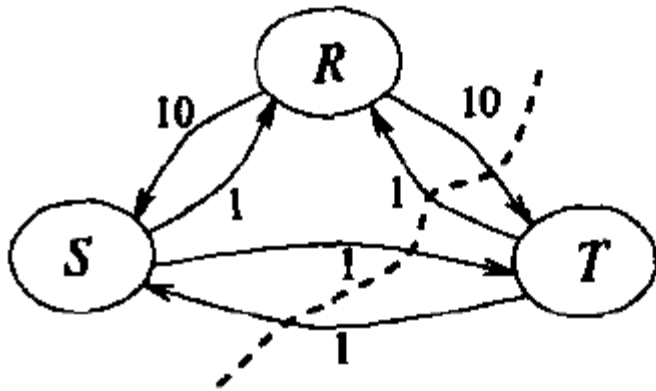


CSE248 Partitioning: Replication Cut, Retiming

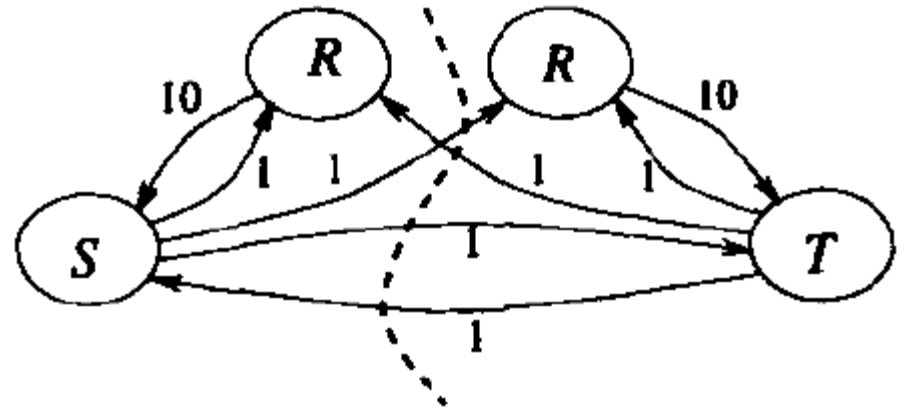
CK Cheng

Dept. of Computer Science and Engineering
University of California, San Diego

Replication Cut: Example

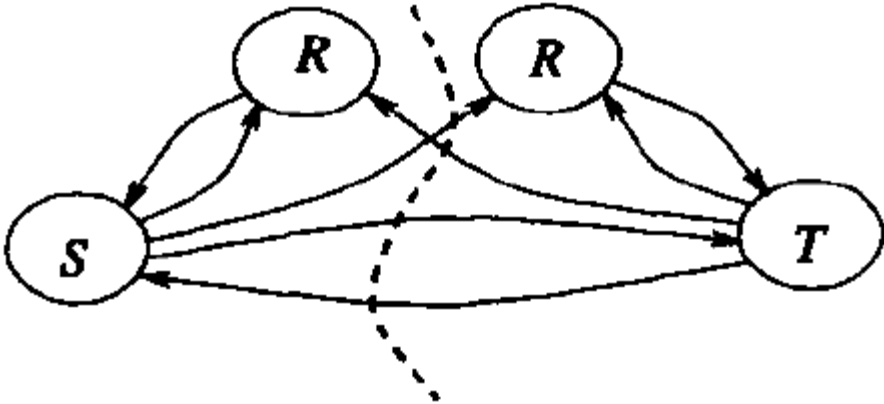
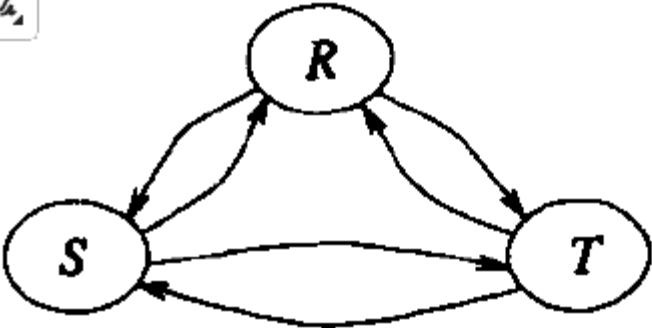


(a)

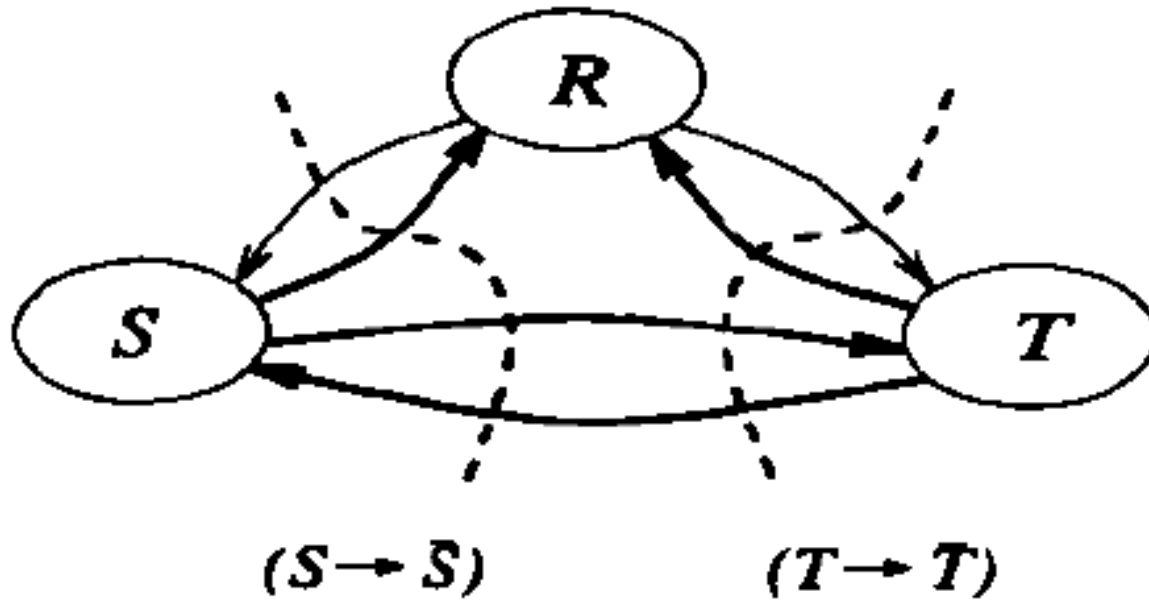


(b)

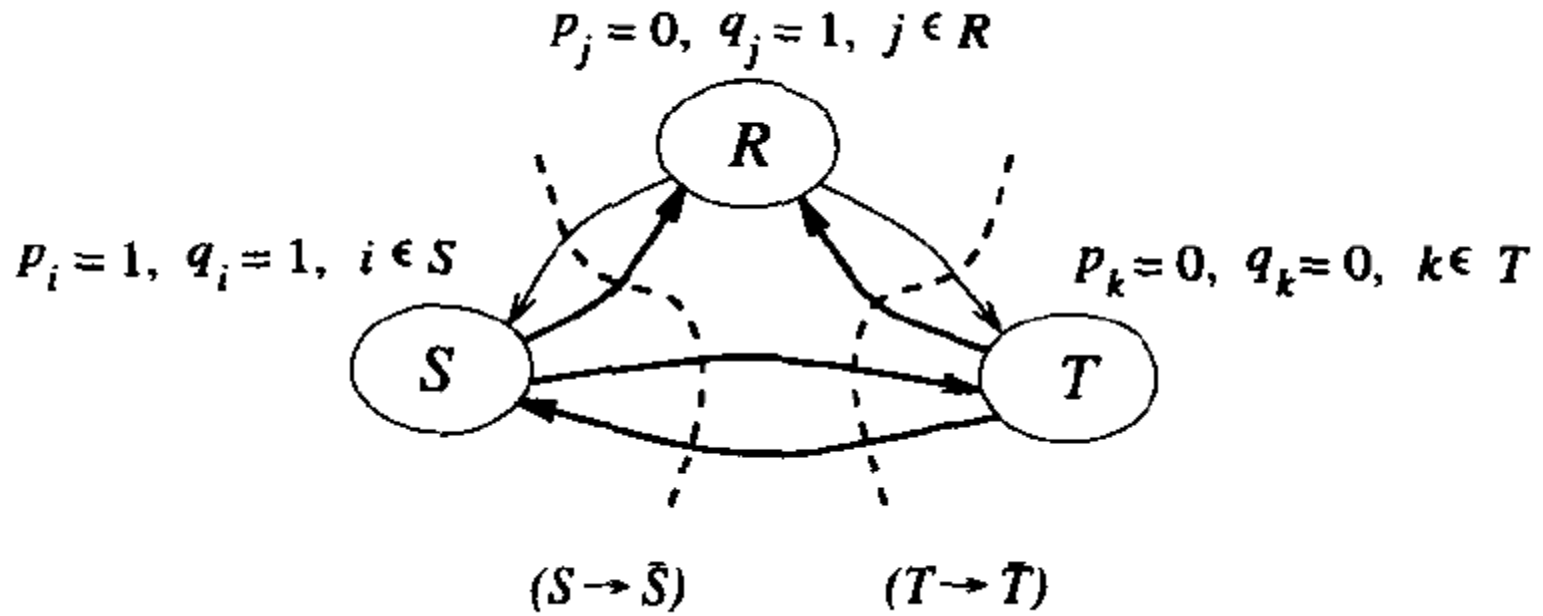
Replication Cut: Operation



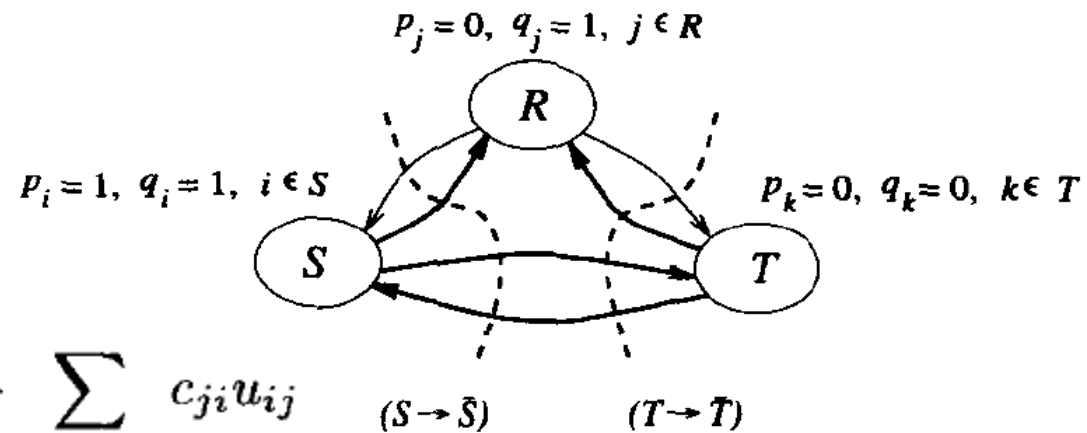
Replication Cut: Two directed cuts (S, \bar{S}) and (T, \bar{T})



Replication Cut: Two directed cuts (S, \bar{S}) and (T, \bar{T})



Replication Cut: Two directed cuts (S, \bar{S}) and (T, \bar{T})



$$\text{Obj: min } \sum_{(i,j) \in E} c_{ij} w_{ij} + \sum_{(j,i) \in E} c_{ji} u_{ij}$$

subject to

$$w_{ij} - p_i + p_j \geq 0 \quad \forall (i, j) \in E$$

$$u_{ij} - q_i + q_j \geq 0 \quad \forall (j, i) \in E$$

$$q_i - p_i \geq 0 \quad \forall i \in V, i \neq s, t$$

$$p_s = 1$$

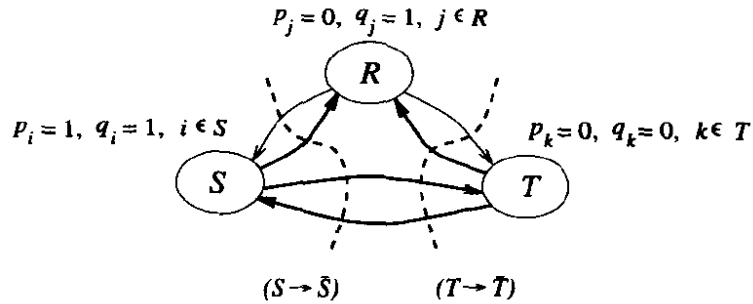
$$q_s = 1$$

$$p_t = 0$$

$$q_t = 0$$

$$w_{ij}, u_{ji} \geq 0 \quad \forall (i, j) \in E.$$

Replication Cut: Two directed cuts (S, \bar{S}) and (T, \bar{T})



$$\text{Obj: } \max a_s + b_s$$

subject to

$$\text{Obj: } \min \sum_{(i,j) \in E} c_{ij} w_{ij} + \sum_{(j,i) \in E} c_{ji} u_{ij}$$

$$x_{ij} \leq c_{ij} \quad \forall (i,j) \in E$$

$$x'_{ij} \leq c_{ji} \quad \forall (j,i) \in E$$

$$\sum_j -x_{ij} + x_{ji} - \lambda_i = 0 \quad \forall i \in V, i \neq s, t$$

$$\sum_j -x'_{ij} + x'_{ji} + \lambda_i = 0 \quad \forall i \in V, i \neq s, t$$

$$\sum_j -x_{sj} + x_{js} + a_s = 0$$

$$\sum_j -x_{tj} + x_{jt} + a_t = 0$$

$$\sum_j -x'_{sj} + x'_{js} + b_s = 0$$

$$\sum_j -x'_{tj} + x'_{jt} + b_t = 0$$

$$\lambda_i, x_{ij}, x'_{ji} \geq 0 \quad \forall i \in V, (i,j) \in E$$

a_s, a_t, b_s, b_t : unrestricted.

subject to

$$w_{ij} - p_i + p_j \geq 0 \quad \forall (i,j) \in E$$

$$u_{ij} - q_i + q_j \geq 0 \quad \forall (j,i) \in E$$

$$q_i - p_i \geq 0 \quad \forall i \in V, i \neq s, t$$

$$p_s = 1$$

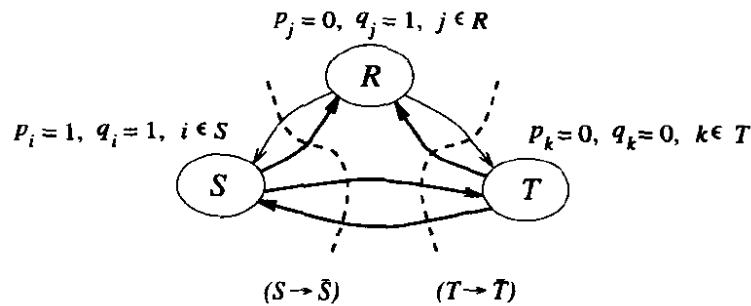
$$q_s = 1$$

$$p_t = 0$$

$$q_t = 0$$

$$w_{ij}, u_{ji} \geq 0 \quad \forall (i,j) \in E.$$

Replication Cut: Two directed cuts (S, \bar{S}) and (T, \bar{T})



$$\text{Obj: } \max a_s + b_s$$

subject to

$$x_{ij} \leq c_{ij} \quad \forall (i, j) \in E$$

$$x'_{ij} \leq c_{ji} \quad \forall (j, i) \in E$$

$$\sum_j -x_{ij} + x_{ji} - \lambda_i = 0 \quad \forall i \in V, i \neq s, t$$

$$\sum_j -x'_{ij} + x'_{ji} + \lambda_i = 0 \quad \forall i \in V, i \neq s, t$$

$$\sum_j -x_{sj} + x_{js} + a_s = 0$$

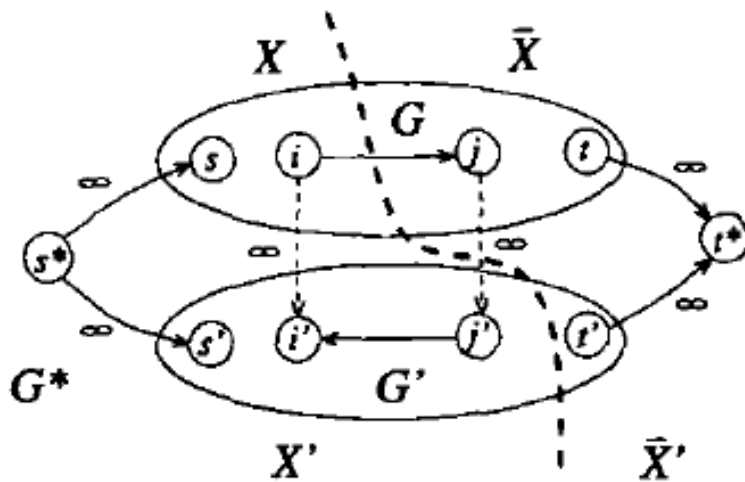
$$\sum_j -x_{tj} + x_{jt} + a_t = 0$$

$$\sum_j -x'_{sj} + x'_{js} + b_s = 0$$

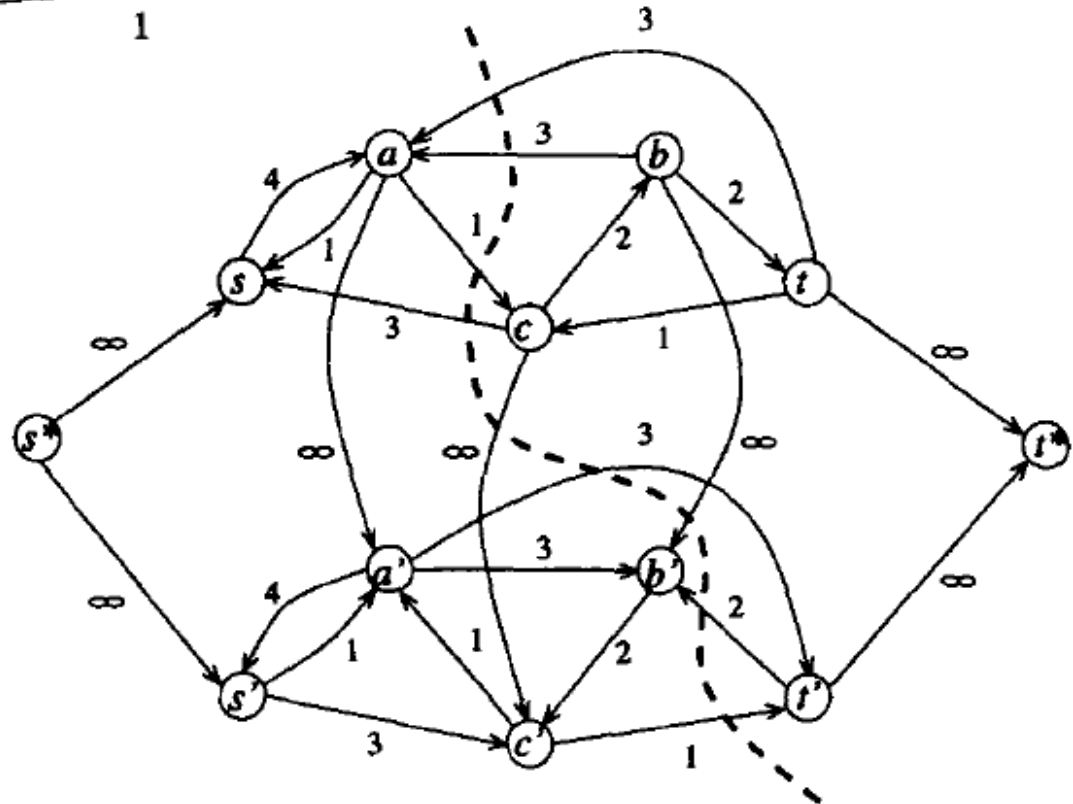
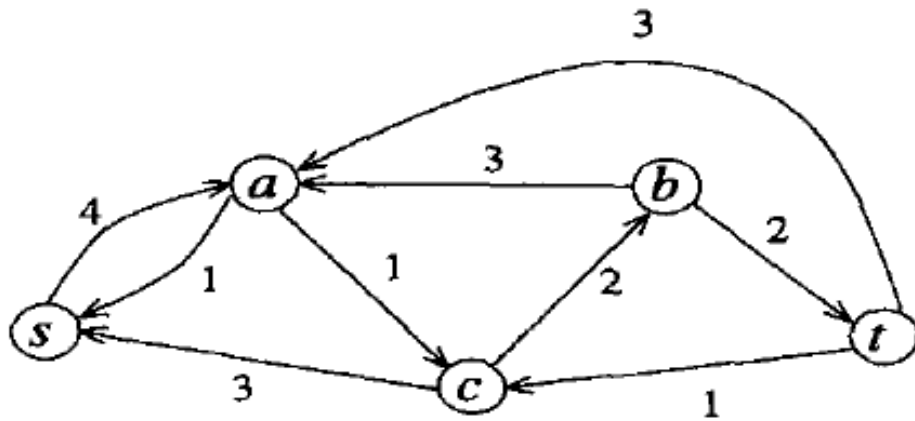
$$\sum_j -x'_{tj} + x'_{jt} + b_t = 0$$

$$\lambda_i, x_{ij}, x'_{ji} \geq 0 \quad \forall i \in V, (i, j) \in E$$

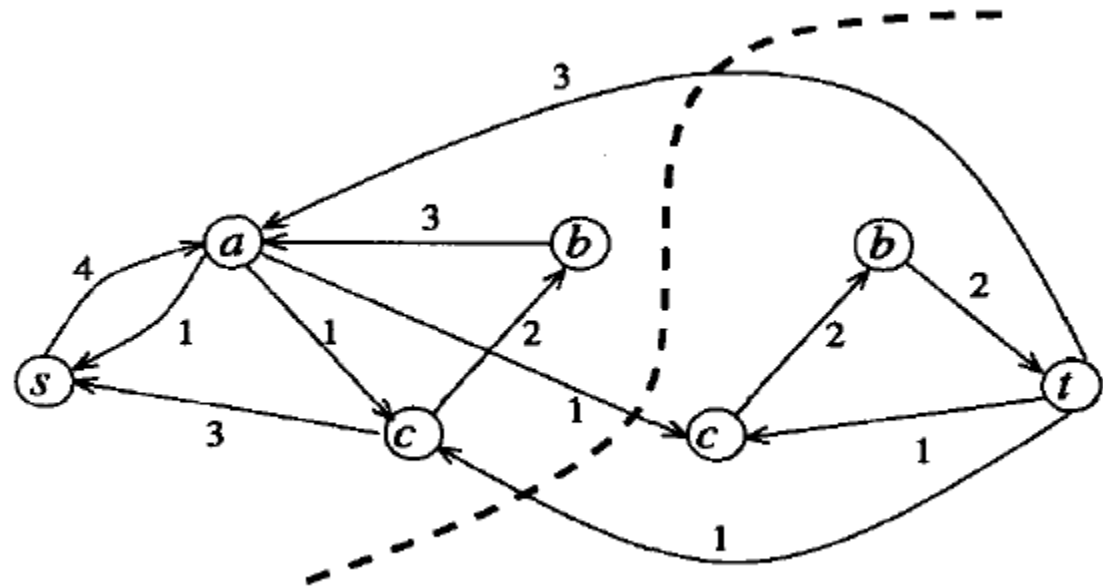
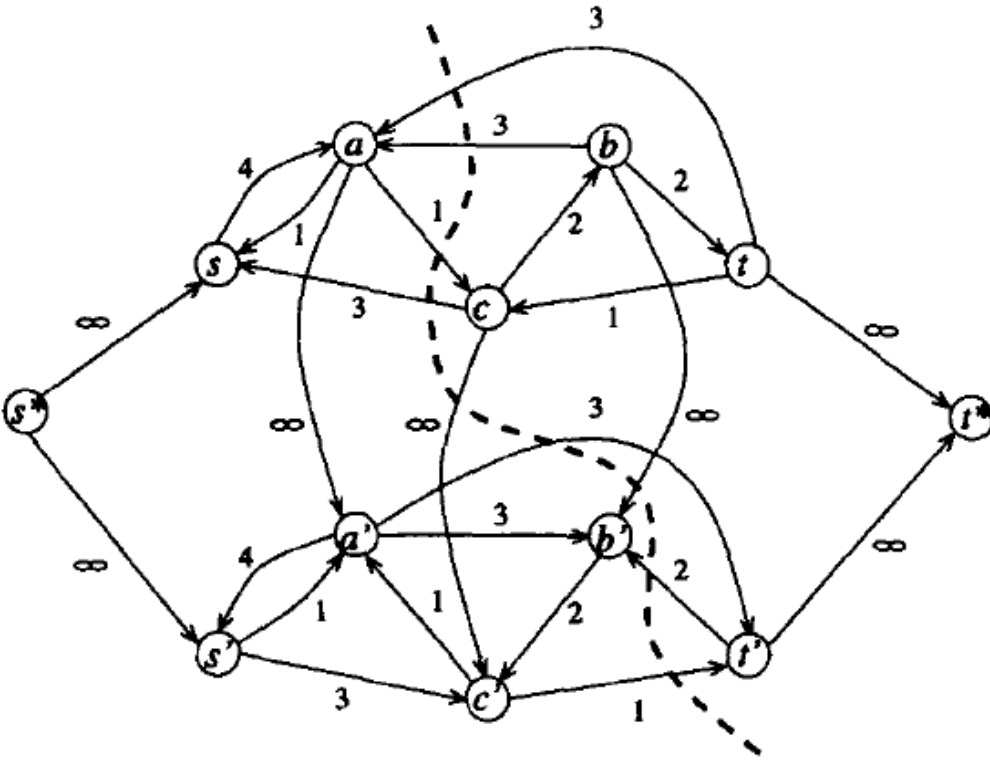
a_s, a_t, b_s, b_t : unrestricted.



Replication Cut: Two directed cuts (S, \bar{S}) and (T, \bar{T})



Replication Cut: Two directed cuts (S, \bar{S}) and (T, \bar{T})



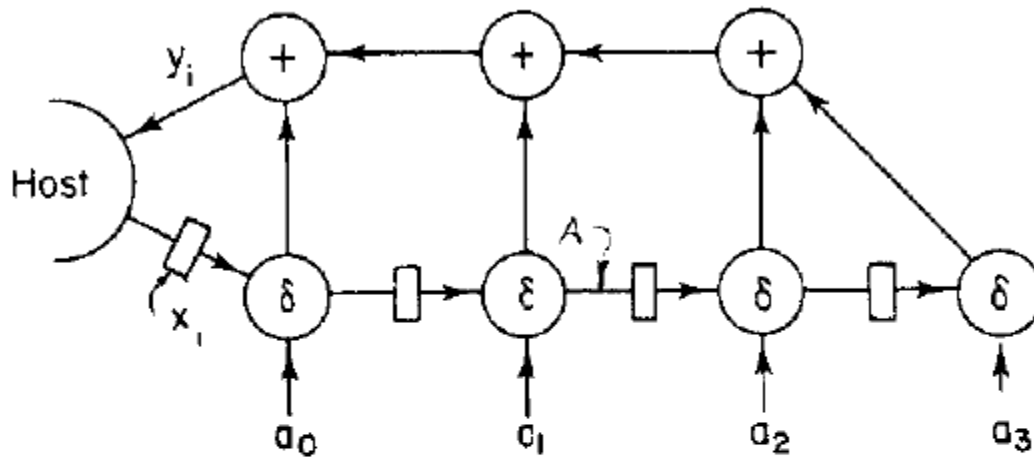
Results: MCNC Test Cases (various overhead limits)

Circuit	RC		FM		DFRG			Improvement	
	Cut	CPU	Cut	CPU	Cut	CPU	Area Overhead	RC	FM
Test02	81	25	88	63	79	221	4.55%	2%	10%
Test03	52	24	71	54	44	97	10.84%	15%	38%
Test04	50	22	62	50	34	139	10.27%	32%	45%
Test05	62	43	66	125	51	385	10.57%	15%	23%
Test06	41	27	41	55	34	157	7.28%	17%	17%
Test07	88	49	91	115	60	497	11.00%	32%	34%

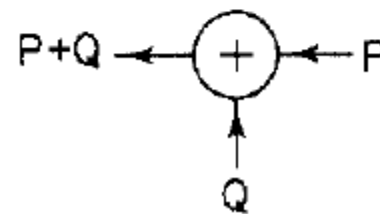
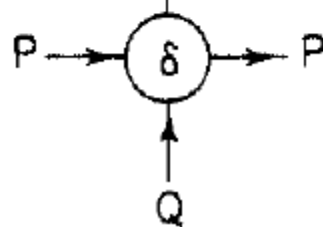
Circuit	RC		FM		DFRG			Improvement	
	Cut	CPU	Cut	CPU	Cut	CPU	Area Overhead	RC	FM
Test02	42	24	47	55	33	77	37.20%	21%	30%
Test03	25	23	30	31	23	58	48.39%	8%	23%
Test04	41	22	44	40	25	68	19.69%	39%	43%
Test05	42	43	42	96	34	146	4.45%	19%	19%
Test06	23	27	25	25	17	75	33.90%	26%	32%
Test07	45	48	54	77	29	142	44.98%	36%	46%

Retiming: Example

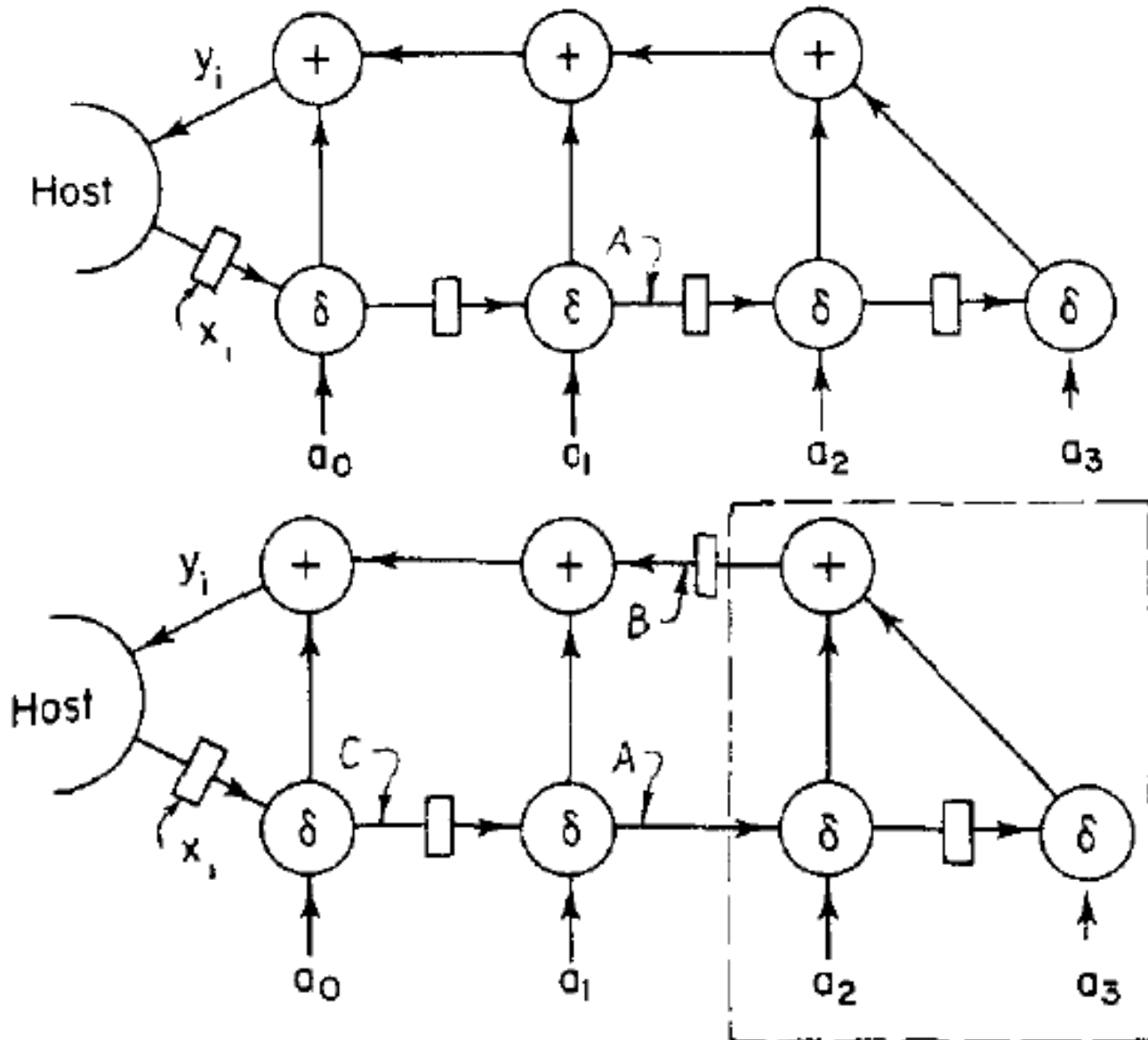
e.g. + takes 7 sec delay, δ takes 3 sec delay



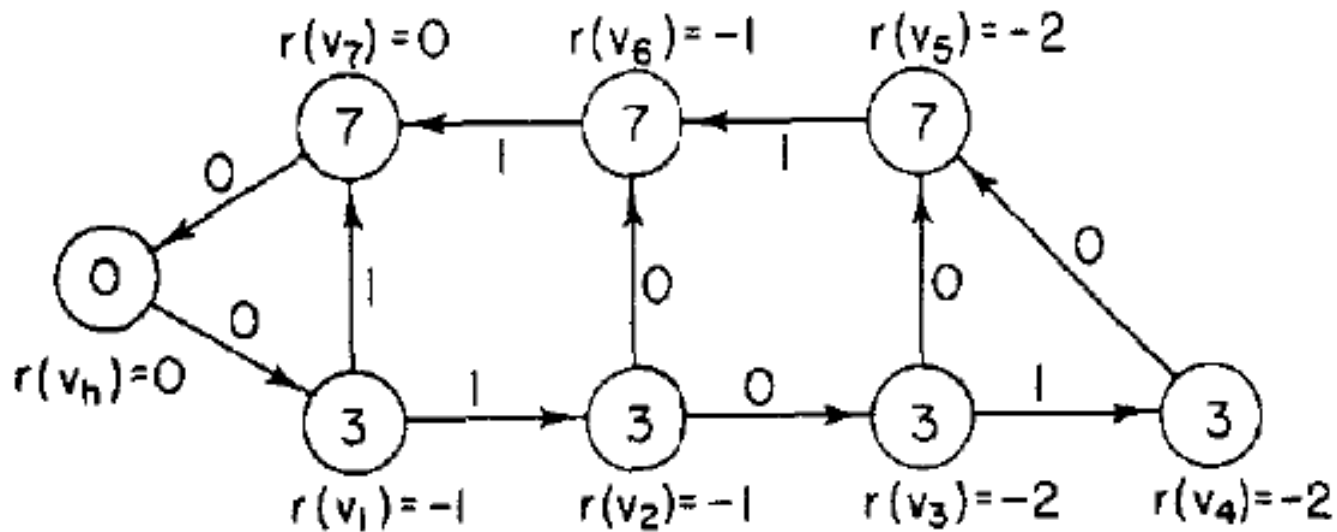
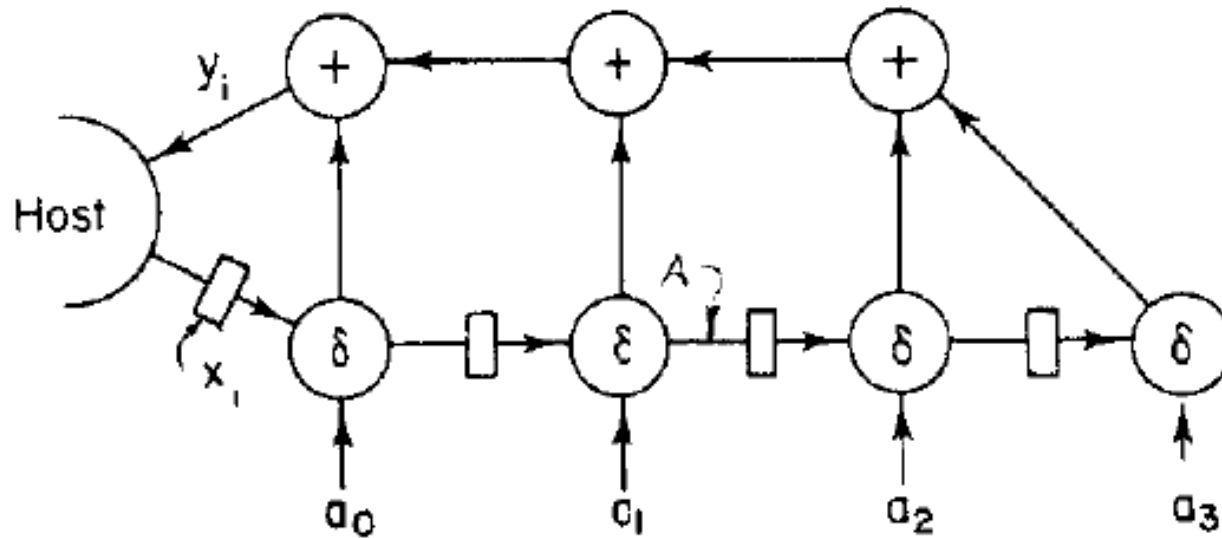
$$\delta(P,Q) = \begin{cases} 0 & P \neq Q \\ 1 & P = Q \end{cases}$$



Retiming: + takes 7 sec delay, δ takes 3 sec delay



Retiming: + takes 7 sec delay, δ takes 3 sec delay



Retiming + Partitioning + Replication: Example

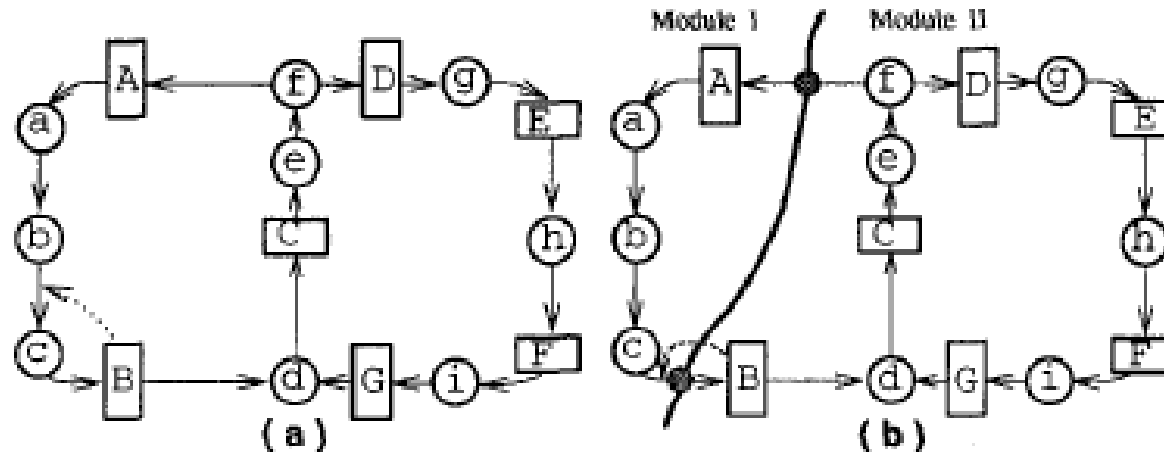


Figure 1: (a) Retiming reduces the delay from 3 units of original circuit to 2 units. (b) Retiming reduces the delay of partitioned circuit from 5 to 4 units which is not bound-optimal.

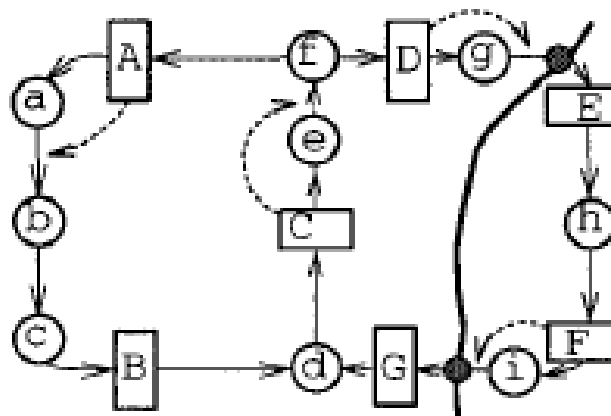


Figure 2: A bound-optimal and timing-optimal partition with 2 units delay using retiming.

Retiming: More explanation via the board