem 8:

At-Least-One Property

Construct an NFA that accepts all strings over $\{a,b\}$ in which at least one a appears before some b .

(Example: $aab, abb, abba$ are in the language, but bbb, aa are not.)

Section 3 – NFAs with Operations (Union, Intersection, Concatenation)

Provide a state diagram and clearly show the start/accept states. No proof needed.

Problem 9:

Intersection via NFA

Let

- $A = \{w \in \{0, 1\}^* \mid w \text{ starts with } 01\}$,
- $B = \{w \in \{0, 1\}^* \mid w \text{ ends with } 10\}$.

Construct an NFA for $A \cap B$.

Problem 10:

Concatenation via NFA

Over $\Sigma = \{0, 1\}$, let

- $L_1 = \{w \mid w \text{ contains exactly one } 0\}$,
- $L_2 = \{w \mid w \text{ ends in } 11\}$.

Construct an NFA for $L_1 \cdot L_2$ (to recall, $L_1 \cdot L_2 = \{xy : x \in L_1, y \in L_2\}$)

Problem 11:

Union via NFA

Build an NFA for $L_1 \cup L_2$, where

- $L_1 = \{w \mid w \text{ has odd length}\}$,
- $L_2 = \{w \mid w \text{ begins with } 1\}$.

Problem 12:

Without constructing a full automaton, explain why closure under union is easier to prove using NFAs than DFAs.

Section 4 – Proving Regularity (Advanced/Reasoning)

Give full proofs in this section.

You may use the fact we will learn later in the class that DFAs and NFAs are equivalent. Namely, to prove that a language is regular, it is sufficient to construct an NFA for it.

Problem 13:

Shuffle Operation

Let L_1, L_2 be regular languages. Define their shuffle:

$$\text{shuffle}(L_1, L_2) = \{x_1y_1x_2y_2\dots x_ny_n \mid x_1\dots x_n \in L_1, y_1\dots y_n \in L_2\}.$$

Prove that the shuffle of two regular languages is regular.

That is, given DFAs for L_1, L_2 , construct a DFA or NFA for $\text{shuffle}(L_1, L_2)$.

Problem 14:

Reverse Operation

Prove that if L is regular, then so is its reverse language L^R , where we reverse all the words (for example, the reverse of cat is tac).

That is, given a DFA for L , construct a DFA or NFA for L^R .

Problem 15:

Reasoning

For each of the following operations, determine whether it always preserves regularity. Give a clear explanation and, if it does not, provide a counterexample.

- Complement
- Intersection
- Reversal

Problem 16:

Let L be a regular language. Define

$$T(L) = \{yx \mid x, y \in \Sigma^*, xy \in L\}$$

Prove that $T(L)$ is regular.

That is, given a DFA for L , construct a DFA or NFA for $T(L)$.