

# Homework 1

CSE 105, Fall 2025

Due: Monday October 6, 11:59pm

Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning, where applicable. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

## Instructions:

- Read each question carefully.
- Selection of questions
  - You are required to answer one question from each topic in this homework.
  - You may choose any one question from the set of questions within each topic.
- Each question is worth 5 points (20 points total).
- Submission guidelines:
  - Upload a single file to Gradescope for each group.
  - All group members' names and PIDs should be on each page of the submission.

## Topics:

1. Computing a string on a DFA
2. Drawing a DFA
3. Describing what a DFA does
4. Formally defining a DFA

### Problem 1:

Consider DFA  $M = (Q = \{q_0, q_1, q_2\}, \Sigma = \{a, b\}, \delta, q_0 = q_0, F = \{q_2\})$  with transitions:

- $\delta(q_0, a) = q_1, \delta(q_0, b) = q_0$
- $\delta(q_1, a) = q_2, \delta(q_1, b) = q_0$
- $\delta(q_2, a) = q_2, \delta(q_2, b) = q_2$

Trace the computation on input  $w = abba$ .

List the sequence of states visited (including start state) and state whether the input  $w$  is accepted.

### Problem 2:

Let DFA  $M (Q = \{q_0, q_1, q_2\}, \Sigma = \{0, 1\}, \delta, q_0 = q_0, F = \{q_2\})$  with transitions:

- $\delta(q_0, 0) = q_0, \delta(q_0, 1) = q_1$
- $\delta(q_1, 0) = q_2, \delta(q_1, 1) = q_0$
- $\delta(q_2, 0) = q_2, \delta(q_2, 1) = q_2$

Trace the computation on input  $w = 10110$ . Does  $M$  accept or reject  $w$ ?

### Problem 3:

Consider DFA  $N (Q = \{s, t\}, \Sigma = \{0, 1\}, \delta, q_0 = s, F = \{t\})$  with transitions:

- $\delta(s, 0) = s, \delta(s, 1) = t$
- $\delta(t, 0) = t, \delta(t, 1) = t$

For the string 0010101, list the states visited and determine acceptance.

Explain in one sentence what feature this DFA accepts (i.e., what property of input makes it accept).

### Problem 4:

Consider a DFA  $X$  over the alphabet  $\{a, b\}$  with states  $\{q_0, q_1\}$ , start state  $q_0$  and accept states  $\{q_0\}$ . Transitions:

- On input  $a$  toggle between  $q_0$  and  $q_1$
- On input  $b$  stay in current state

Trace the machine on  $aababa$ . Is the string accepted? Explain.

Provide a state diagram and clearly show the start/accept states.

**Problem 5:**

Draw a DFA over the alphabet  $\{0,1\}$  that accepts all strings containing an even number of 0's.

**Problem 6:**

Draw a DFA accepting the following language over the alphabet  $\{0, 1\}$ :  
 $\{w \mid w \text{ contains at least three 1s}\}$ :

**Problem 7:**

Draw a DFA accepting the following language over the alphabet  $\{0, 1\}$ :  
 $\{w \mid w \text{ does not contain the substring } 110\}$ :

**Problem 8:**

Draw a DFA over  $\{0,1\}$  that accepts strings with exactly two 1's.

**Problem 9:**

Consider DFA  $S$  ( $Q = \{r, s\}, \Sigma = \{0, 1\}, \delta, q_0 = r, F = \{s\}$ )

- $\delta(r, 0) = r, \delta(r, 1) = s$
- $\delta(s, 0) = r, \delta(s, 1) = s$

Describe in words the set of strings accepted by  $S$ .  
Give two examples of strings  $S$  accepts and two it rejects.

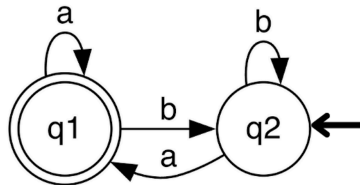
**Problem 10:**

A DFA  $M$  has three states and accepts exactly the language of strings whose length is divisible by 3. Which of the following is true? Briefly explain your reasoning.

- a.  $M$  must have at least 3 states
- b.  $M$  must be non-deterministic
- c. A 2-state DFA can recognize this language
- d. Construction of  $M$  is not possible

**Problem 11:**

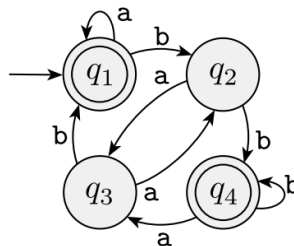
Consider the DFA over  $\{a, b\}$  below. Explain what type of strings this DFA accepts?



*Hint: List down 5 strings this DFA accepts. Do you notice a pattern?*

**Problem 12:**

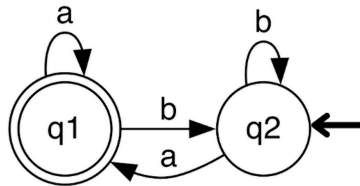
The following is a state diagram of a DFA. Answer the following questions about this machine:



- (a) What is the start state?
- (b) What is the set of accept states?
- (c) Does the machine accept the string  $\epsilon$  ?

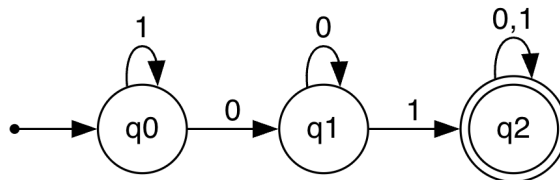
**Problem 13:**

Given the DFA below, write its 5-tuple  $(Q, \Sigma, \delta, q_0, F)$  explicitly. Use a table to define  $\delta$ .



**Problem 14:**

Given the DFA below, write its 5-tuple  $(Q, \Sigma, \delta, q_0, F)$  explicitly. Use a table to define  $\delta$ .



**Problem 15:**

Consider the description of a DFA as follows:

$$Q = \{s\} \mid \Sigma = \{0,1\} \mid \delta(s,0) = \emptyset \mid q_0 = s \mid F = \{s\}$$

Explain what's wrong (if anything) and how to fix it to be a valid DFA.

**Problem 16:**

Describe what constraints the transition function ( $\delta$ ) must satisfy for a DFA.

Give an example of an invalid transition specification.