Geometric Image Formation

Computer Vision I

CSE 252A

Lecture 2

Announcements

- Assignment 0 will be released today
  - Due Oct 11, 11:59 PM

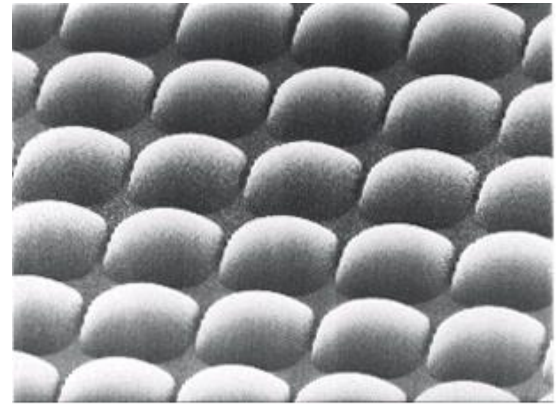
Earliest Surviving Photograph

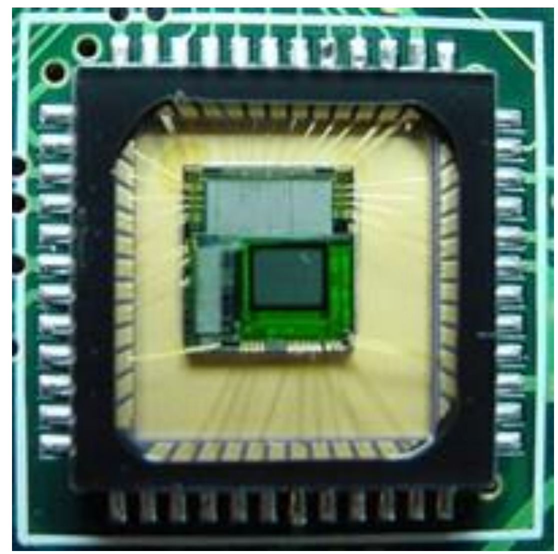
- First photograph on record, “la table service” by Nicephore Niepce in 1822
- Note: First photograph by Niepce was in 1816

How Cameras Produce Images

- Basic process:
  - photons hit a detector
  - the detector becomes charged
  - the charge is read out as brightness

- Sensor types:
  - CCD (charge-coupled device)
    - high sensitivity
    - high power
    - cannot be individually addressed
    - blooming
  - CMOS
    - simple to fabricate (cheap)
    - lower sensitivity, lower power
    - can be individually addressed





Images are two-dimensional patterns of brightness values.

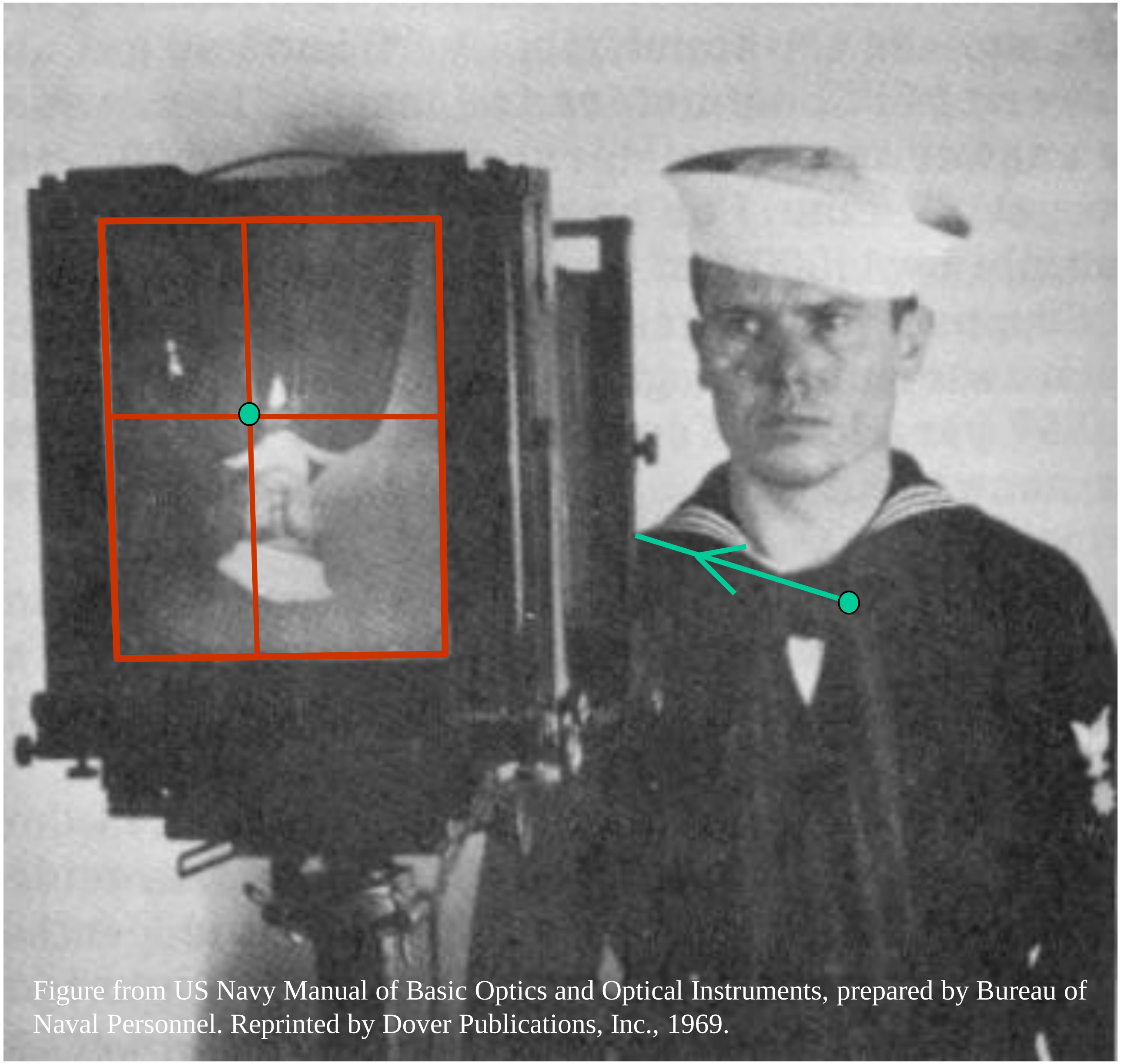
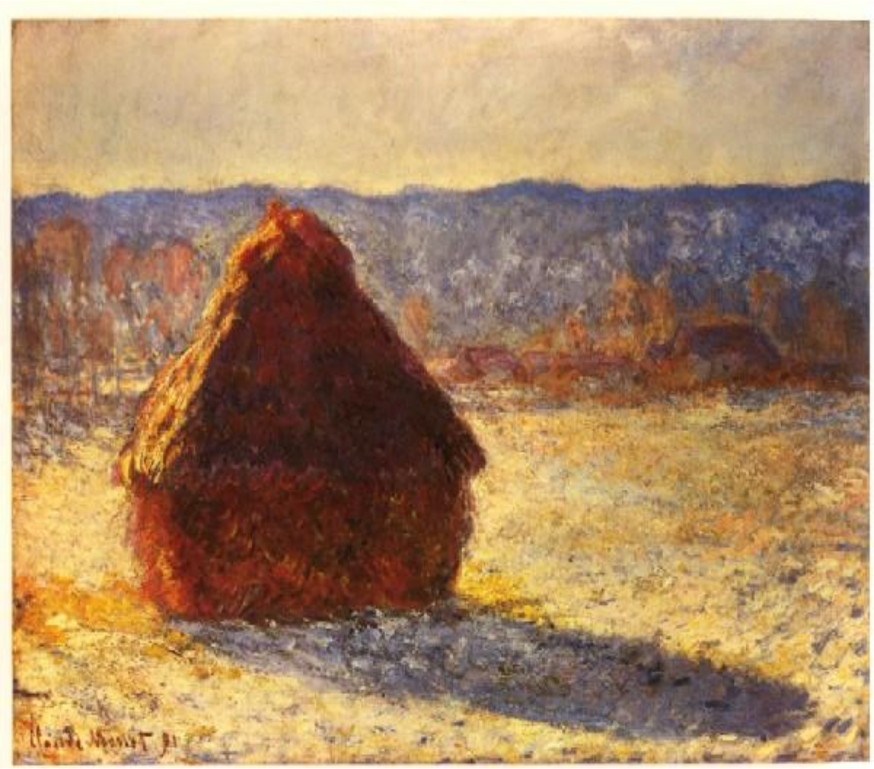


Figure from US Navy Manual of Basic Optics and Optical Instruments, prepared by Bureau of Naval Personnel. Reprinted by Dover Publications, Inc., 1969.

They are formed by the projection of 3D objects.

Lighting Affects Appearance: Monet



Viewpoint Affects Appearance: Monet

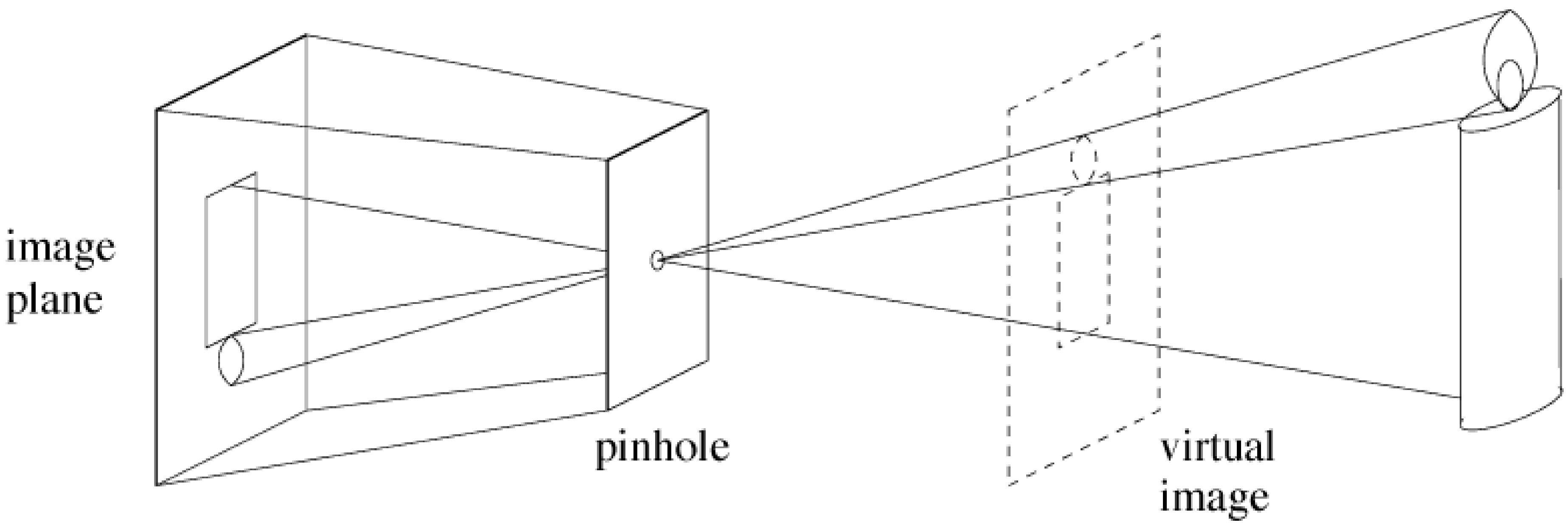
Haystack at Chailly at sunrise (1865)

Image Formation: Outline

- Geometric image formation
  - How do 3D world points project to 2D image points?
- Photometric image formation
  - What color is the projected point?

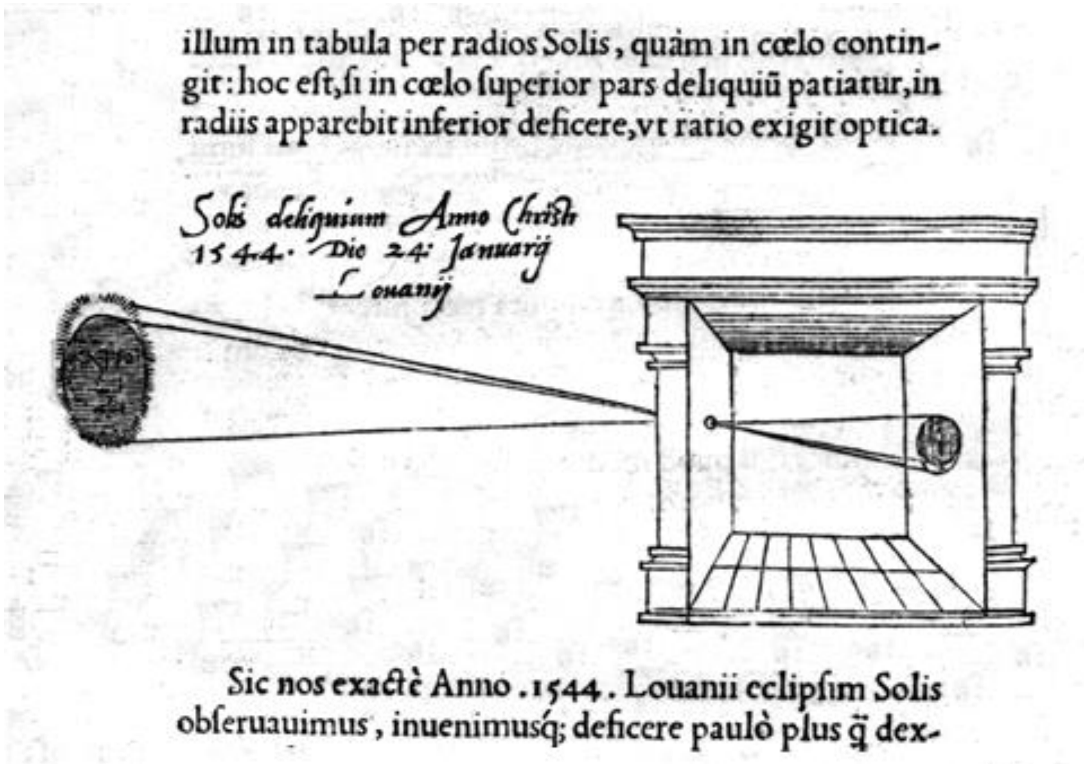
Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it



Forsyth&Ponce

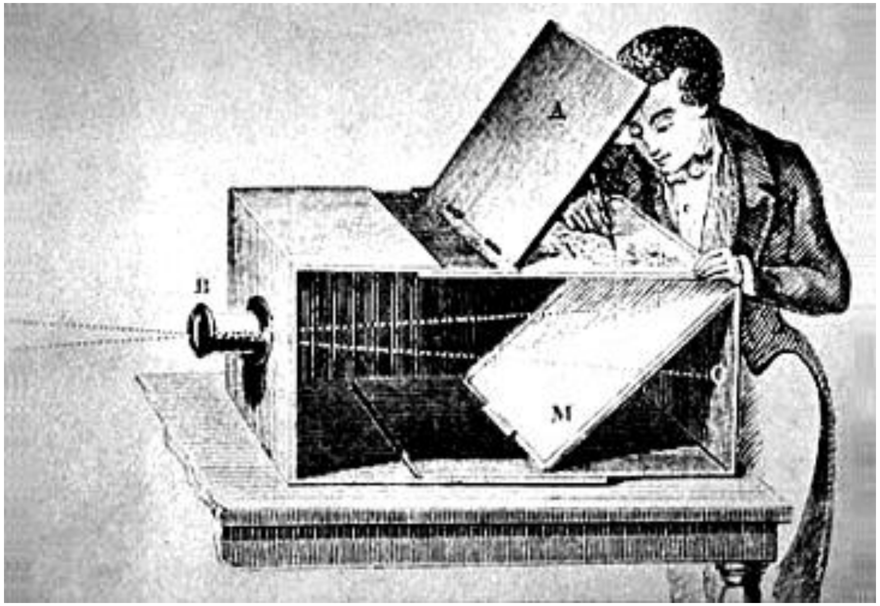
Camera Obscura



"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays”. --- Leonardo *Da Vinci*

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)

Camera Obscura



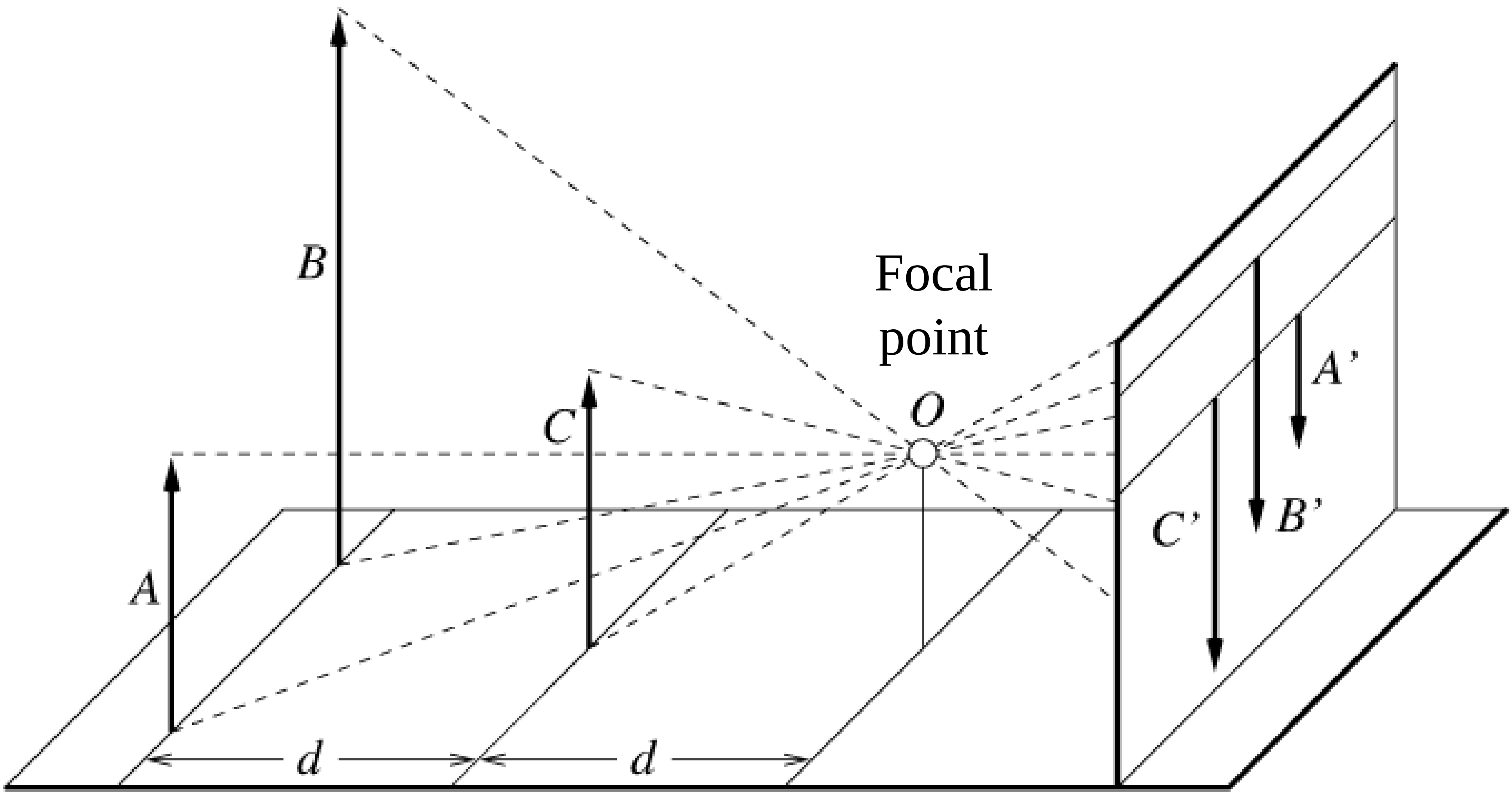
- Used to observe eclipses (e.g., Bacon, 1214-1294)
- By artists (e.g., Vermeer).

Camera Obscura

Jetty at Margate England, 1898.

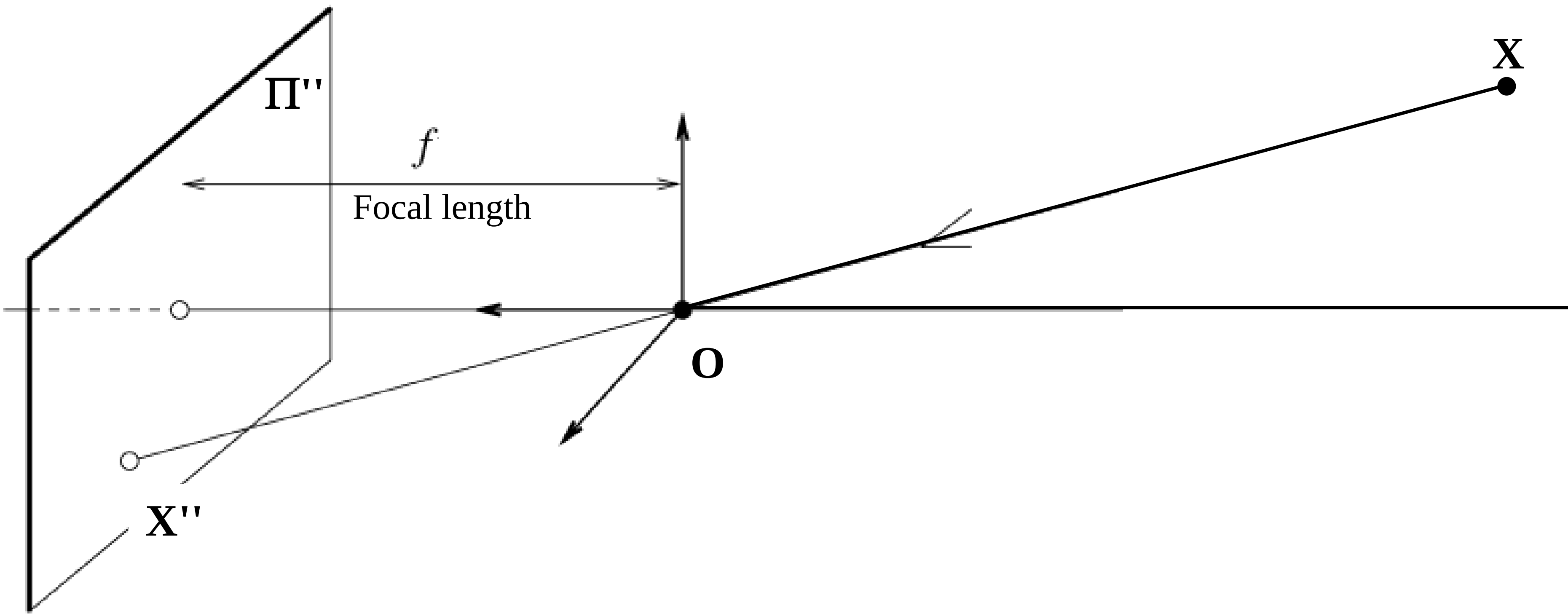
http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)

A and C are same size, but A is further from camera, so its image A’ is smaller



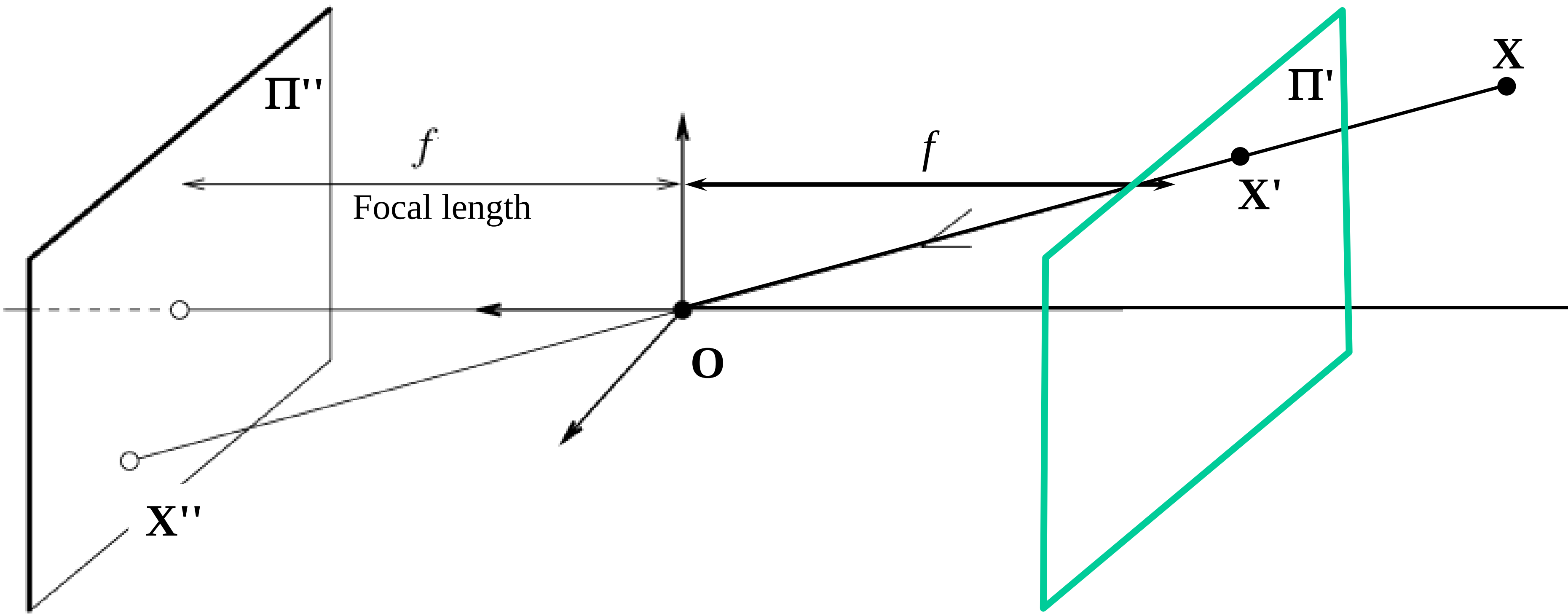
(Forsyth & Ponce)

Purely Geometric View of Perspective



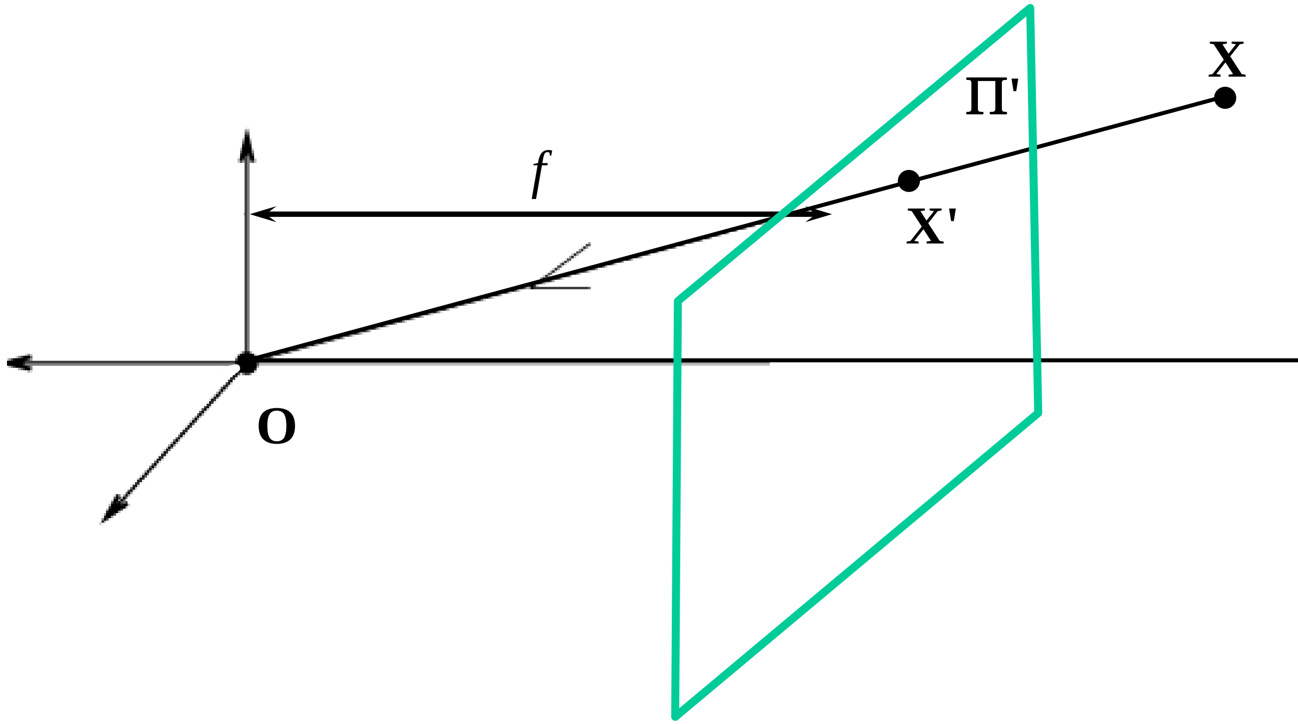
The projection of the point **X** on the image plane **Π''** is given by the point of intersection **X''** of the ray defined by **XO** with the plane **Π''**

Virtual image plane



- Virtual image plane in front of optical center
- Image is 'upright'

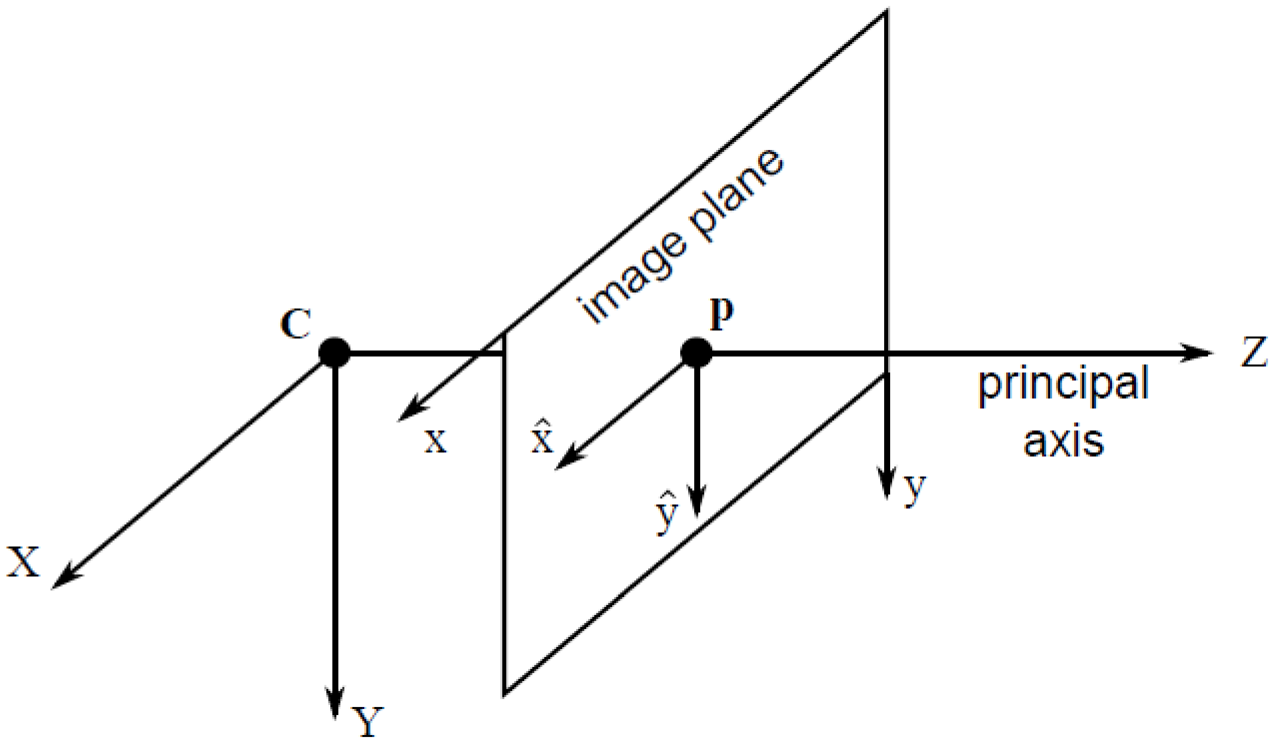
Virtual image plane



- Virtual image plane in front of optical center
- Image is ‘upright’

Geometric image formation

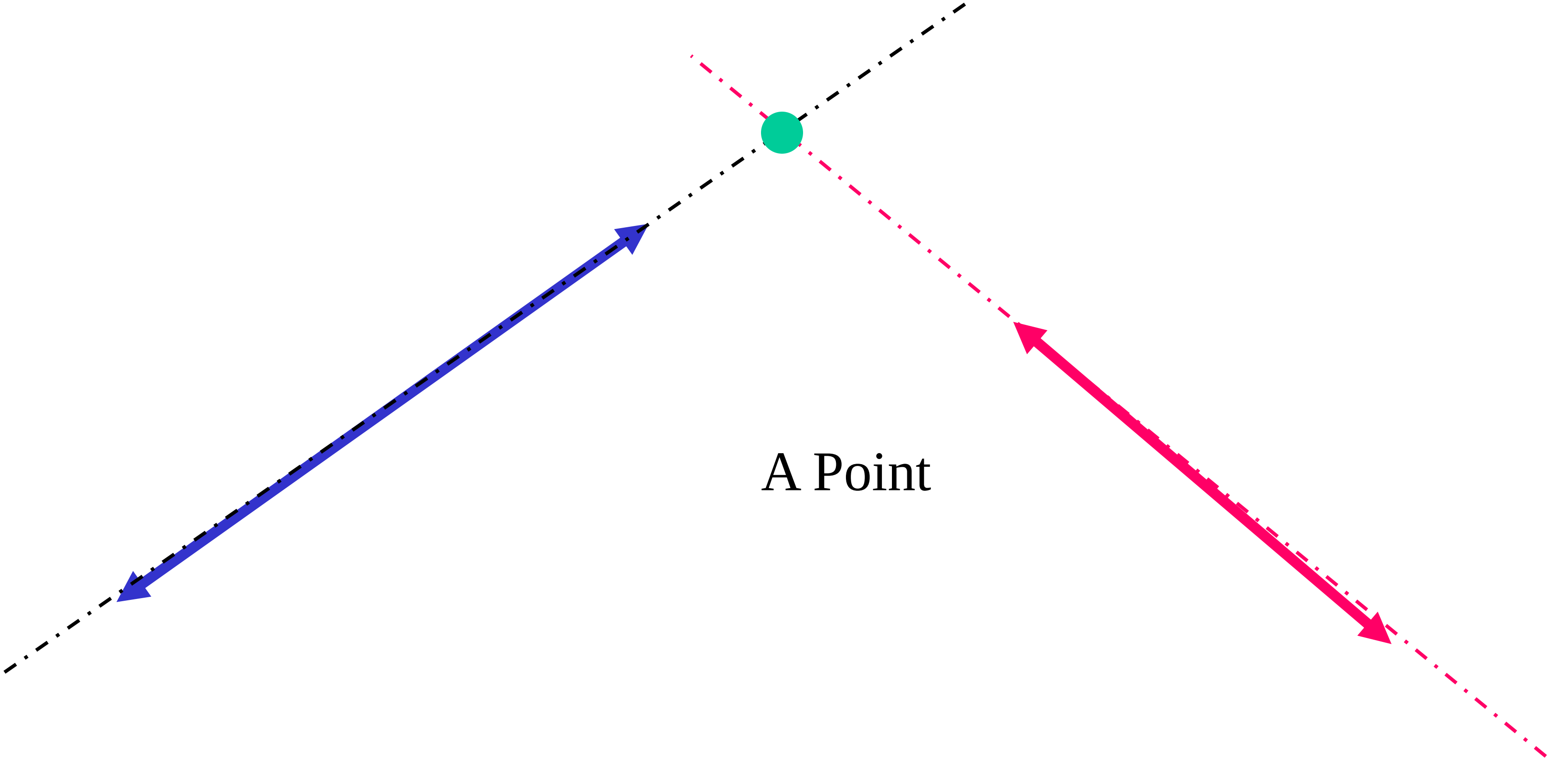
- How do 3D world points project to 2D image points?



A Digression

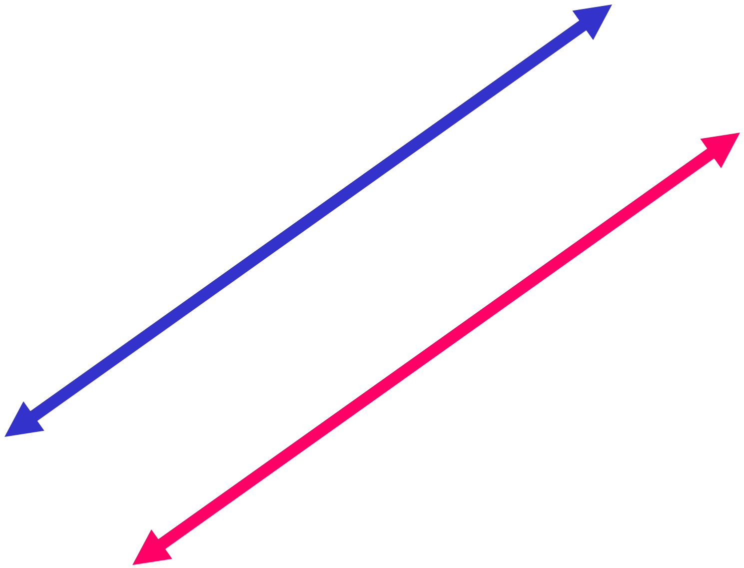
Projective Geometry
and
Homogenous Coordinates

What is the intersection of two lines in a plane?



A Point

Do two lines in the plane always intersect at a point?



No, Parallel lines don't meet at a point.

Can the perspective projection of parallel lines in 3D meet at a point in an image?

YES

Projective geometry provides an elegant means for handling these different situations in a unified way

Homogenous coordinates are a way to represent entities (points & lines) in projective spaces

Geometric primitives

- 2D and 3D points
- 2D lines
- Joint and intersection of 2D points and lines

Points

- Inhomogeneous coordinates

2D

$$\tilde{\mathbf{x}} = (\tilde{x}, \tilde{y})^\top$$

3D

$$\tilde{\mathbf{X}} = (\tilde{X}, \tilde{Y}, \tilde{Z})^\top$$

- Homogeneous (i.e., defined up to nonzero scale) coordinates

2D

$$\mathbf{x} = (x, y, w)^\top$$

3D

$$\mathbf{X} = (X, Y, Z, T)^\top$$

Points

- Homogeneous coordinates to inhomogeneous coordinates

2D

$$\mathbf{x} = (x, y, w)^\top \mapsto \tilde{\mathbf{x}} = (x/w, y/w)^\top$$

3D

$$\mathbf{X} = (X, Y, Z, T)^\top \mapsto \tilde{\mathbf{X}} = (X/T, Y/T, Z/T)^\top$$

- Inhomogeneous coordinates to homogeneous coordinates

2D

$$\tilde{\mathbf{x}} = (\tilde{x}, \tilde{y})^\top \mapsto \mathbf{x} = (\tilde{x}, \tilde{y}, 1)^\top$$

3D

$$\tilde{\mathbf{X}} = (\tilde{X}, \tilde{Y}, \tilde{Z})^\top \mapsto \mathbf{X} = (\tilde{X}, \tilde{Y}, \tilde{Z}, 1)^\top$$

Points

- Homogeneous (i.e., defined up to nonzero scale) coordinates to inhomogeneous coordinates

2D

$$\mathbf{x} = \lambda\mathbf{x}, \text{ for nonzero } \lambda$$

$$\begin{aligned}
\mathbf{x} = (x, y, w)^\top &\mapsto \tilde{\mathbf{x}} = (x/w, y/w)^\top \\
\lambda\mathbf{x} = (\lambda x, \lambda y, \lambda w)^\top &\mapsto \tilde{\mathbf{x}} = (\lambda x/\lambda w, \lambda y/\lambda w)^\top \\
& \tilde{\mathbf{x}} = (x/w, y/w)^\top
\end{aligned}$$

3D

$$\mathbf{X} = \lambda\mathbf{X}, \text{ for nonzero } \lambda$$

$$\begin{aligned}
\mathbf{X} = (X, Y, Z, T)^\top &\mapsto \tilde{\mathbf{X}} = (X/T, Y/T, Z/T)^\top \\
\lambda\mathbf{X} = (\lambda X, \lambda Y, \lambda Z, \lambda T)^\top &\mapsto \tilde{\mathbf{X}} = (\lambda X/\lambda T, \lambda Y/\lambda T, \lambda Z/\lambda T)^\top \\
& \tilde{\mathbf{X}} = (X/T, Y/T, Z/T)^\top
\end{aligned}$$

2D and 3D points

Definitions

$$\infty = \lim_{x \to 0^+} \frac{1}{x}$$

$$0 = \lim_{x \to \infty} \frac{1}{x}$$

Homogeneous coordinates allow for points at infinity to be treated like other points
Set of 2D and 3D points at infinity

$$\mathbf{x}_\infty = (x, y, 0)^\top$$

$$\mathbf{X}_\infty = (X, Y, Z, 0)^\top$$

2D lines

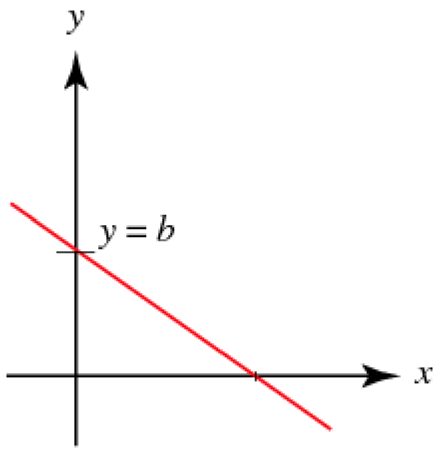
Equation of a 2D line, slope-intercept form

$$y = mx + b$$

with y-intercept b and slope

$$m = \frac{\Delta y}{\Delta x}$$

Slope of horizontal line is 0
Slope of vertical line is undefined (division by zero)



Do not use this

2D lines

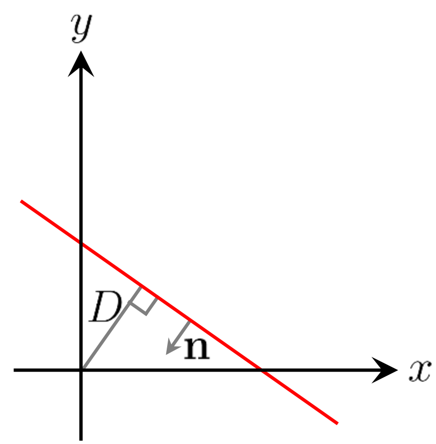
Equation of a 2D line, standard form

$$
\begin{aligned}
a\tilde{x} + b\tilde{y} + c &= 0 \\
a(x/w) + b(y/w) + c &= 0 \\
ax + by + cw &= 0 \\
\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} &= 0 \\
\boldsymbol{\ell}^\top \mathbf{x} &= 0
\end{aligned}
$$

2D lines

2D line in homogeneous coordinates

$$\boldsymbol{\ell} = (a, b, c)^\top$$
$$\boldsymbol{\ell} = (\mathbf{n}^\top, c)^\top$$



where normal vector

$$\mathbf{n} = (a, b)^\top$$

Signed distance from origin

$$D = \frac{c}{\sqrt{a^2 + b^2}} = \frac{c}{\|\mathbf{n}\|}$$

Set of 2D lines at infinity

$$\boldsymbol{\ell}_\infty = (0, 0, 1)^\top$$

2D points and lines

2D points and lines in homogeneous coordinates

$$\mathbf{x}^\top \boldsymbol{\ell} = 0$$
$$\boldsymbol{\ell}^\top \mathbf{x} = 0$$

if and only if $\mathbf{x}$ is on $\boldsymbol{\ell}$

2D points and lines

Two points on 2D line is left null space of line
2D line joining two points in general position is (right) null space of matrix

$$\begin{bmatrix} \mathbf{x}_1^\top \\ \mathbf{x}_2^\top \end{bmatrix} \boldsymbol{\ell} = \mathbf{0}$$

$$[\boldsymbol{\ell}]^\perp = \begin{bmatrix} \mathbf{x}_1^\top \\ \mathbf{x}_2^\top \end{bmatrix}$$

Two 2D lines intersecting at a point is the left null space of point
2D point at intersection of two lines in general position is (right) null space of matrix

$$\begin{bmatrix} \boldsymbol{\ell}_1^\top \\ \boldsymbol{\ell}_2^\top \end{bmatrix} \mathbf{x} = \mathbf{0}$$

$$[\mathbf{x}]^\perp = \begin{bmatrix} \boldsymbol{\ell}_1^\top \\ \boldsymbol{\ell}_2^\top \end{bmatrix}$$

2D points and lines

For 3-vectors, the cross product is the (right) null space. Note these assume points and lines are in general position.

Join

$$\begin{aligned} \boldsymbol{\ell} &= \mathbf{x}_1 \times \mathbf{x}_2 \\ \boldsymbol{\ell} &= [\mathbf{x}_1]_\times \mathbf{x}_2 = (\mathbf{x}_1^\top [\mathbf{x}_2]_\times)^\top \\ [\boldsymbol{\ell}]_\times &= \mathbf{x}_1 \mathbf{x}_2^\top - \mathbf{x}_2 \mathbf{x}_1^\top \end{aligned}$$

Intersection

$$\begin{aligned} \mathbf{x} &= \boldsymbol{\ell}_1 \times \boldsymbol{\ell}_2 \\ \mathbf{x} &= [\boldsymbol{\ell}_1]_\times \boldsymbol{\ell}_2 = (\boldsymbol{\ell}_1^\top [\boldsymbol{\ell}_2]_\times)^\top \\ [\mathbf{x}]_\times &= \boldsymbol{\ell}_1 \boldsymbol{\ell}_2^\top - \boldsymbol{\ell}_2 \boldsymbol{\ell}_1^\top \end{aligned}$$

3x3 skew symmetric matrices

3-vector is skew symmetric matrix form

$$\mathbf{a} = (a_1, a_2, a_3)^\top$$

$$[\mathbf{a}]_\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

3×3 skew symmetric matrices are rank 2

Skew symmetric matrix (or antisymmetric matrix)

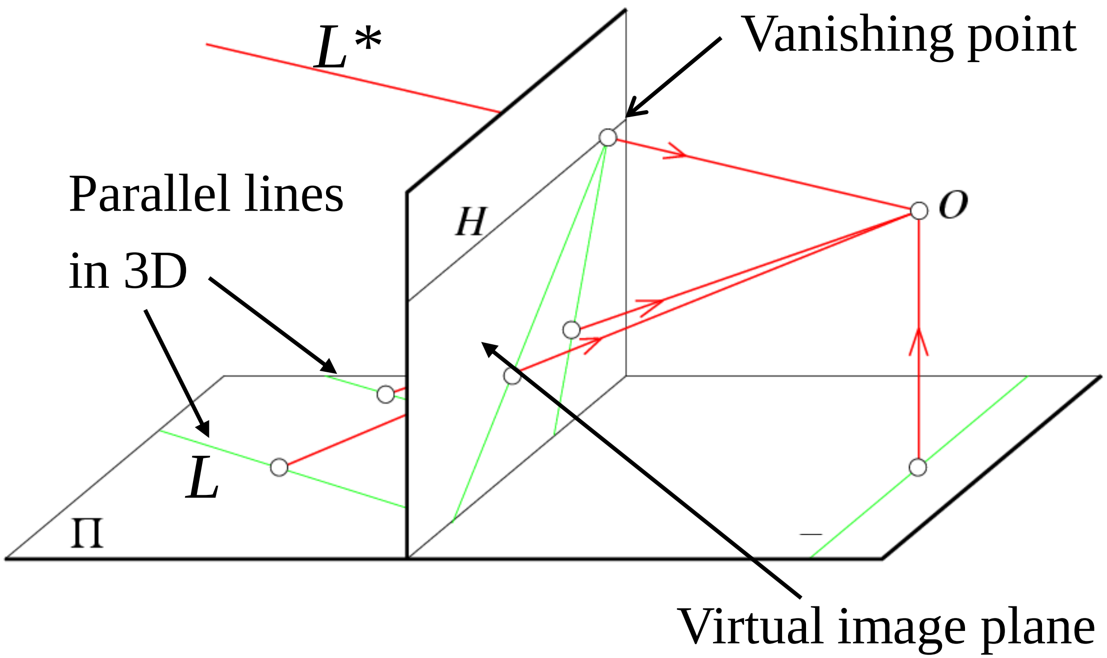
$$\mathtt{A} = -\mathtt{A}^\top$$

End of the Digression

In a perspective image, parallel lines meet at a point, called the vanishing point

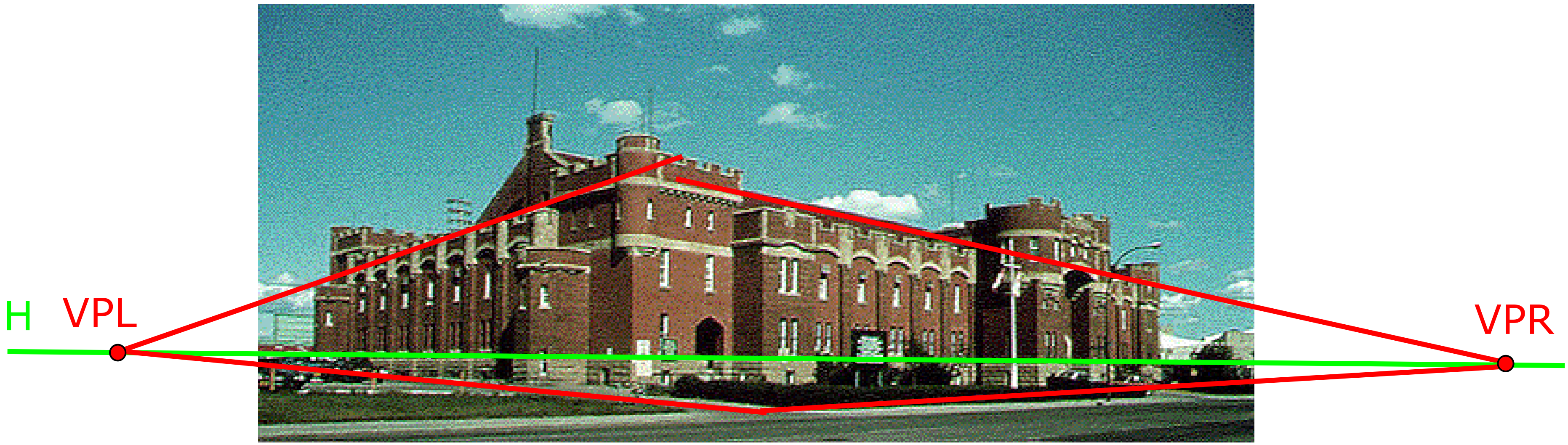
Doesn’t need to be near the center of the image

Parallel lines meet in the image

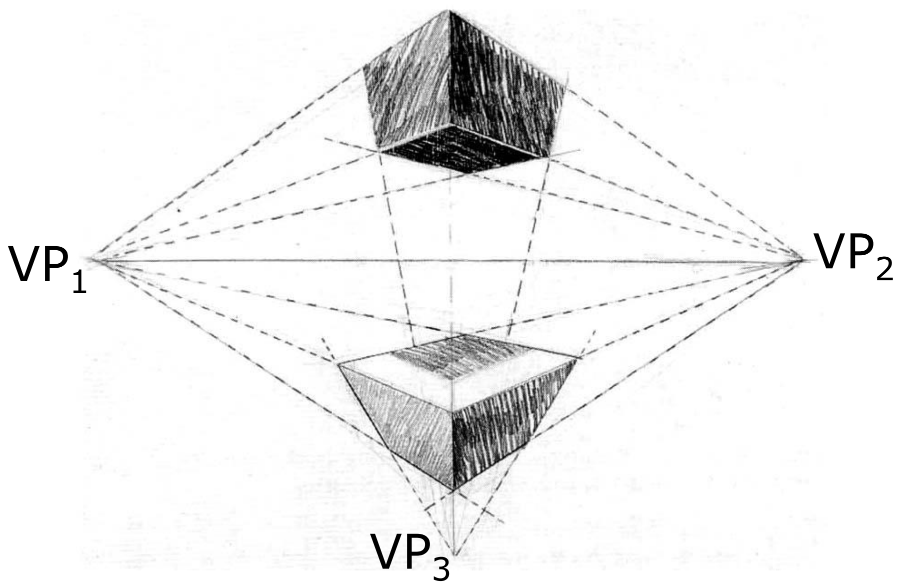


- A single line *L* can have a vanishing point.
- Vanishing point location: Intersection of image plane with a 3-D line *L** through optical center O parallel to *L*

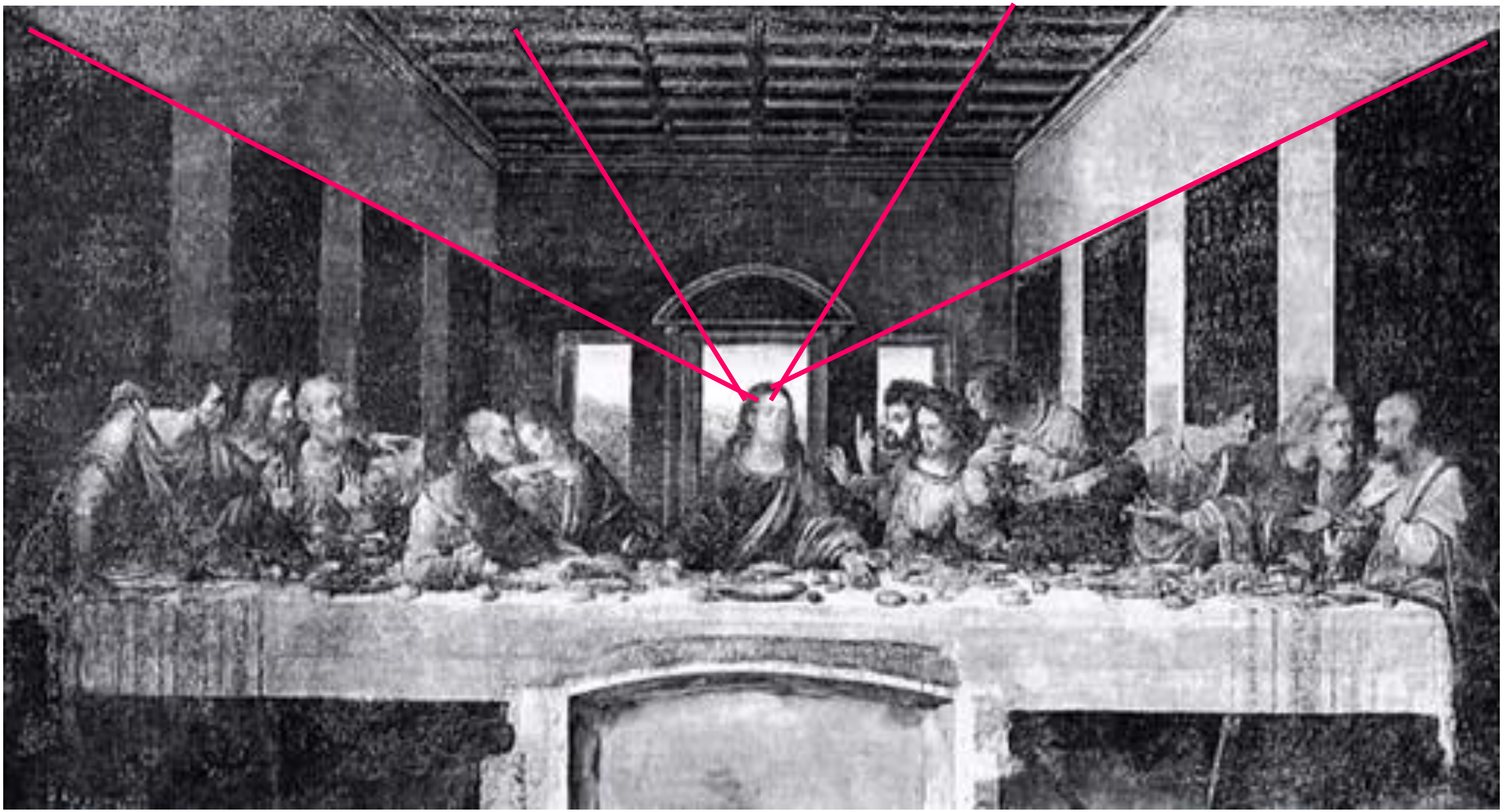
Vanishing points



- A scene can have more than one vanishing point
- Different 3-D directions correspond different vanishing points



Vanishing Points



Vanishing Point

- In the **projective plane**, parallel lines meet at a point at infinity

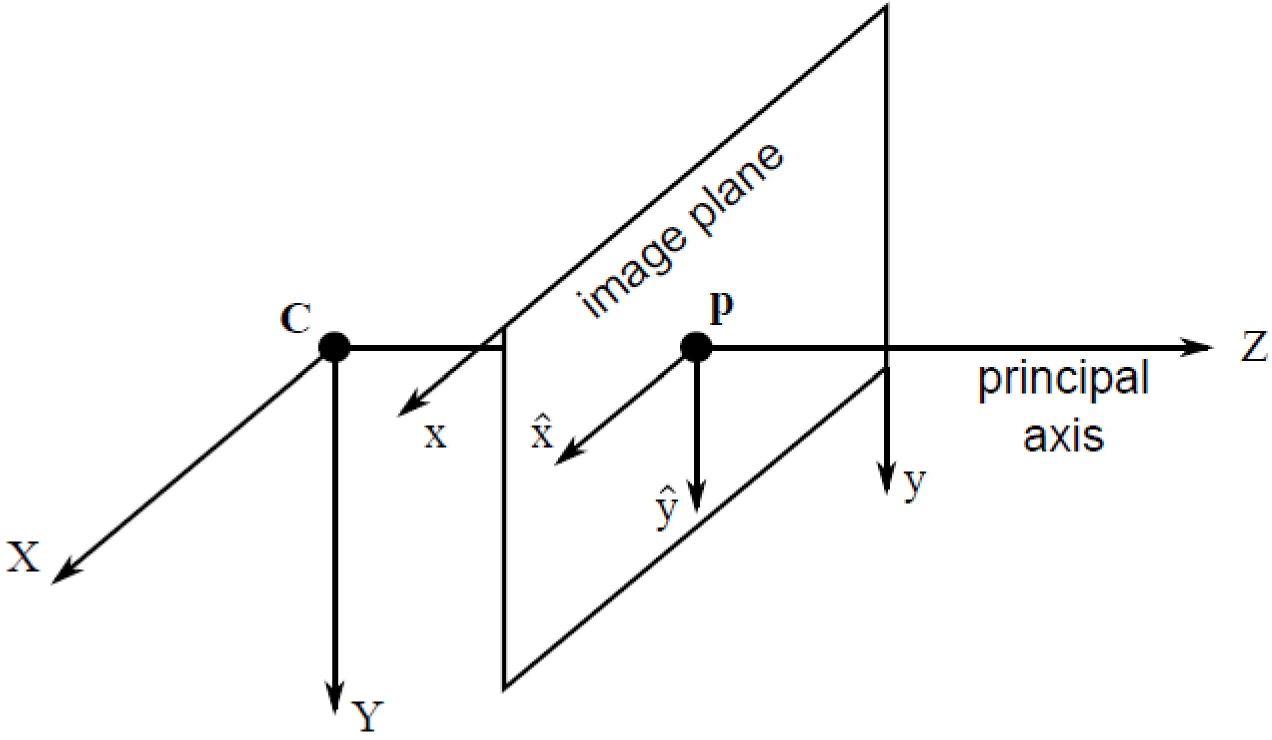
- The 2D vanishing point in the image is the perspective projection of this 3D point at infinity

What is a Camera?

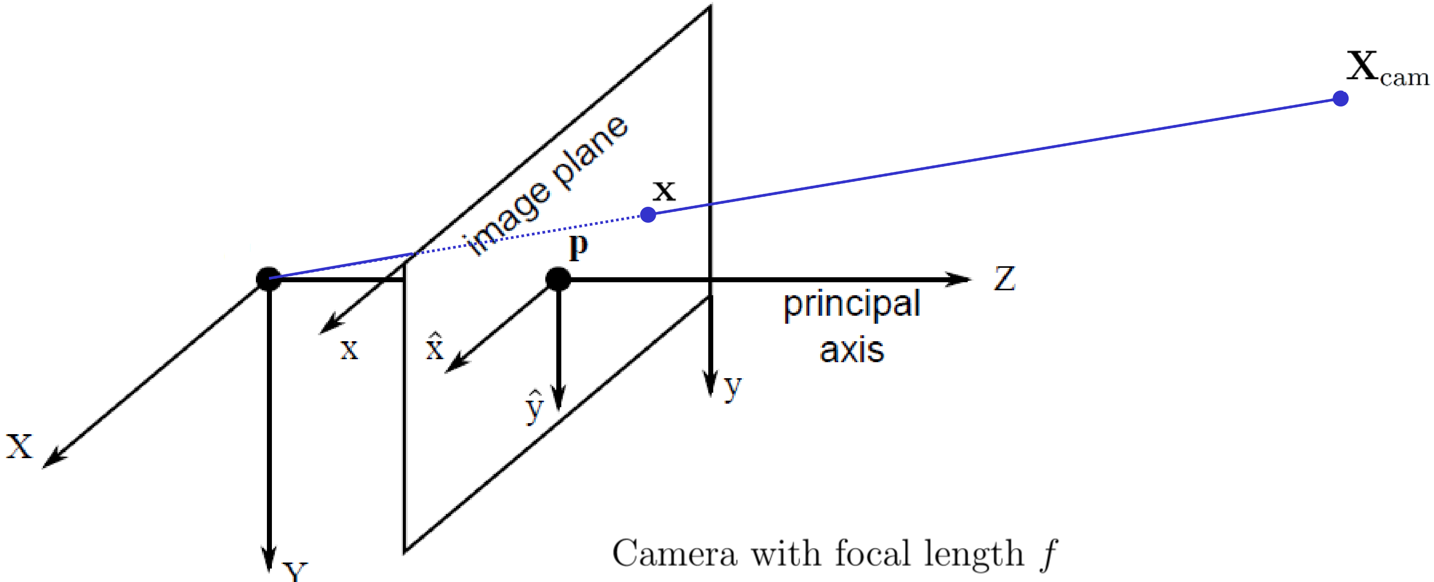
- A mathematical expression that relates points in 3D to points in an image for different types of physical cameras or imaging situations

Geometry

- How do 3D world points project to 2D image points?



Camera projection



Camera with focal length f

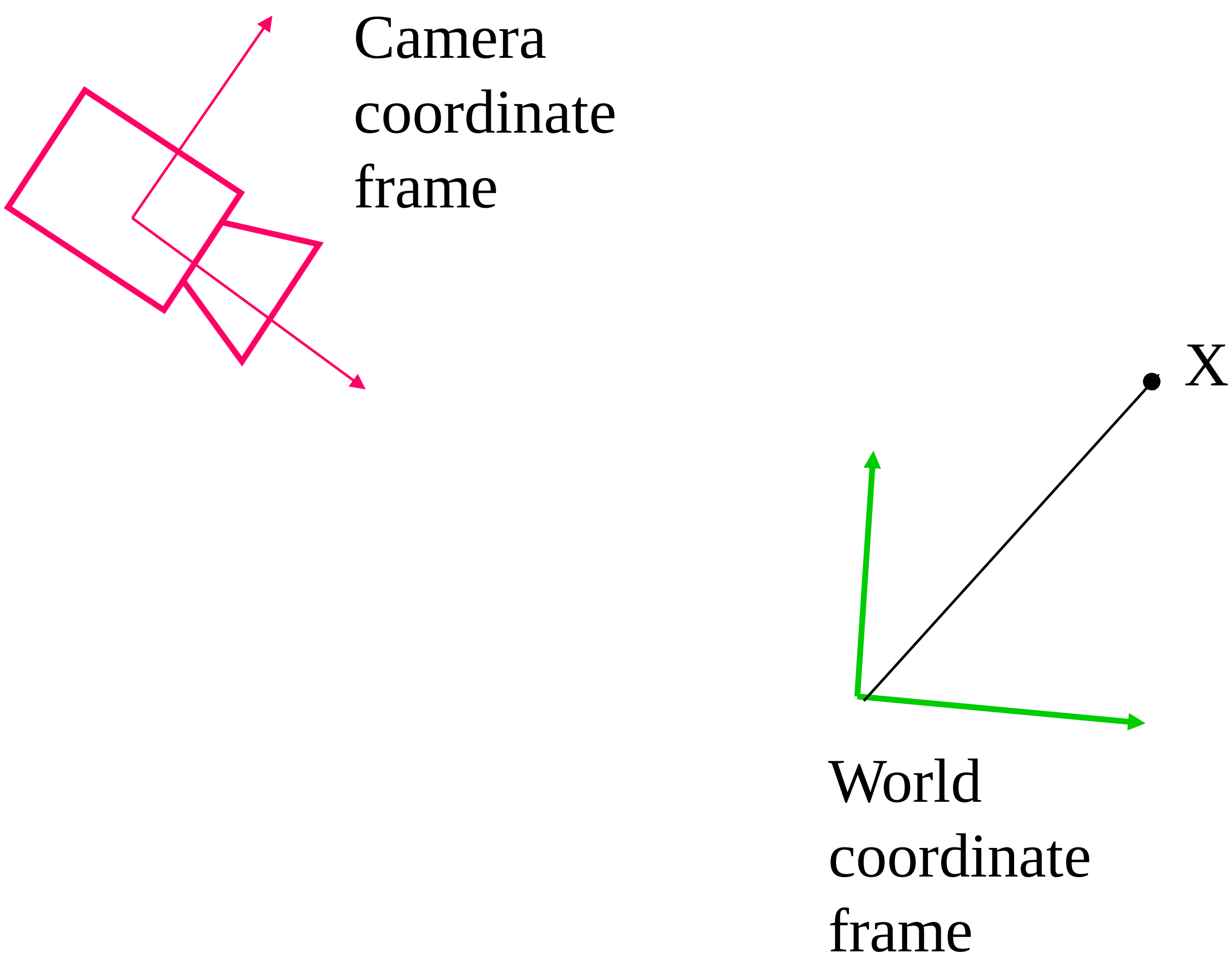
Inhomogeneous coordinates

$$(\tilde{X}_{\text{cam}}, \tilde{Y}_{\text{cam}}, \tilde{Z}_{\text{cam}}) \mapsto \left(f\frac{\tilde{X}_{\text{cam}}}{\tilde{Z}_{\text{cam}}}, f\frac{\tilde{Y}_{\text{cam}}}{\tilde{Z}_{\text{cam}}} \right)$$

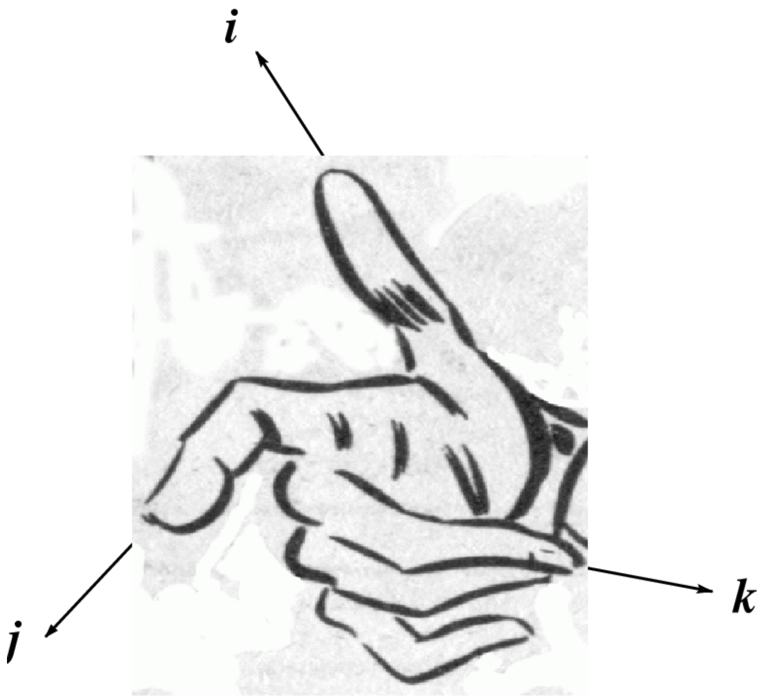
Homogeneous coordinates

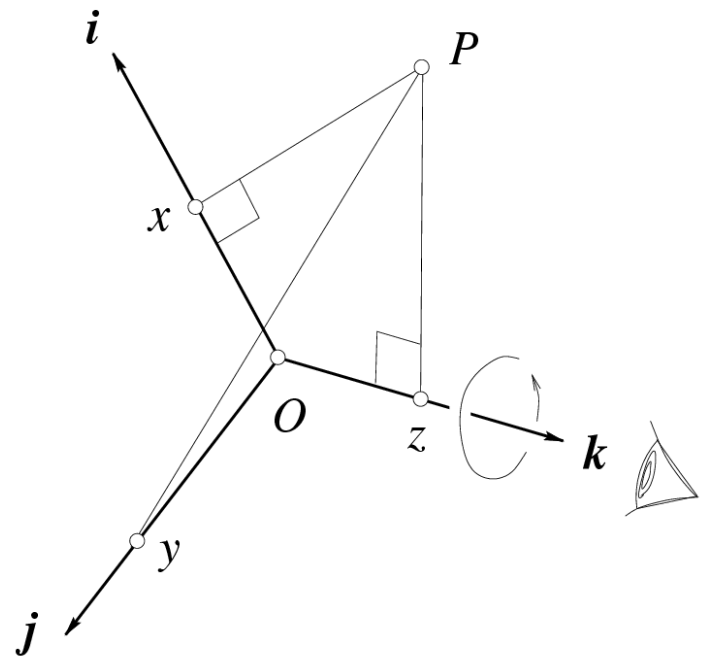
$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_{\text{cam}} \\ Y_{\text{cam}} \\ Z_{\text{cam}} \\ T_{\text{cam}} \end{bmatrix}$$

What if camera coordinate system differs from world coordinate system?

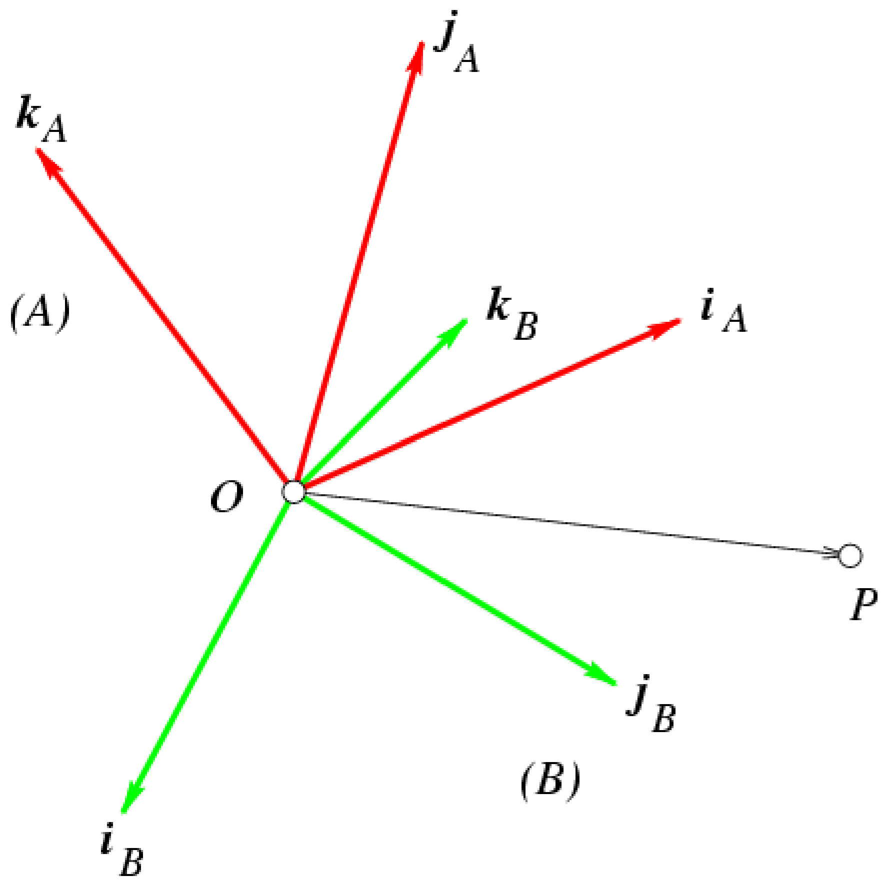


Euclidean Coordinate Systems





Coordinate Change: Rotation Only



$$\begin{bmatrix} \tilde{X}' \\ \tilde{Y}' \\ \tilde{Z}' \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix}$$

$$\tilde{\mathbf{X}}' = \mathtt{R}\tilde{\mathbf{X}}$$

$$\begin{bmatrix} \tilde{\mathbf{X}}' \\ 1 \end{bmatrix} = \begin{bmatrix} \mathtt{R} & \mathbf{0} \\ \mathbf{0}^\top & 1 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}} \\ 1 \end{bmatrix}$$

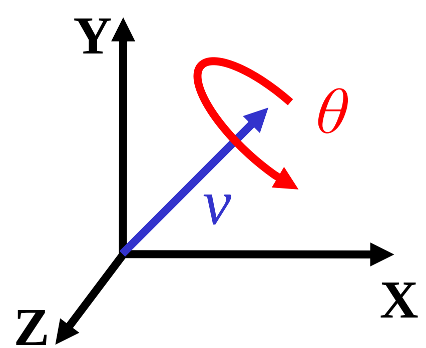
$$\mathbf{X}' = \begin{bmatrix} \mathtt{R} & \mathbf{0} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{X}$$

Some points about SO(n)

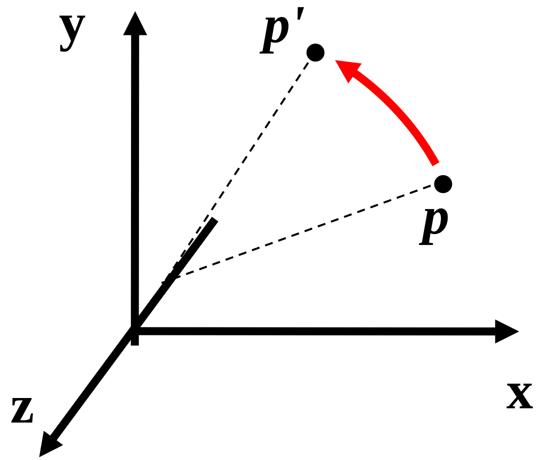
- $SO(n) = \{ R \in \Re^{nxn} : R^T R = I, \det(R) = 1\}$
  - SO(2): rotation matrices in plane $\Re^2$
  - SO(3): rotation matrices in 3-space $\Re^3$
- Forms a Group under matrix product operation:
  - Identity
  - Inverse
  - Associative
  - Closure
- Closed (finite intersection of closed sets)
- Bounded $R_{i,j} \in [-1, +1]$
- Does not form a vector space.
- Manifold of dimension n(n-1)/2
  - Dim(SO(2)) = 1
  - Dim(SO(3)) = 3

Parameterizations of SO(3)

- Even though a rotation matrix is 3x3 with nine elements, it only has three degrees of freedom. It can be parameterized with three elements. There are many parameterizations.
- Other common parameterizations
  - Euler Angles
  - Axis Angle
  - Quaternions
    - four parameters; homogeneous



3-D Rotation about the Z axis



$$\begin{bmatrix} \tilde{X}' \\ \tilde{Y}' \\ \tilde{Z}' \end{bmatrix} = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix}$$

$$\tilde{\mathbf{X}}' = \mathtt{R}_Z(\gamma)\tilde{\mathbf{X}}$$

3-D rotations about X and Y axes

- About X axis:

$$\begin{bmatrix}\tilde{X}' \\ \tilde{Y}' \\ \tilde{Z}'\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha\end{bmatrix} \begin{bmatrix}\tilde{X} \\ \tilde{Y} \\ \tilde{Z}\end{bmatrix}$$

$$\tilde{\mathbf{X}}' = \mathtt{R}_X(\alpha)\tilde{\mathbf{X}}$$

- About Y axis:

$$\begin{bmatrix}\tilde{X}' \\ \tilde{Y}' \\ \tilde{Z}'\end{bmatrix} = \begin{bmatrix}\cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta\end{bmatrix} \begin{bmatrix}\tilde{X} \\ \tilde{Y} \\ \tilde{Z}\end{bmatrix}$$

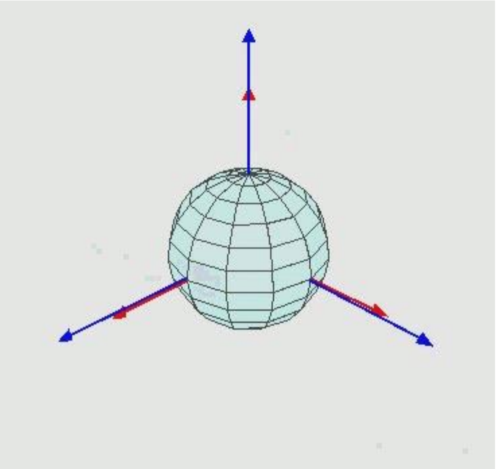
$$\tilde{\mathbf{X}}' = \mathtt{R}_Y(\beta)\tilde{\mathbf{X}}$$

3D rotation, Euler angles

- A sequence of 3 elemental rotations
- 12 possible sequences

X-Y-X	Y-X-Y	Z-X-Y
X-Y-Z	Y-X-Z	Z-X-Z
X-Z-X	Y-Z-X	Z-Y-X
X-Z-Y	Y-Z-Y	Z-Y-Z

Tait-Bryan angles, also



  - Example: Roll-Pitch-Yaw (ZYX convention)
    - Rotation about X-axis, followed by rotation about Y-axis, followed by rotation about Z-axis

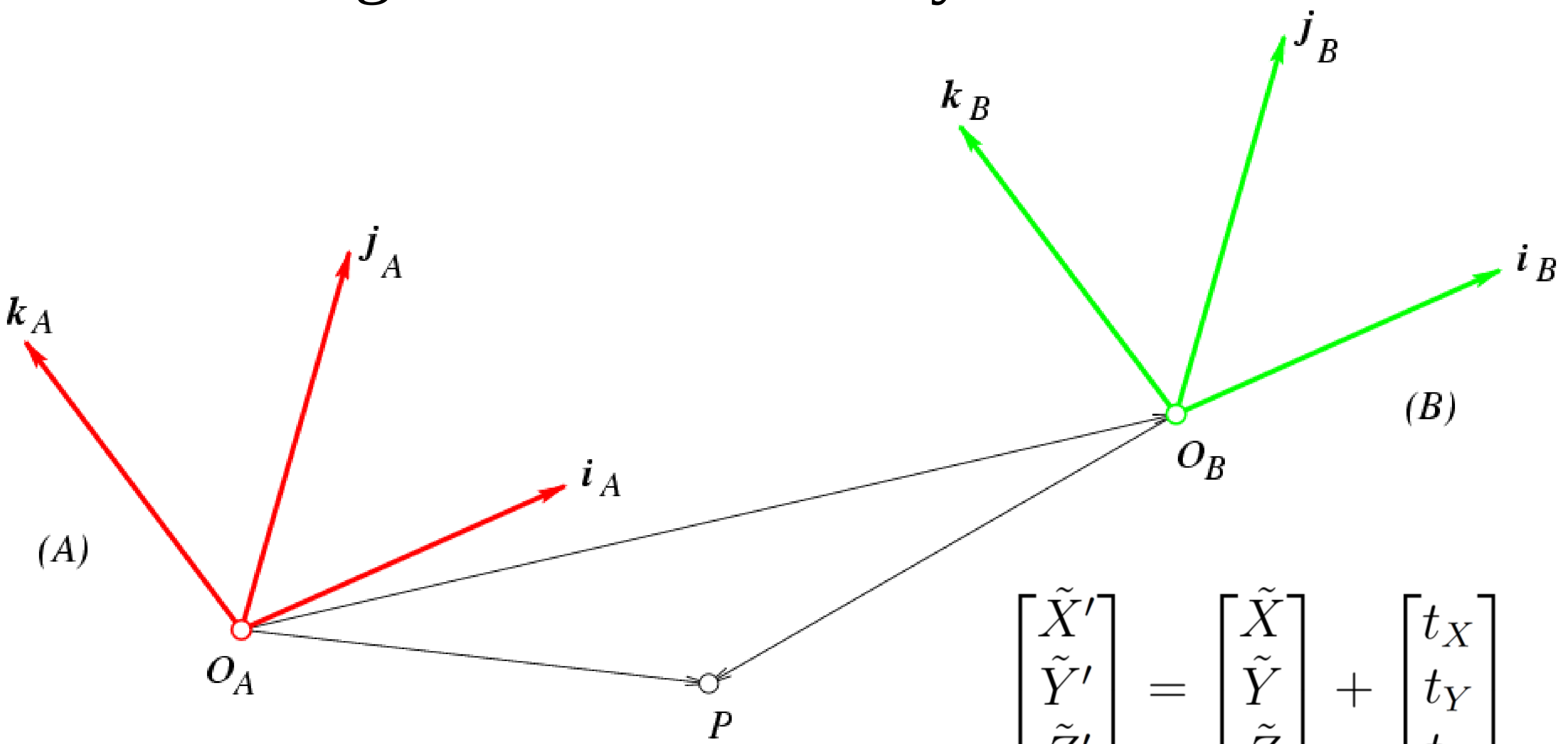
Euler Angles: Roll-Pitch-Yaw

- Composition of rotations

$$R = R_Z(\gamma)\, R_Y(\beta)\, R_X(\alpha)$$

$$R = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix}$$

Coordinate Change: Translation Only



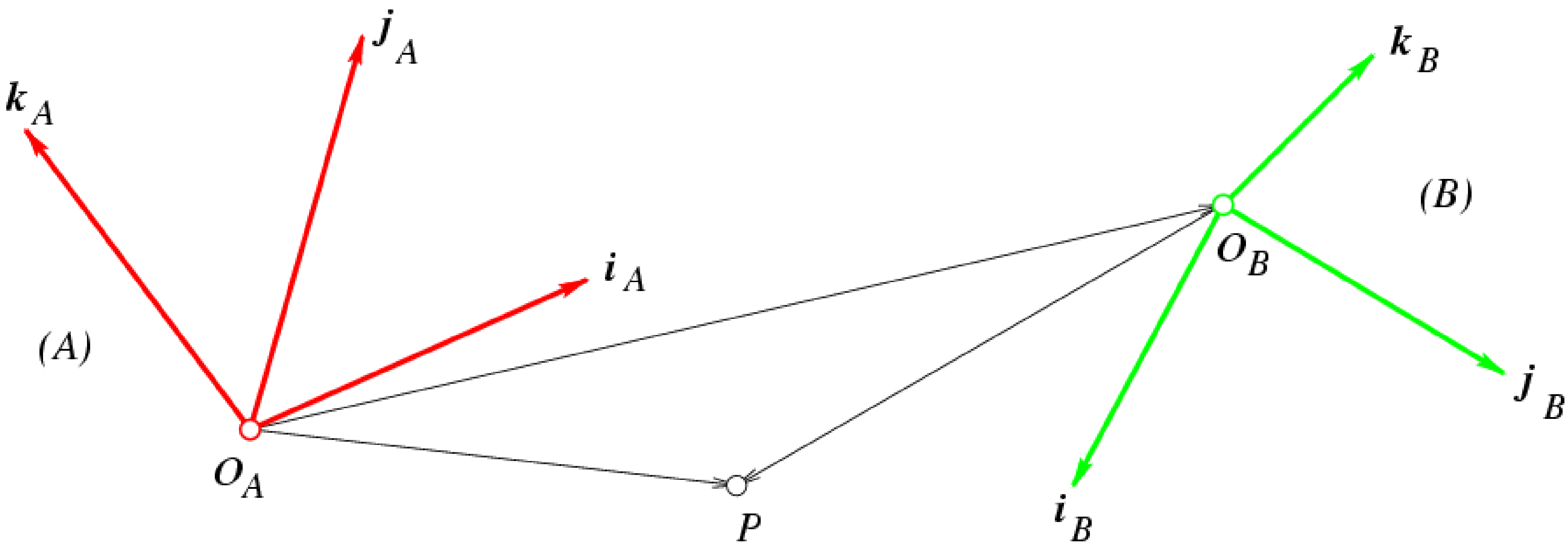
$$\begin{bmatrix} \tilde{X}' \\ \tilde{Y}' \\ \tilde{Z}' \end{bmatrix} = \begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix} + \begin{bmatrix} t_X \\ t_Y \\ t_Z \end{bmatrix}$$

$$\tilde{\mathbf{X}}' = \tilde{\mathbf{X}} + \mathbf{t}$$

$$\begin{bmatrix} \tilde{\mathbf{X}}' \\ 1 \end{bmatrix} = \begin{bmatrix} \mathtt{I} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}} \\ 1 \end{bmatrix}$$

$$\mathbf{X}' = \begin{bmatrix} \mathtt{I} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{X}$$

Coordinate Changes: Rotation and Translation

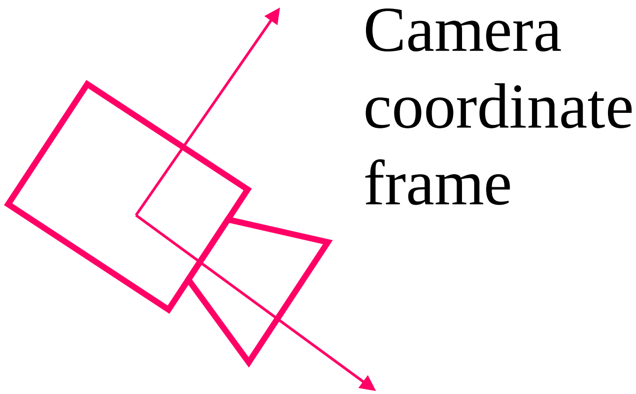


$$\tilde{\mathbf{X}}' = \mathtt{R}\tilde{\mathbf{X}} + \mathbf{t}$$

$$\begin{bmatrix} \tilde{\mathbf{X}}' \\ 1 \end{bmatrix} = \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}} \\ 1 \end{bmatrix}$$

$$\mathbf{X}' = \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{X}$$

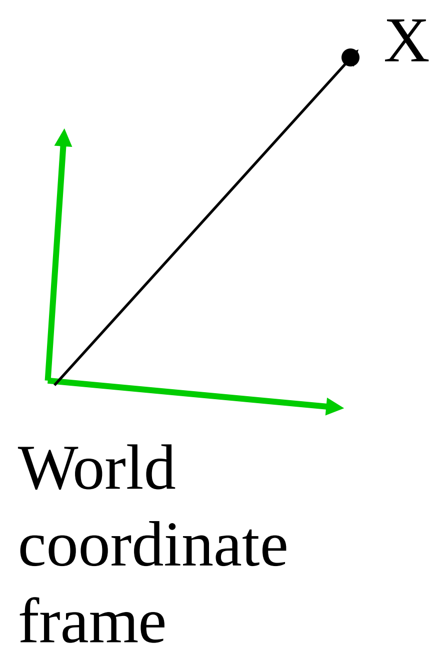
What if camera coordinate system differs from world coordinate system?



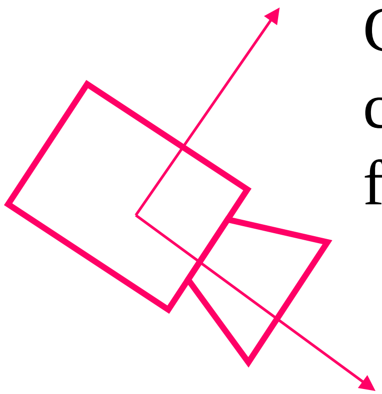
$$\tilde{\mathbf{X}}_{\text{camera}} = \mathtt{R}\tilde{\mathbf{X}}_{\text{world}} + \mathbf{t}$$

$$\begin{bmatrix} \tilde{\mathbf{X}}_{\text{camera}} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}}_{\text{world}} \\ 1 \end{bmatrix}$$

$$\mathbf{X}_{\text{camera}} = \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{X}_{\text{world}}$$

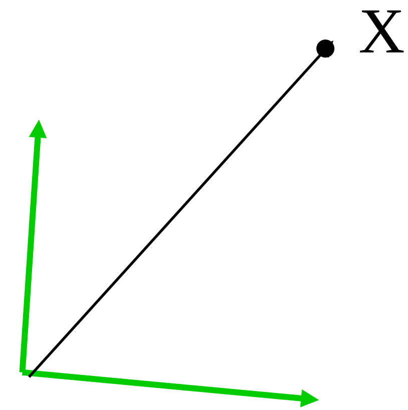


Where is the camera center in the world coordinate system?



Camera coordinate frame

$$\boxed{\begin{aligned} \tilde{\mathbf{0}}_{\text{cam}} &= \mathtt{R}\tilde{\mathbf{C}} + \mathbf{t} \\ -\mathtt{R}\tilde{\mathbf{C}} &= \mathbf{t} \\ \tilde{\mathbf{C}} &= -\mathtt{R}^\top \mathbf{t} \end{aligned}}$$



$$\tilde{\mathbf{X}}_{\text{camera}} = \mathtt{R}\tilde{\mathbf{X}}_{\text{world}} + \mathbf{t}$$

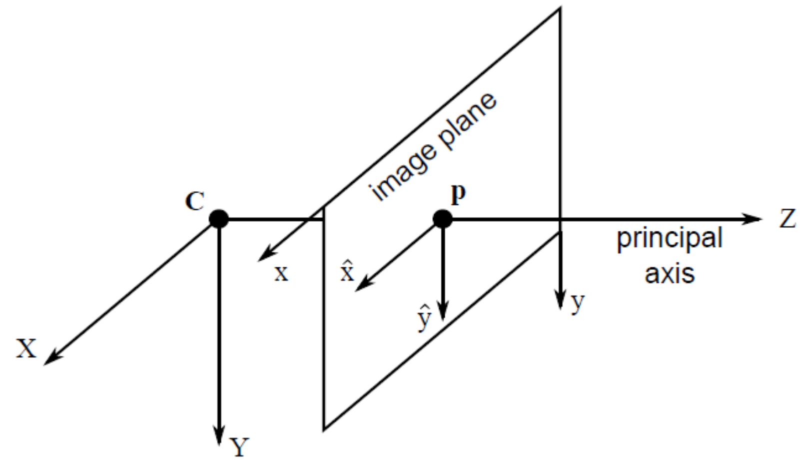
$$\begin{bmatrix} \tilde{\mathbf{X}}_{\text{camera}} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}}_{\text{world}} \\ 1 \end{bmatrix}$$

$$\mathbf{X}_{\text{camera}} = \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{X}_{\text{world}}$$

World coordinate frame

Intrinsic parameters

$$\mathtt{K} = \begin{bmatrix} \alpha_x & 0 & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\left.\begin{aligned} \alpha_x &= f m_x \\ \alpha_y &= f m_y \end{aligned}\right\} \text{focal length in terms of pixel dimensions}$$

$$\left.\begin{aligned} x_0 &= m_x p_x \\ y_0 &= m_y p_y \end{aligned}\right\} \text{principal point in terms of pixel dimensions}$$

m_x and m_y are number of pixels per unit distance in x and y directions, respectively

Perspective projection camera model

- Extrinsic parameters: Euclidean transformation from the world coordinate frame to the camera coordinate frame
- Intrinsic parameters: Camera calibration matrix embodying focal length and principal point (and pixel aspect ratio and skew)

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} \alpha_x & 0 & x_0 \\ 0 & \alpha_y & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_X \\ r_{21} & r_{22} & r_{23} & t_Y \\ r_{31} & r_{32} & r_{33} & t_Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix}$$

$$\mathbf{x} = \mathtt{K}[\mathtt{I} \mid \mathbf{0}] \begin{bmatrix} \mathtt{R} & \mathbf{t} \\ \mathbf{0}^\top & 1 \end{bmatrix} \mathbf{X}$$

$$\mathbf{x} = \mathtt{K}[\mathtt{R} \mid \mathbf{t}]\mathbf{X}$$

$$\mathbf{x} = \mathtt{P}\mathbf{X}$$

Camera projection matrix

Linear projection of points in homogeneous coordinates

$$\mathbf{x} = \mathtt{P}\mathbf{X}$$

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ T \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} Xp_{11} + Yp_{12} + Zp_{13} + Tp_{14} \\ Xp_{21} + Yp_{22} + Zp_{23} + Tp_{24} \\ Xp_{31} + Yp_{32} + Zp_{33} + Tp_{34} \end{bmatrix}$$

Nonlinear projection of points in inhomogeneous coordinates

$$\tilde{x} = \frac{x}{w} = \frac{Xp_{11} + Yp_{12} + Zp_{13} + Tp_{14}}{Xp_{31} + Yp_{32} + Zp_{33} + Tp_{34}} \text{ and}$$
$$\tilde{y} = \frac{y}{w} = \frac{Xp_{21} + Yp_{22} + Zp_{23} + Tp_{24}}{Xp_{31} + Yp_{32} + Zp_{33} + Tp_{34}}$$

Do not forget this is a nonlinear projection

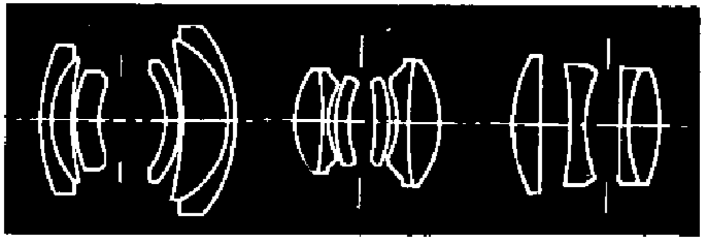
Deviations from the lens model

Deviations from this ideal are *aberrations*

Two types

1. geometrical
   - spherical aberration
   - astigmatism
   - distortion
   - coma
2. chromatic

Aberrations are reduced by combining lenses

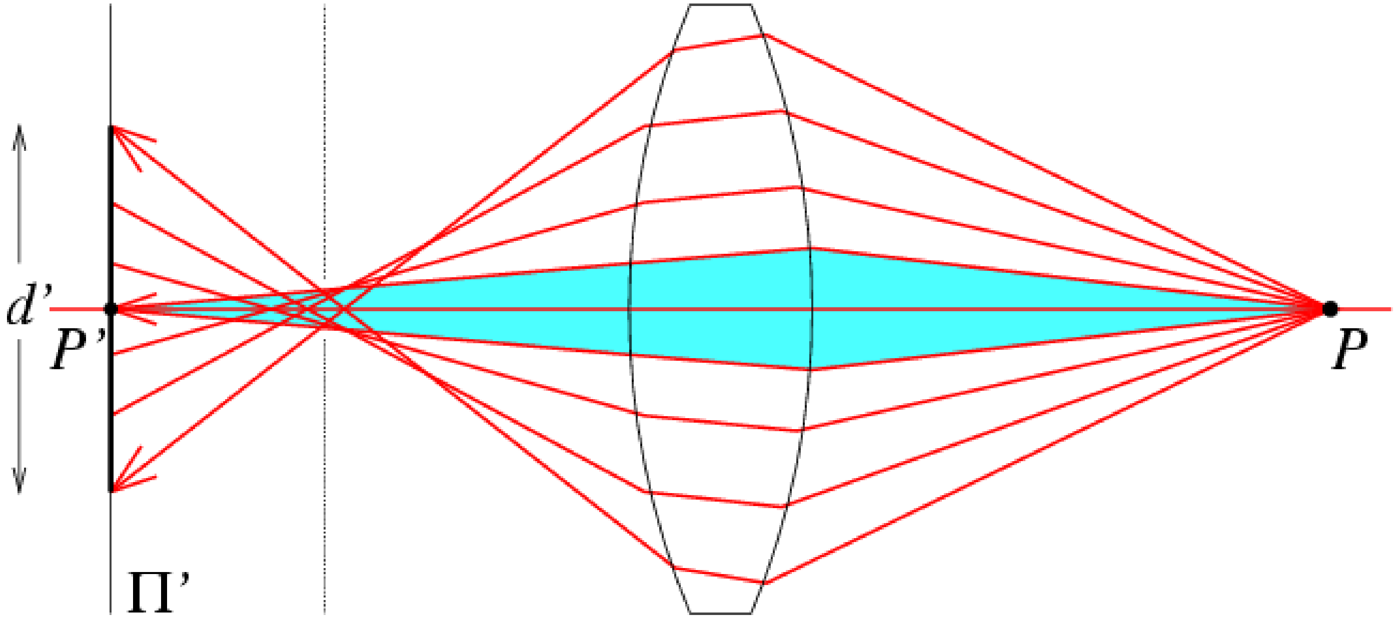


Compound lenses

Spherical aberration

Rays parallel to the axis do not converge

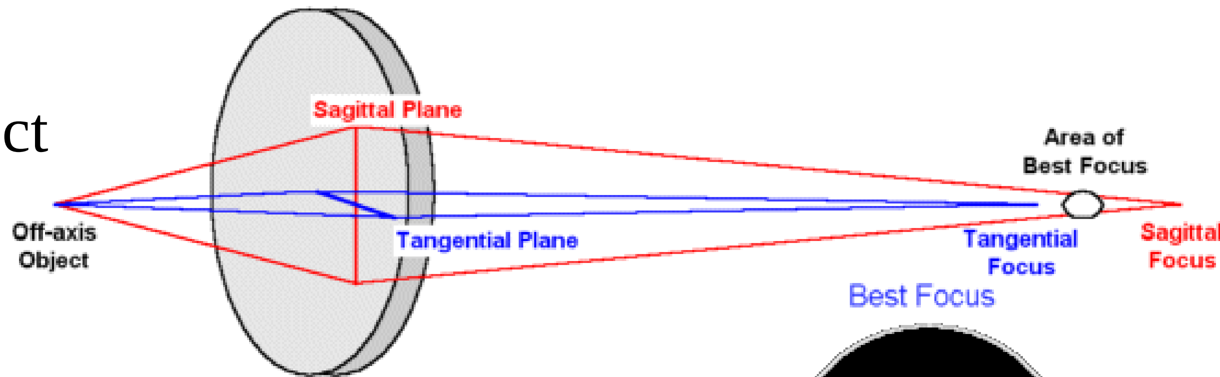
Outer portions of the lens yield smaller focal lengths

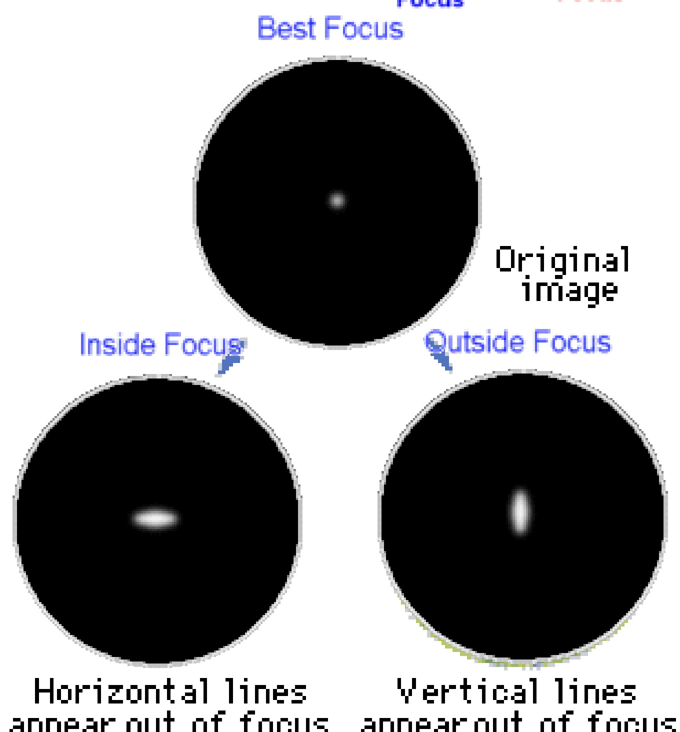


Astigmatism

An optical system with astigmatism is one where rays that propagate in two perpendicular planes have different focus. If an optical system with astigmatism is used to form an image of a cross, the vertical and horizontal lines will be in sharp focus at two different distances.

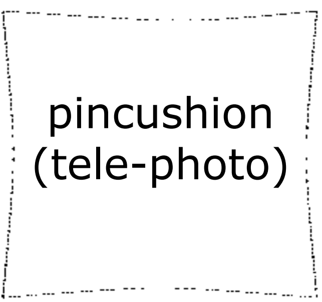
object

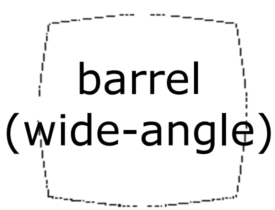


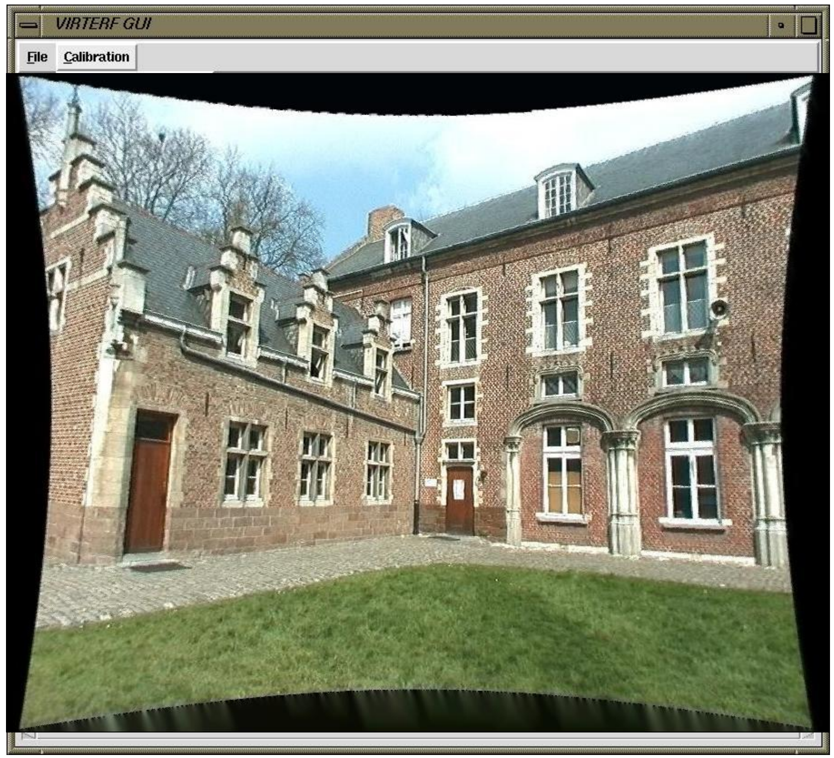


Distortion

magnification/focal length different
for different angles of inclination



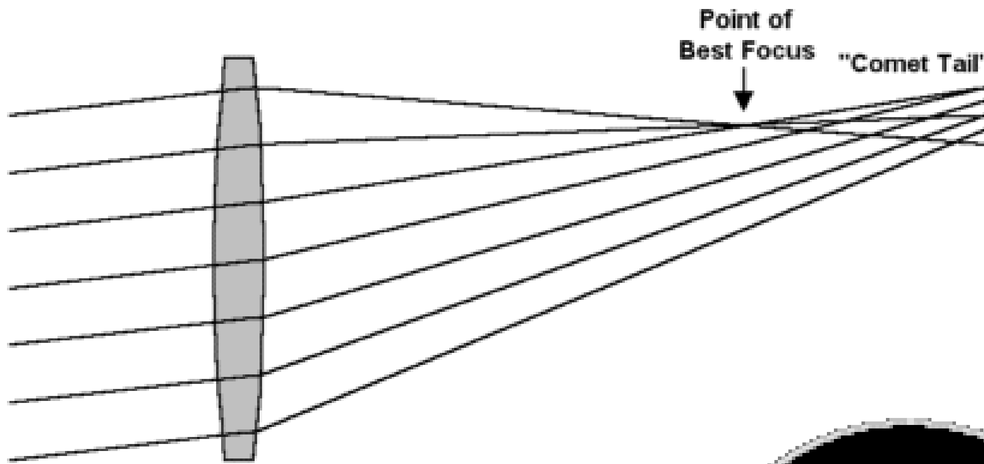


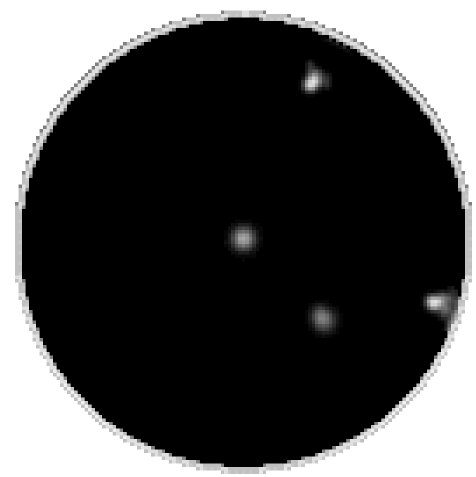


Can be corrected! (if parameters are know)

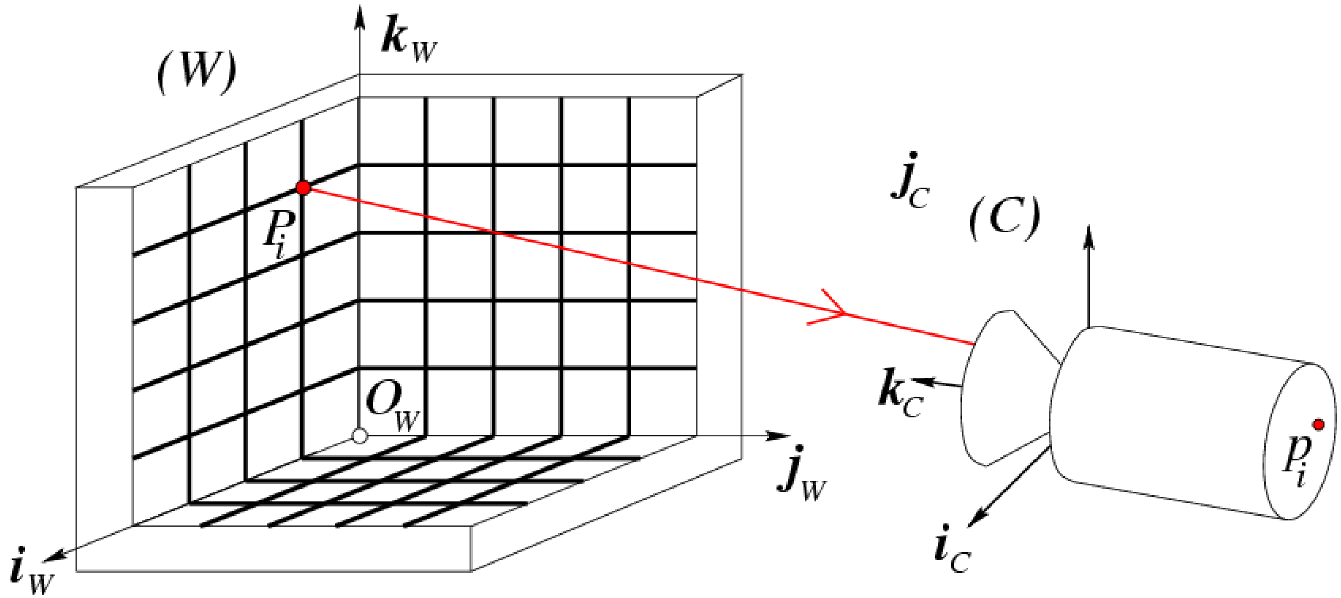
Coma

point off the axis depicted as comet shaped blob





Camera Calibration

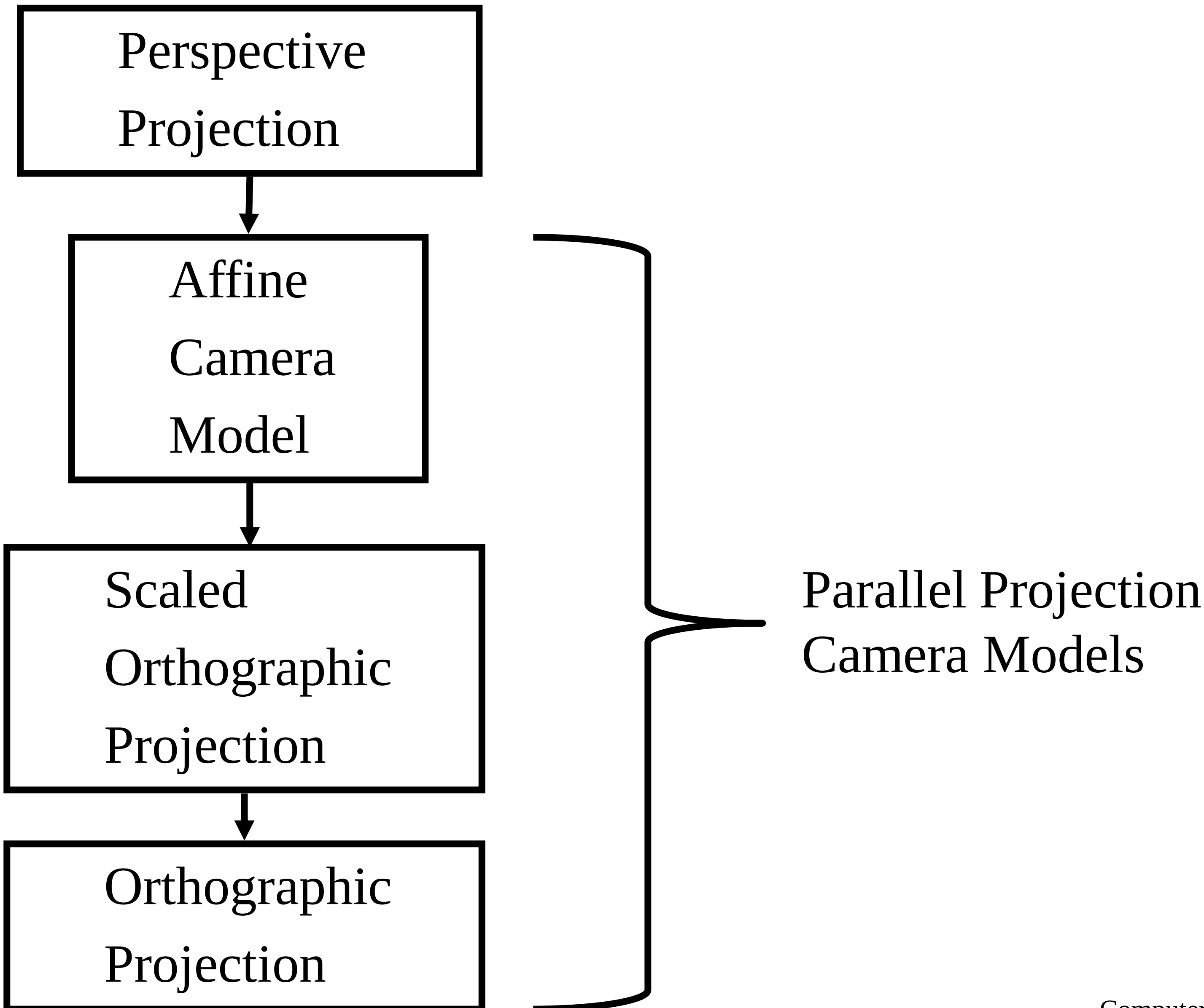


- Given n points $\mathbf{P}_1, \ldots, \mathbf{P}_n$ with known 3-D position and their pixel coordinates $\mathbf{x}_1, \ldots, \mathbf{x}_n$, estimate intrinsic **K** and extrinsic camera parameters and lens distortion parameters
- See textbook for details.
- Camera Calibration Toolbox for Matlab (Bouguet)

Archived

http://www.vision.caltech.edu/bouguetj/calib_doc/

Camera Models

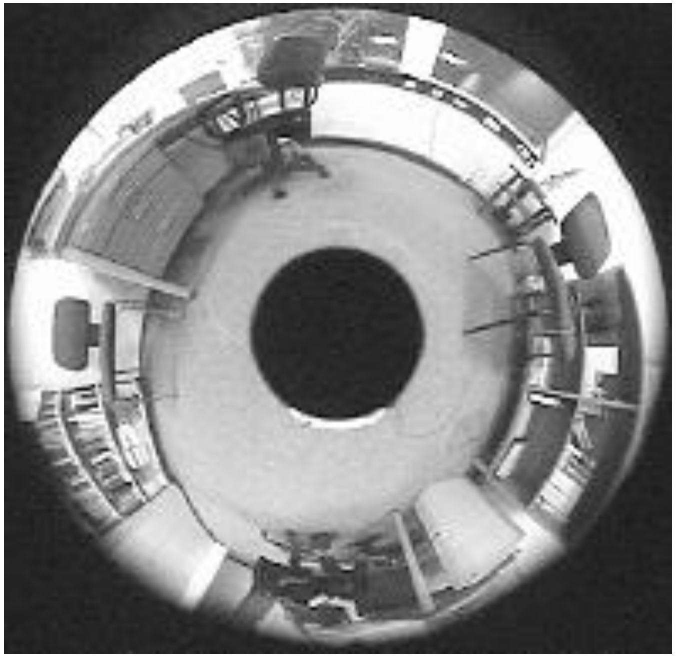


For all cameras?

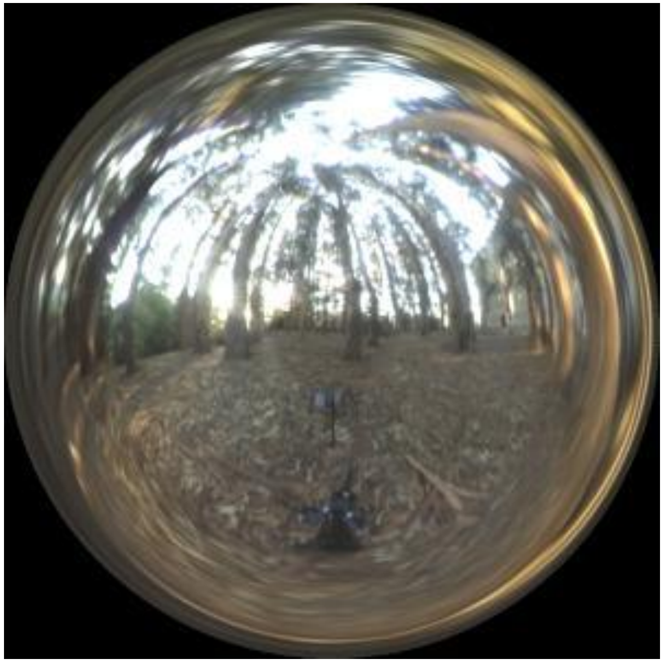
Other camera models

- Generalized camera – maps points lying on rays and maps them to points on the image plane.

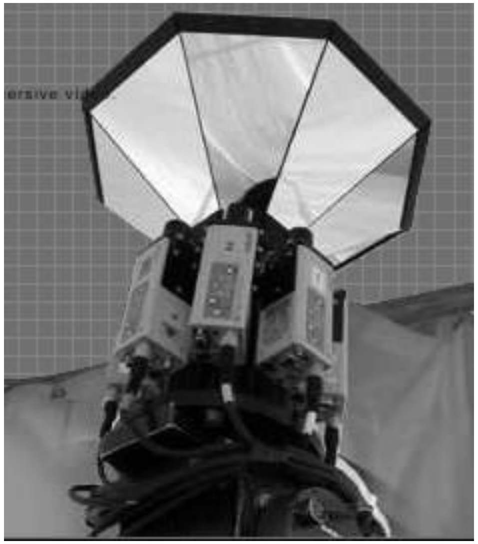
Omnicam (hemispherical)

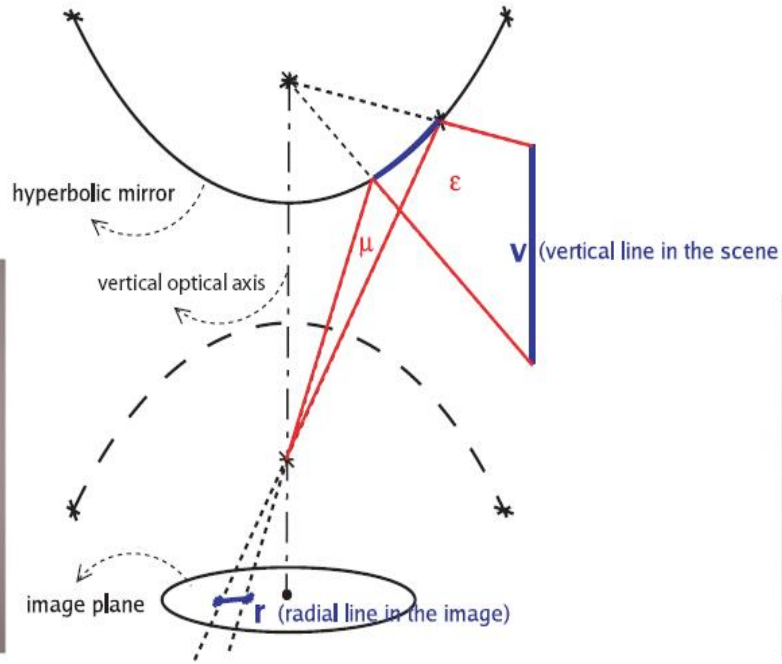


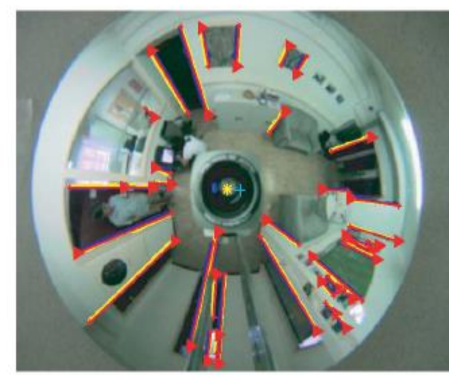
Light Probe (spherical)



Some Alternative “Cameras”







Next Lecture

- Photometric image formation