Calibrated Stereo (Part 1)

Computer Vision I CSE 252A Lecture 7

CSE 252A, Fall 2023

Announcements

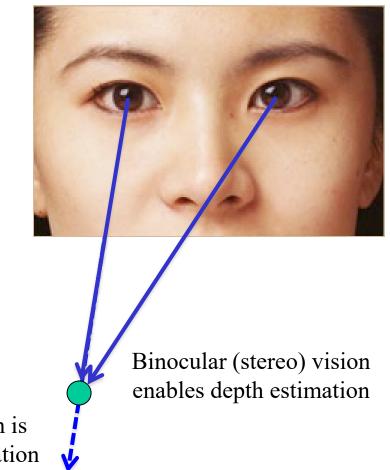
- Assignment 1 is due Oct 25, 11:59 PM
- Assignment 2 will be released Oct 25
 Due Nov 8, 11:59 PM

Why Do We Have Two Eyes?



- 1. Redundancy If we lose one, we're not blind
- 2. Larger field of view
- 3. Ability to recover depth for some points

Why Do We Have Two Eyes?



Depth information is lost in image formation

Stereo Vision

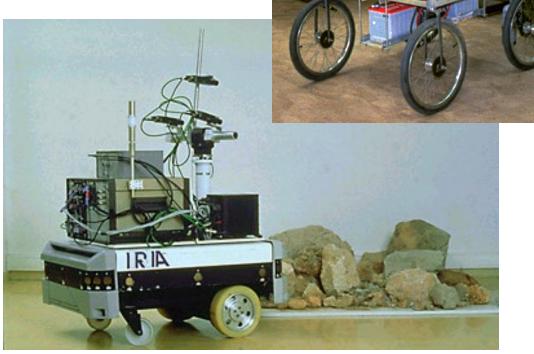


Holmes Stereoscope

An Application: Mobile Robot Navigation



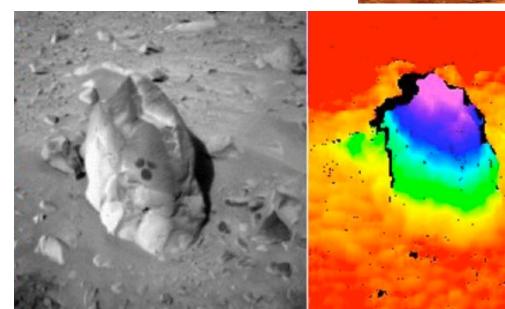
The Stanford Cart, H. Moravec, 1979



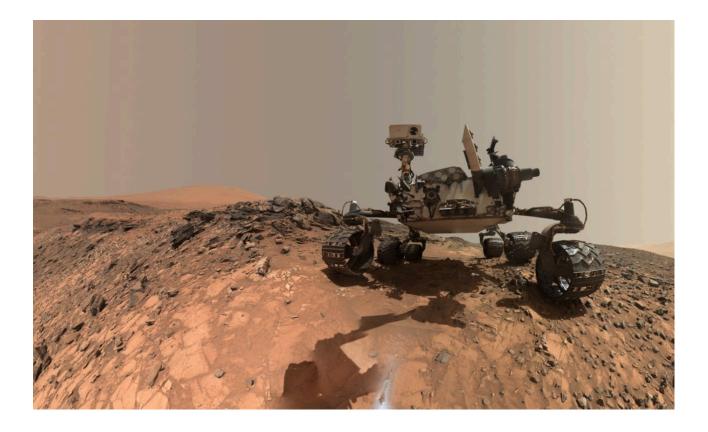
Mobi, Stanford, 1987 CSE 252A, Fall 2023 INRIA Mobile Robot 1990

Mars Exploratory Rovers: Spirit and Opportunity, 2004



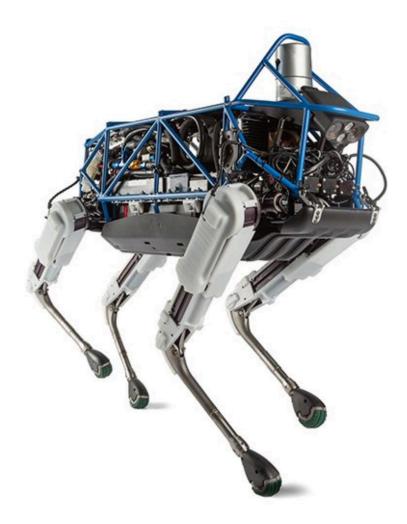


Curiosity Rover (2012)

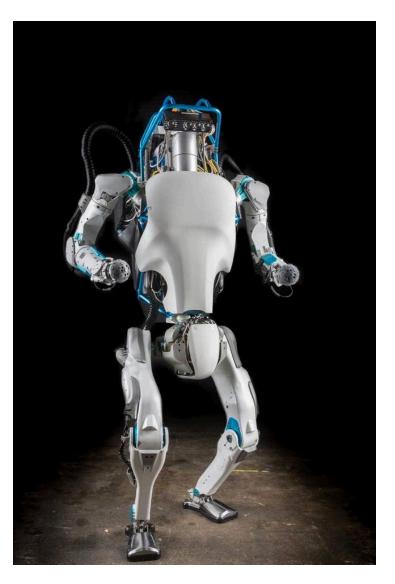


- Navigation cameras (Navcams) B&W, 45° field of view
- Hazard avoidance cameras (hazcams), 4 pairs, 120° field of view

Boston Dynamics



Stereo + Lidar

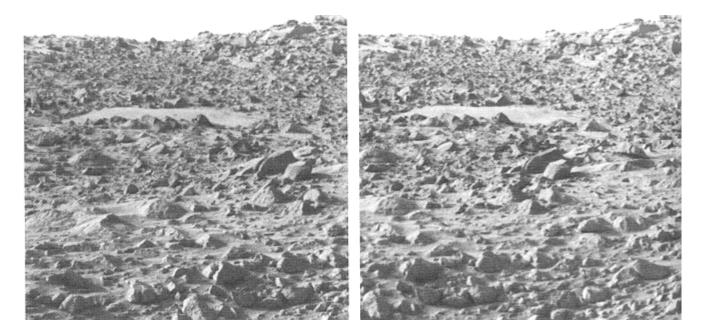


Commercial Stereo Heads



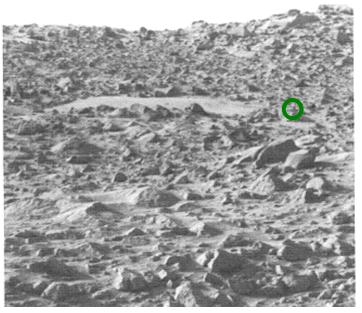


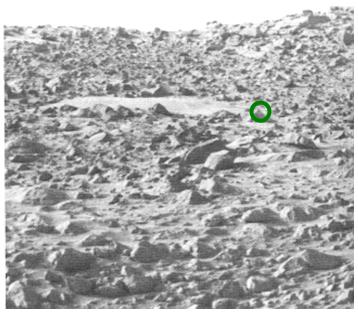
Binocular Stereopsis: Mars Given two images of a scene where relative locations of cameras are known, estimate depth of all common scene points.



Two images of Mars (Viking Lander)

Matching complexity (naïve)





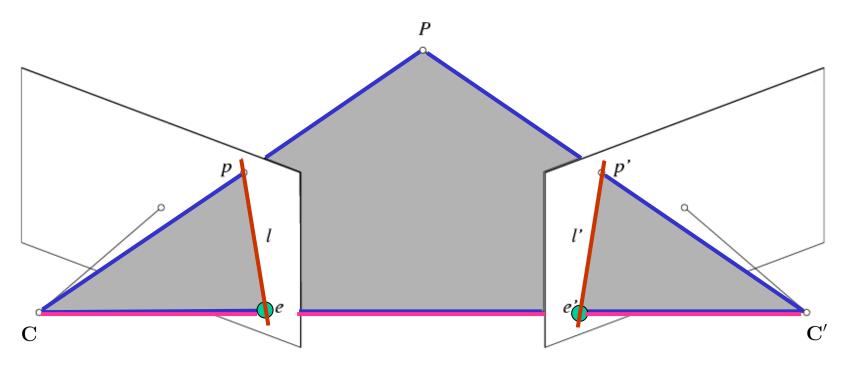
Input: two images that are n x n pixels

For a given point in the left image, where do we look in the right image? For each point in left mage, there are $O(n^2)$ possible matching points in right image.

With n² pixels in left image, complexity of matching is O(n⁴)

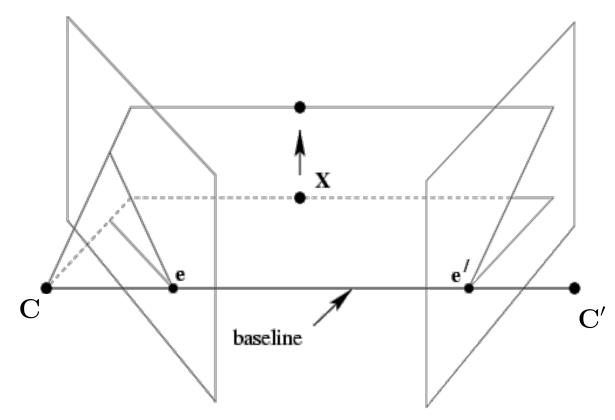
Can we do better?

Epipolar Geometry Terminology



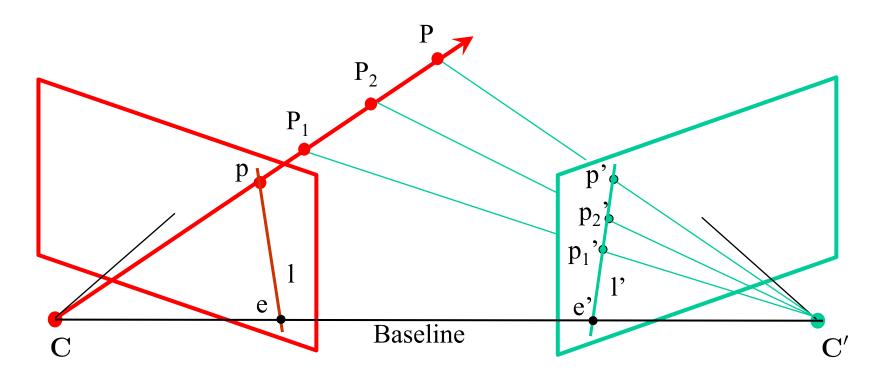
- Baseline: line connecting camera centers (of projection) C and C'
- Epipoles (e, e'): Two intersection points of baseline with image planes
- Epipolar Plane: Any plane that contains the baseline
- Epipolar Lines (I, I'): Pair of lines from intersection of an epipolar plane with the two image planes CSE 252A, Fall 2023

Family of Epipolar Planes



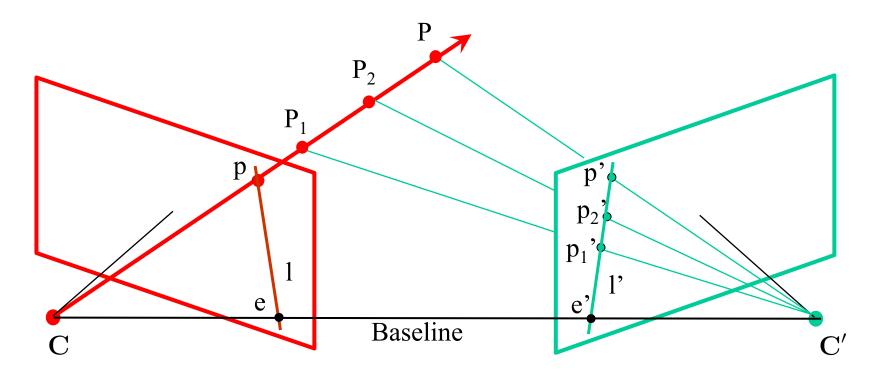
- Epipolar Plane: Any plane that contains the baseline
- The set of epipolar planes is a family of all planes passing through the baseline and can be parameterized by the angle about baseline

Epipolar matching



- Potential matches for **p** have to lie on the corresponding epipolar line **l**'
- Epipolar line l' passes through epipole e', the intersection of the baseline with the image plane
- Potential matches for **p**' have to lie on the corresponding epipolar line **I**

Epipolar matching complexity



Using epipolar matching, complexity is reduced from $O(n^4)$ to $O(n^3)$. Why?

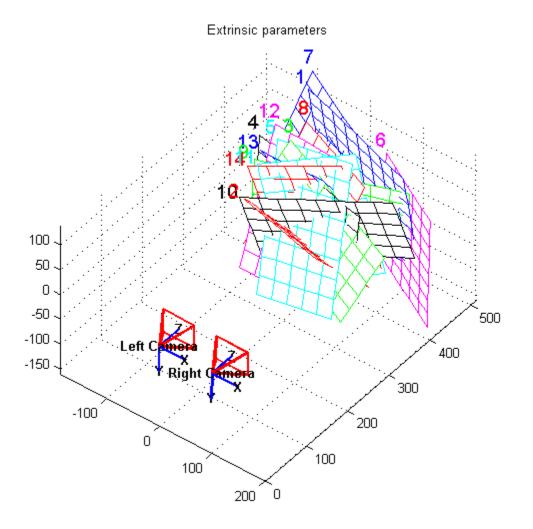
- There are n² points in the left image
- For each point in the left image, all candidate matches are on an epipolar line in the right image, and the length of the epipolar line is O(n)
- Therefore, match complexity is $O(n^{2*}n) = O(n^3)$

Stereo Vision Outline

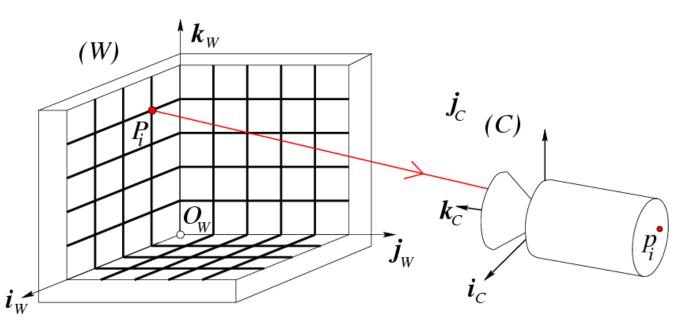
- Offline
 - Calibration of stereo cameras
- Online
 - 1. Acquire stereo images
 - 2. Epipolar rectify stereo images
 - 3. Establish correspondence
 - 4. Estimate depth

Calibration of stereo cameras

- From images of known calibration fixture, determine intrinsic parameters K₁, K₂ and extrinsic relation of two cameras R₁, t₁ and R₂, t₂
- 2. Compute the relative rotation R and translation t of the two cameras from R₁, t₁ and R₂, t₂
 3. Compute the essential matrix E



Camera calibration



- Given *n* points P₁,..., P_n with known 3-D position and their pixel coordinates x₁,..., x_n, estimate intrinsic K and extrinsic camera parameters and lens distortion parameters
- See textbook for details
- Camera Calibration Toolbox for Matlab (Bouguet)



http://www.vision.caltech.edu/bouguetj/calib_doc/

Compute the rotation and translation of the second camera relative to the first one

$$\begin{split} \mathbf{x}_1 &= \mathtt{K}_1[\mathtt{I} \mid \mathbf{0}] \begin{bmatrix} \mathtt{R}_1 & \mathbf{t}_1 \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{X} \\ \mathbf{x}_1 &= \mathtt{K}_1[\mathtt{I} \mid \mathbf{0}] \mathbf{X}_{\mathrm{cam},1} \end{split}$$

where
$$\mathbf{X}_{\text{cam},1} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{t}_1 \\ \mathbf{0}^{\top} & 1 \end{bmatrix} \mathbf{X}$$

 $\begin{bmatrix} \mathbf{R}_1 & \mathbf{t}_1 \\ \mathbf{0}^{\top} & 1 \end{bmatrix}^{-1} \mathbf{X}_{\text{cam},1} = \mathbf{X}$

$$\begin{aligned} \mathbf{x}_{2} &= \mathtt{K}_{2}[\mathtt{I} \mid \mathbf{0}] \begin{bmatrix} \mathtt{R}_{2} & \mathtt{t}_{2} \\ \mathtt{0}^{\top} & \mathtt{1} \end{bmatrix} \mathtt{X} \\ \mathbf{x}_{2} &= \mathtt{K}_{2}[\mathtt{I} \mid \mathtt{0}] \begin{bmatrix} \mathtt{R}_{2} & \mathtt{t}_{2} \\ \mathtt{0}^{\top} & \mathtt{1} \end{bmatrix} \begin{bmatrix} \mathtt{R}_{1} & \mathtt{t}_{1} \\ \mathtt{0}^{\top} & \mathtt{1} \end{bmatrix}^{-1} \mathtt{X}_{\mathrm{cam},1} \\ \mathbf{x}_{2} &= \mathtt{K}_{2}[\mathtt{I} \mid \mathtt{0}] \begin{bmatrix} \mathtt{R}_{2} & \mathtt{t}_{2} \\ \mathtt{0}^{\top} & \mathtt{1} \end{bmatrix} \begin{bmatrix} \mathtt{R}_{1}^{\top} & -\mathtt{R}_{1}^{\top} \mathtt{t}_{1} \\ \mathtt{0}^{\top} & \mathtt{1} \end{bmatrix} \mathtt{X}_{\mathrm{cam},1} \\ \mathbf{x}_{2} &= \mathtt{K}_{2}[\mathtt{I} \mid \mathtt{0}] \begin{bmatrix} \mathtt{R}_{2}\mathtt{R}_{1}^{\top} & \mathtt{t}_{2} - \mathtt{R}_{2}\mathtt{R}_{1}^{\top} \mathtt{t}_{1} \\ \mathtt{0}^{\top} & \mathtt{1} \end{bmatrix} \mathtt{X}_{\mathrm{cam},1} \\ \mathbf{x}_{2} &= \mathtt{K}_{2}[\mathtt{I} \mid \mathtt{0}] \begin{bmatrix} \mathtt{R}_{2}\mathtt{R}_{1}^{\top} & \mathtt{t}_{2} - \mathtt{R}_{2}\mathtt{R}_{1}^{\top} \mathtt{t}_{1} \\ \mathtt{0}^{\top} & \mathtt{1} \end{bmatrix} \mathtt{X}_{\mathrm{cam},1} \\ \mathbf{x}_{2} &= \mathtt{K}_{2}[\mathtt{I} \mid \mathtt{0}] \begin{bmatrix} \mathtt{R} & \mathtt{t} \\ \mathtt{0}^{\top} & \mathtt{1} \end{bmatrix} \mathtt{X}_{\mathrm{cam},1} \end{aligned}$$

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Image points

• Image points in pixel coordinates

$$\begin{split} \mathbf{x} &= \mathtt{K}[\mathtt{I} \mid \mathbf{0}] \begin{bmatrix} \mathtt{R} & \mathtt{t} \\ \mathtt{0}^\top & \mathtt{1} \end{bmatrix} \mathbf{X} \\ \mathbf{x} &= \mathtt{K}[\mathtt{R} \mid \mathtt{t}] \mathbf{X} \end{split}$$

• Image points in normalized coordinates

$$\begin{aligned} \mathbf{x} &= \mathtt{K}[\mathtt{R} \,|\, \mathbf{t}] \mathbf{X} \\ \mathtt{K}^{-1} \mathbf{x} &= [\mathtt{R} \,|\, \mathbf{t}] \mathbf{X} \\ \hat{\mathbf{x}} &= [\mathtt{R} \,|\, \mathbf{t}] \mathbf{X} \text{ where } \hat{\mathbf{x}} = \mathtt{K}^{-1} \mathbf{x} \end{aligned}$$

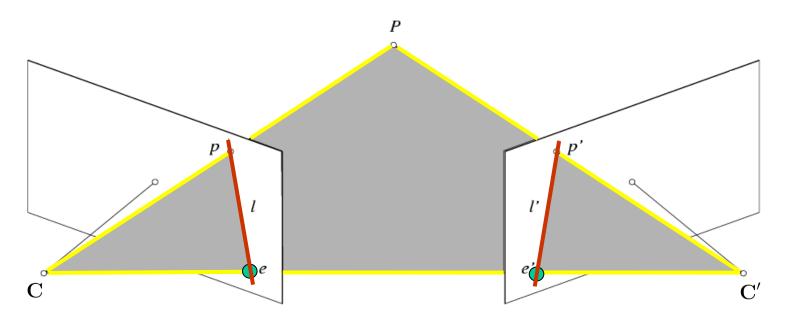
Image points in normalized coordinates

$$\begin{split} \mathbf{x} &= \mathtt{K}[\mathtt{I} \mid \mathbf{0}] \begin{bmatrix} \mathtt{R} & \mathtt{t} \\ \mathtt{0}^{\top} & \mathtt{1} \end{bmatrix} \mathtt{X} \\ \mathtt{K}^{-1} \mathbf{x} &= [\mathtt{I} \mid \mathtt{0}] \begin{bmatrix} \mathtt{R} & \mathtt{t} \\ \mathtt{0}^{\top} & \mathtt{1} \end{bmatrix} \mathtt{X} \\ \hat{\mathbf{x}} &= [\mathtt{I} \mid \mathtt{0}] \begin{bmatrix} \mathtt{R} & \mathtt{t} \\ \mathtt{0}^{\top} & \mathtt{1} \end{bmatrix} \mathtt{X} \text{ where } \hat{\mathbf{x}} &= \mathtt{K}^{-1} \mathtt{x} \\ \hat{\mathbf{x}} &= [\mathtt{I} \mid \mathtt{0}] \mathtt{X}_{\text{cam}} \\ \hat{\mathbf{x}} &= [\mathtt{I} \mid \mathtt{0}] \begin{bmatrix} \tilde{\mathtt{X}}_{\text{cam}} \\ \mathtt{1} \end{bmatrix} \\ \hat{\mathbf{x}} &= \tilde{\mathtt{X}}_{\text{cam}} \text{ (up to nonzero scale)} \end{split}$$

The essential matrix

3D point $\tilde{\mathbf{X}}' = \lambda' \hat{\mathbf{x}}'$ in the second camera coordinate frame. 3D point $\tilde{\mathbf{X}} = \lambda \hat{\mathbf{x}}$ in the first camera coordinate frame. Map $\tilde{\mathbf{X}}$ from first camera coordinate frame to second camera coordinate frame.

$$egin{aligned} & ilde{\mathbf{X}}' = \mathtt{R} ilde{\mathbf{X}} + \mathbf{t} \ & ilde{\mathbf{X}}' = \mathtt{R}(\lambda \hat{\mathbf{x}}) + \mathbf{t} \ & ilde{\mathbf{X}}' = \lambda \mathtt{R} \hat{\mathbf{x}} + \mathbf{t} \end{aligned}$$



The essential matrix

3D point $\tilde{\mathbf{X}}' = \lambda' \hat{\mathbf{x}}'$ in the second camera coordinate frame. 3D point $\tilde{\mathbf{X}} = \lambda \hat{\mathbf{x}}$ in the first camera coordinate frame. Map $\tilde{\mathbf{X}}$ from first camera coordinate frame to second camera coordinate frame.

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$$\begin{split} \lambda R \hat{\mathbf{x}} + \mathbf{t} &= \lambda' \hat{\mathbf{x}}' \\ [\mathbf{t}]_{\times} (\lambda R \hat{\mathbf{x}} + \mathbf{t}) &= [\mathbf{t}]_{\times} (\lambda' \hat{\mathbf{x}}') \\ \lambda [\mathbf{t}]_{\times} R \hat{\mathbf{x}} &= \lambda' [\mathbf{t}]_{\times} \hat{\mathbf{x}}' \\ \hat{\mathbf{x}}'^{\top} (\lambda [\mathbf{t}]_{\times} R \hat{\mathbf{x}}) &= \hat{\mathbf{x}}'^{\top} (\lambda' [\mathbf{t}]_{\times} \hat{\mathbf{x}}') \\ \lambda \hat{\mathbf{x}}'^{\top} [\mathbf{t}]_{\times} R \hat{\mathbf{x}} &= 0 \\ \hat{\mathbf{x}}'^{\top} [\mathbf{t}]_{\times} R \hat{\mathbf{x}} &= 0 \end{split}$$
The epipolar constraint $\hat{\mathbf{x}}'^{\top} E \hat{\mathbf{x}} = 0$ where $\mathbf{E} = [\mathbf{t}]_{\times} R$
Essential Matrix

(Longuet-Higgins, 1981)

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Cross product using a skew symmetric matrix

The cross product a × b of two 3-vectors a = (a₁, a₂, a₃)[⊤] and b = (b₁, b₂, b₃)[⊤] can be expressed a matrix-vector product [a]_×b, where [a]_× is the 3x3 skew symmetric matrix

$$[\mathbf{a}]_{\times} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

- A matrix **S** is skew symmetric if and only if $S = -S^T$
- The determinant of a skew symmetric matrix is 0

The essential matrix

• Maps a point (in normalized coordinates) in the first image to its corresponding epipolar line (in normalized coordinates) in the second image

$$\hat{oldsymbol{\ell}}' = \mathtt{E}\hat{\mathbf{x}}$$

• The epipolar line passes through the corresponding point in the second image

$$\hat{\mathbf{x}}^{\prime \top} \hat{\boldsymbol{\ell}}^{\prime} = 0$$

 $\hat{\mathbf{x}}^{\prime \top} \mathbf{E} \hat{\mathbf{x}} = 0$ The epipolar constraint

• Every epipolar line passes through the epipole

$$\hat{\mathbf{e}}^{\prime \top} \hat{\boldsymbol{\ell}}^{\prime} = 0$$

The essential matrix

• Maps a point (in normalized coordinates) in the second image to its corresponding epipolar line (in normalized coordinates) in the first image $\hat{\ell} = \mathbf{E}^{\top} \hat{\mathbf{x}}'$

$$\hat{\boldsymbol{\ell}}^{\top} = \hat{\mathbf{x}}^{\prime op} \mathbf{E}$$

• The epipolar line passes through the corresponding point in the first image

$$\hat{\boldsymbol{\ell}}^{\top}\hat{\mathbf{x}}=0$$

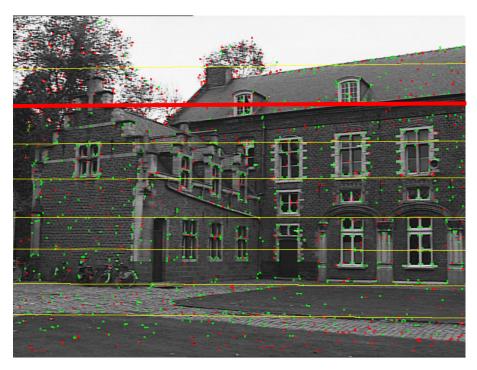
 $\hat{\mathbf{x}}^{\prime \top} \mathbf{E} \hat{\mathbf{x}} = 0$ The epipolar constraint

• Every epipolar line passes through the epipole

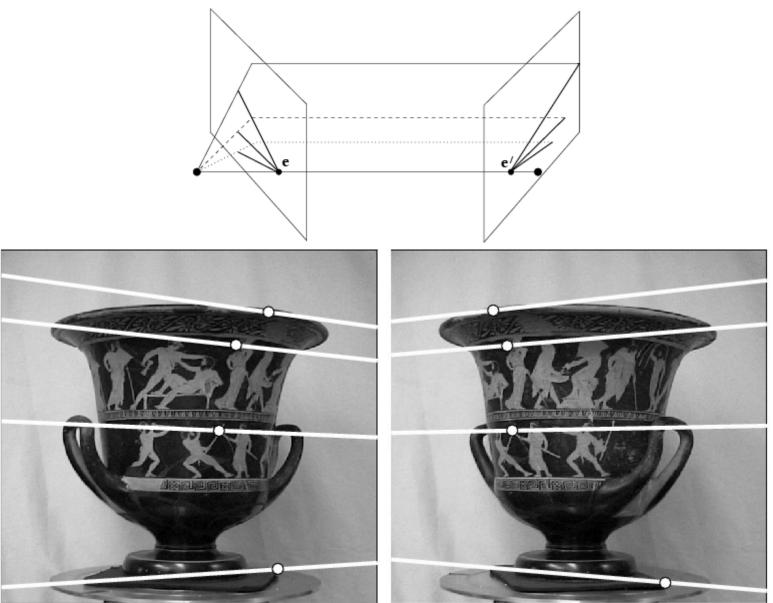
$$\hat{\boldsymbol{\ell}}^{\top} \hat{\mathbf{e}} = 0$$

Example of using the essential matrix





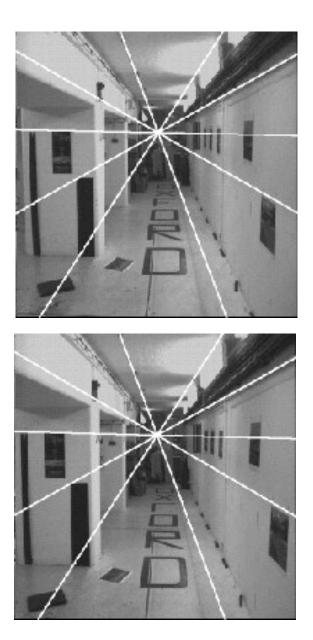
Another example: converging cameras

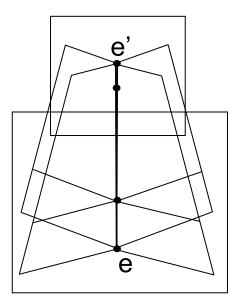


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courtesy of Andrew Zisserman

Another example: second camera in front of first camera





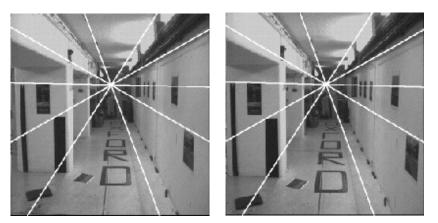
courtesy of Andrew Zisserman Computer Vision I

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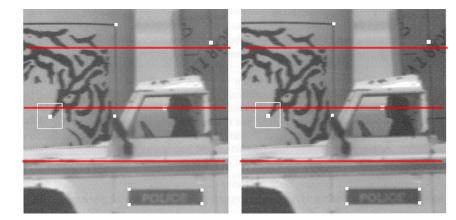
Epipolar rectify stereo images

- Epipolar geometry reduces matching complexity from $O(n^4)$ to $O(n^3)$
- But matching requires comparing points across pairs of epipolar lines which may have arbitrary orientation. That can be costly to index.
- Is there a more convenient epipolar geometry?

VS



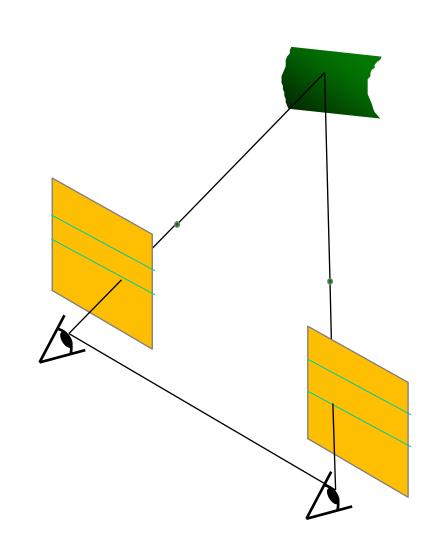
Slanted epipolar lines



Horizontal, row aligned epipolar lines

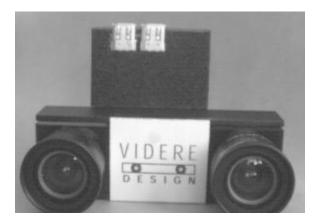
Cameras with a convenient epipolar geometry

- When two cameras have parallel optical axes and these axis are orthogonal to the baseline, the epipolar lines are parallel
- When rows of the two images are parallel to the baseline, the epipolar lines are horizontal rows of the two images



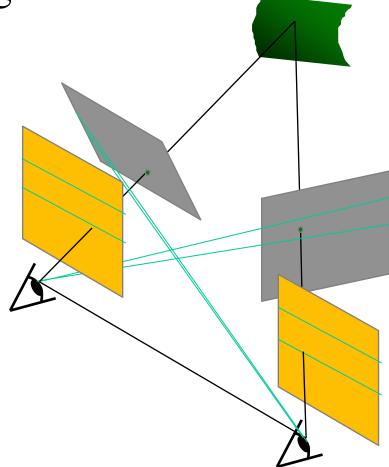
Cameras with a convenient epipolar geometry

- When two cameras have parallel optical axes and these axis are orthogonal to the baseline, the epipolar lines are parallel
- When rows of the two images are parallel to the baseline, the epipolar lines are horizontal rows of the two images



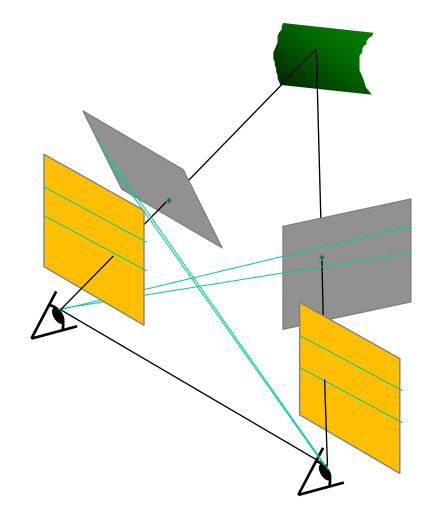


What if stereo geometry is not convenient? Rectification: Given a pair of images, transform both images so that epipolar lines are image rows



Epipolar rectification

- Given a pair of images, transform both images so that epipolar lines are image rows
- Create pair of virtual cameras
 - The virtual cameras have the same camera centers as real cameras
 - Both virtual cameras have the same:
 - Camera rotation matrix R
 - Camera calibration matrix K



- Given calibrated stereo cameras (i.e., K₁, R₁, t₁, K₂, R₂, t₂) determine the (same) rotation matrix R and (same) calibration matrix K of the virtual cameras
- To minimize image distortion:
 - For the calibration matrix
 - For principal point, set x_0 and y_0 to average of input x_0 and y_0 values, respectively
 - Set two focal length parameters to average of all four input focal length parameters (results in square pixels)
 - Set skew to 0
 - For the rotation matrix **R**, interpolate halfway between the two 3D rotations embodied by \mathbf{R}_1 and \mathbf{R}_2

- For the rotation matrix **R**, interpolate halfway between the two 3D rotations embodied by **R**₁ and **R**₂
 - Rotating a camera does not change its center C, but does change its translation t

$$\begin{split} \tilde{\mathbf{0}}_{cam} &= \mathtt{R}\tilde{\mathbf{C}} + \mathtt{t} & \tilde{\mathbf{C}} = -\mathtt{R}_{virtual}^{\top} \mathtt{t}_{virtual} \\ -\mathtt{R}\tilde{\mathbf{C}} &= \mathtt{t} & \tilde{\mathbf{C}} = -\mathtt{R}_{virtual}^{\top} \mathtt{t}_{virtual} \\ \tilde{\mathbf{C}} &= -\mathtt{R}_{real}^{\top} \mathtt{t}_{real} \\ -\mathtt{R}_{virtual}^{\top} \mathtt{t}_{virtual} &= -\mathtt{R}_{real}^{\top} \mathtt{t}_{real} \\ \mathbf{x} &= \mathtt{K}[\mathtt{R} \mid \mathtt{t}] \mathtt{X} & \mathtt{t}_{virtual} = \mathtt{R}_{virtual} \mathtt{R}_{real}^{\top} \mathtt{t}_{real} \\ \mathbf{x} &= \mathtt{K}[\mathtt{R} \mid -\mathtt{R}\tilde{\mathbf{C}}] \mathtt{X} & \mathtt{t}_{virtual} = \mathtt{R}_{virtual} \mathtt{R}_{real}^{\top} \mathtt{t}_{real} \end{split}$$

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Rectification transformation matrices

• Transformation from image acquired by real camera to image acquired by virtual camera

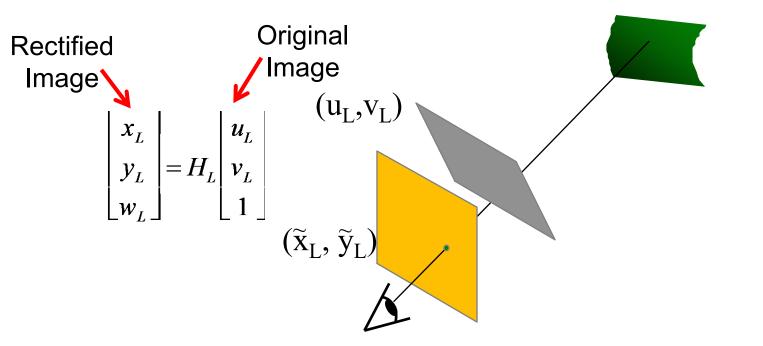
$$\mathbf{x}_{ ext{real}} = \mathtt{K}_{ ext{real}} \mathtt{R}_{ ext{real}}^{-1} [\mathtt{I} \mid - \tilde{\mathbf{C}}] \mathbf{X}$$

 $\mathtt{R}_{ ext{real}}^{ op} \mathtt{K}_{ ext{real}}^{-1} \mathbf{x}_{ ext{real}} = [\mathtt{I} \mid - \tilde{\mathbf{C}}] \mathbf{X}$

$$\begin{aligned} \mathbf{x}_{\text{virtual}} &= \mathsf{K}_{\text{virtual}} \mathsf{R}_{\text{virtual}} \mathsf{R}_{\text{real}}^{\top} \mathsf{I} \big| - \tilde{\mathbf{C}} \big] \mathbf{X} \\ \mathbf{x}_{\text{virtual}} &= \mathsf{K}_{\text{virtual}} \mathsf{R}_{\text{virtual}} \mathsf{R}_{\text{real}}^{\top} \mathsf{K}_{\text{real}}^{-1} \mathbf{x}_{\text{real}} \\ \mathbf{x}_{\text{virtual}} &= \mathsf{H} \mathbf{x}_{\text{real}}, \text{ where } \mathsf{H} = \mathsf{K}_{\text{virtual}} \mathsf{R}_{\text{virtual}} \mathsf{R}_{\text{real}}^{\top} \mathsf{K}_{\text{real}}^{-1} \\ \begin{bmatrix} x_{\text{virtual}} \\ y_{\text{virtual}} \\ w_{\text{virtual}} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_{\text{real}} \\ y_{\text{real}} \\ w_{\text{real}} \end{bmatrix} \end{aligned}$$

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Under perspective projection, the mapping from a plane to a plane is given by a 2D projective transformation (homography)



Under perspective projection, the mapping from a plane to a plane is given by a 2D projective transformation (homography)

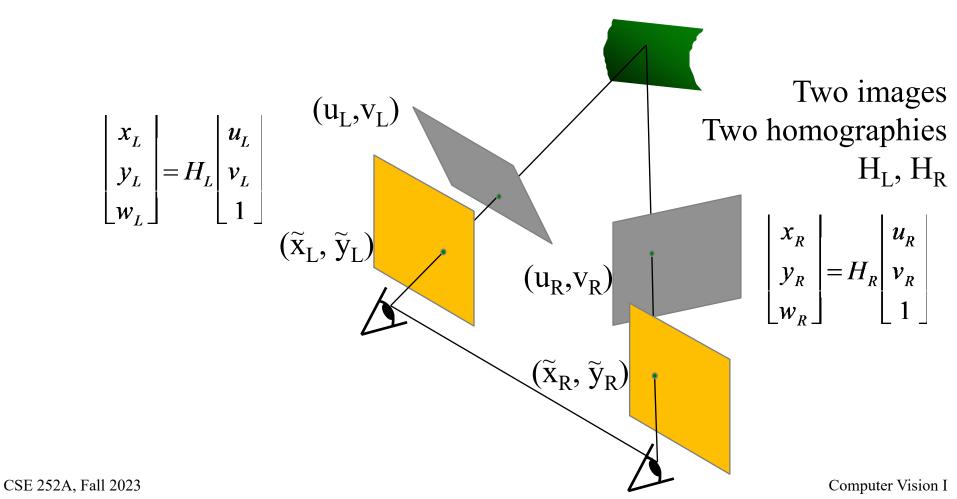
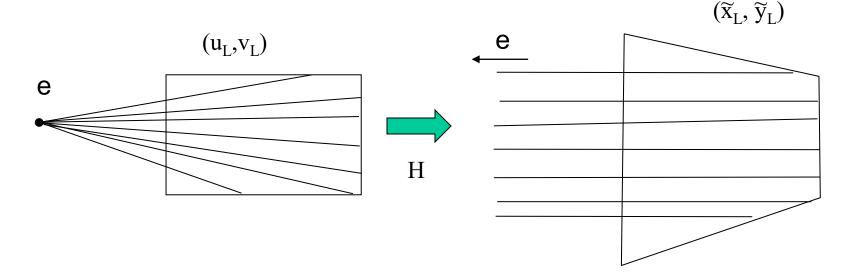


Image pair rectification

Apply projective transformation so that epipolar lines correspond to horizontal scanlines



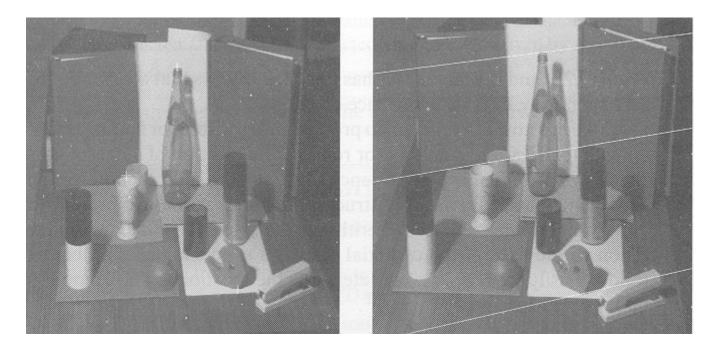
H should map epipole e to (1,0,0), a point at infinity on the x-axis H should minimize image distortion

Note that rectified images are usually not rectangular

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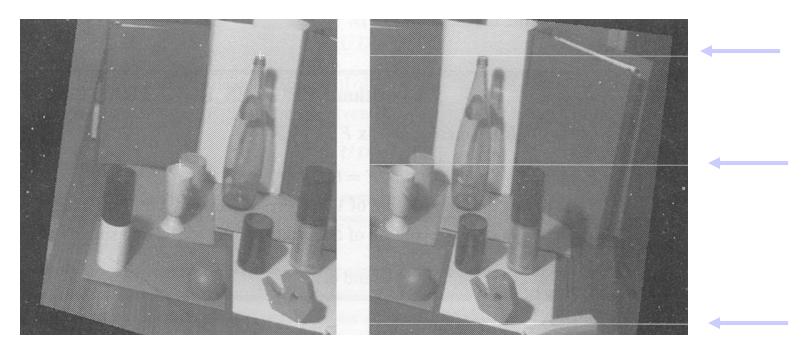
=He

Given a pair of images, transform both images so that epipolar lines are scan lines.



Input Images

Given a pair of images, transform both images so that epipolar lines are scan lines.



Rectified Images

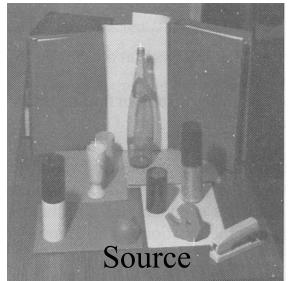
epipolar lines run parallel with the *x*-axis and are aligned between two views (no *y* disparity)

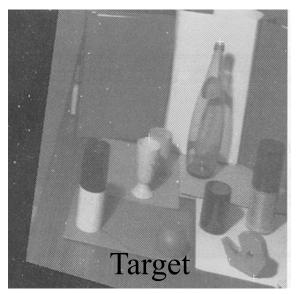
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- Input: Source image I and Rectification matrix H
- Forward "warping" method
 - For each pixel coordinate \mathbf{x}_{real} in source image, map it to pixel coordinate $\mathbf{x}_{virtual}$ in target image

 $\mathbf{x}_{\mathrm{virtual}} = \mathtt{H}\mathbf{x}_{\mathrm{real}}$

- Problem: there is no guarantee that every pixel in target Image will be written to
 - If target image is larger than source image or target image is highly stretched, then there may be missing points that appear as speckles or lines





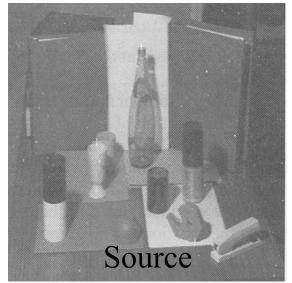
Computer Vision I

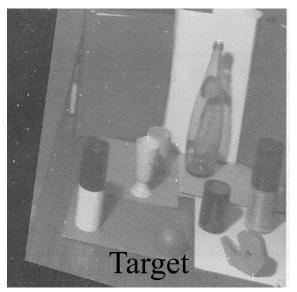
- Input: Source image I and Rectification matrix H
- Backward "warping" method
 - For each pixel coordinate $\mathbf{x}_{virtual}$ in target image, map it to pixel coordinate \mathbf{x}_{real} in source image

 $\mathbf{x}_{real} = \mathtt{H}^{-1} \mathbf{x}_{virtual}$

- Interpolate pixel values in source image to determine pixel value in destination image
- Problem: some of the pixel coordinates in the target image map to pixel coordinates outside of the source image
 - You'll solve this problem in a homework assignment by determining where the pixel coordinates of the corners of the source image map to pixel coordinates in the destination image using

$$\mathbf{x}_{ ext{virtual}} = \mathtt{H}\mathbf{x}_{ ext{real}}$$





Computer Vision I



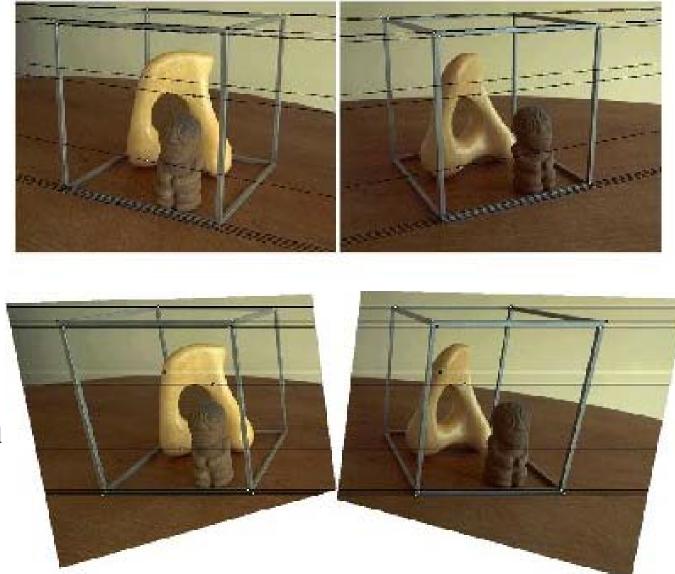
Original

Rectified



• Epipolar lines





Original

Rectified

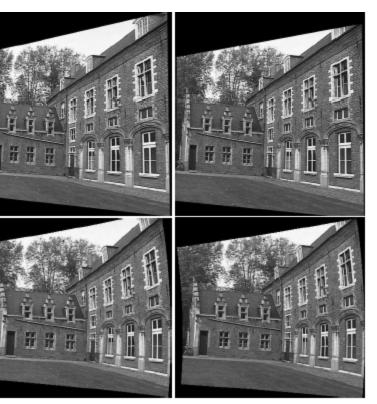
CSE 252A, Fall 2023

Polar Rectification

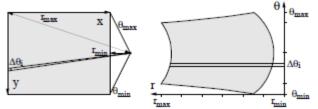


Homography-based Rectification

Polar Rectification



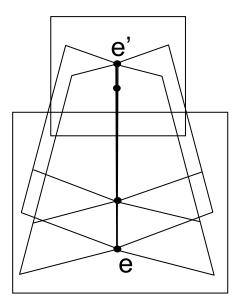
Alternative epipolar rectification method that minimizes pixel distortion



Computer Vision I

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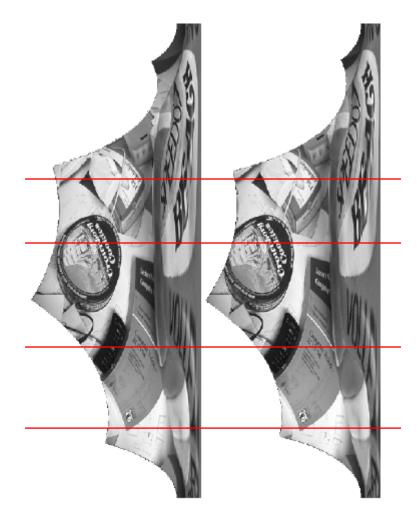
Polar Rectification



Epipoles are in images (white dot on ball)



Homography-based rectification is not possible



Features on same epipolar line



Next Lecture

• Calibrated stereo and feature matching