# Graph-Based Transistor Network Generation Method for Supergate Design 

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#### Abstract

Transistor network optimization represents an effective way of improving VLSI circuits. This paper proposes a novel method to automatically generate networks with minimal transistor count, starting from an irredundant sum-of-products expression as the input. The method is able to deliver both series-parallel (SP) and non-SP switch arrangements, improving speed, power dissipation, and area of CMOS gates. Experimental results demonstrate expected gains in comparison with related approaches.


Index Terms-Automated synthesis, CMOS gates, digital circuit, switching theory, transistor network.

## I. Introduction

IN VLSI digital design, the signal delay propagation, power dissipation, and area of circuits are strongly related to the number of transistors (switches) [1]-[3]. Hence, transistor arrangement optimization is of special interest when designing standard cell libraries and custom gates [4], [5]. Switchbased technologies, such as CMOS, FinFET [6], and carbon nanotubes [7], can take advantage of such an improvement. Therefore, efficient algorithms to automatically generate optimized transistor networks are quite useful for designing digital integrated circuits (ICs).

Several methods have been presented in the literature for generating and optimizing transistor networks. Most traditional solutions are based on factoring Boolean expressions, in which only series-parallel (SP) associations of transistors can be obtained from factored forms [8]-[11]. On the other hand, graph-based methods are able to find SP and also non-SP (NSP) arrangements with potential reduction in transistor count [12]-[15].

Despite the efforts of previous works, there is still a room for improving the generation of transistor networks.

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Fig. 1. Transistor networks corresponding to (1). (a) SP solution from factored form. (b) NSP from existing graph-based generation methods. (c) Optimum NSP solution.

For instance, consider a given function represented by the following equation:

$$
\begin{equation*}
F=a \cdot b+a \cdot c+a \cdot d+b \cdot c \cdot d \tag{1}
\end{equation*}
$$

For this function, factorization methods are able to deliver the SP network shown in Fig. 1(a), comprising seven transistors. Existing graph-based methods, in turn, are able to provide the NSP solution shown in Fig. 1(b), also with seven transistors. However, the optimal arrangement composed of only five transistors, as shown in Fig. 1(c), is not found by any of these methods [8]-[15].

The proposed method starts from a sum-ofproducts (SOP) form $F$ and produces a reduced transistor network. It comprises two main modules: 1) kernel identification and 2) network composition. The former aims to find efficient SP and NSP switch networks through graph structures called kernels. The latter receives the partial networks obtained from the first module and performs switch sharing, resulting in a single network representing $F$. Results have shown a significant reduction in transistor count when compared with other approaches [10]-[14]. Experiments have also demonstrated an improvement in performance, power dissipation, and area of CMOS gates as a consequence of such a device saving.

This paper is organized as follows. Section II reviews some fundamentals and definitions. Section III describes the novel method for transistor network generation. Section IV presents different execution modes of the proposed method considering the restriction of devices in series. Section V demonstrates the efficiency of the proposed approach by providing experimental results regarding transistor count, area estimation, gate performance, and power dissipation. Finally, the conclusion is drawn in Section VI.


Fig. 2. Representation of (a) switch element (ideal MOS transistor), where $x$ controls the connection between $T$ and $T_{1}$ terminals, and associations. (b) Series. (c) Parallel. (d) SP. (e) NSP.

## II. Definition and Preliminaries

A Boolean function $f(X)$ defined over the variable set $X=\left\{x_{0}, \ldots, x_{n-1}\right\}$ is a function defined as $f(X): B^{n} \rightarrow B$, where $B=\{0,1\}$ and $n=|X|$, i.e., the number of variables in $X$. The AND, or, and NOT operations are denoted by $\cdot,+$, and !, respectively. A literal is a variable or its complement (e.g., $x_{i}$ or ! $x_{i}$ ), whereas a cube is a product of literals. An irredundant SOP (ISOP) is a SOP where neither a literal nor a cube can be removed without changing the represented function. Let $f$ be a Boolean function given in ISOP form $F=c_{1}+\cdots+c_{m}$, where $m$ denotes the number of cubes in $F$. Considering that $i, j \leq m$, the union of the cubes $c_{i}$ and $c_{j}$, denoted by $c_{i} \cup c_{j}$, returns the literals that belong to cube $c_{i}$ or cube $c_{j}$. For instance, $a \cdot b \cup a \cdot c=\{a, b, c\}$. Notice that both positive and negative literals of the same variable can be returned, for example $a \cdot b \cup a \cdot c=\{a,!a, b, c\}$. An intersection of cubes $c_{i}$ and $c_{j}$, denoted by $c_{i} \cap c_{j}$, returns literals that belong to both cubes, for example $a \cdot b \cap a \cdot c=a$. Notice that an empty cube can also be returned, e.g., $a \cdot b \cap d \cdot e=\emptyset[16]$.
A switch is a device composed by one control terminal and two contact terminals. The control terminal determines if there is a connection between the contact terminals, as shown in Fig. 2(a). In this sense, an ideal MOS transistor device acts as a switch. For this reason, the terms transistor and switch are used as synonymous in this paper. Moreover, series association of switches, as shown in Fig. 2(b), represents an AND operation, whereas parallel association, as seen in Fig. 2(c), corresponds to an OR operation. An SP switch network is obtained by iteratively connecting contact terminals in series and/or in parallel. An example of SP network is shown in Fig. 2(d). An NSP switch network is an arrangement that cannot be achieved by connecting terminal contacts in series and/or in parallel, as observed in Fig. 2(e). Notice that the function represented by a given switch network corresponds to the sum of all cubes associated to the paths between the contact terminals [17].

## III. Switch Network Synthesis Method

The proposed method comprises two main modules: 1) the kernel identification and 2) the switch network composition. The former receives an ISOP $F$ and identifies individual NSP and SP switch networks, representing subfunctions of $f$. The latter composes those networks into a single network by performing logic sharing. The provided output is an optimized switch network representing the target function $f$. The execution flow of the method is presented in Fig. 3.


Fig. 3. Execution flow of the proposed method.

```
Algorithm 1 Pseudocode of the Kernel Identification Module
    kernelIdentification \((F)\)
    \(S \leftarrow \emptyset\)
    \(S \leftarrow S \cup\) NSPKernelFinder \((F)\)
    \(F_{1} \leftarrow\) removeImplementedCubes \((F, S)\)
    \(S \leftarrow S \cup\) SPKernelFinder \(\left(F_{l}\right)\)
    \(F_{2} \leftarrow\) removeImplementedCubes \(\left(F_{1}, S\right)\)
    \(S \leftarrow S \cup\) redundantCubeInsertion \(\left(F_{2}\right)\)
    \(F_{3} \leftarrow\) removeImplementedCubes \(\left(F_{2}, S\right)\)
    \(S \leftarrow S \cup\) branchedNetworkGeneration \(\left(F_{3}\right)\)
    return \(S\)
    end
```


## A. Kernel Identification

During the kernel identification module, an intermediate data structure called kernel is used to search for possible SP and NSP networks. A kernel of an ISOP $F$ with $m$ cubes is an undirected graph $G=(V, E)$, where vertices in $V=\left\{v_{1}, v_{2}, \ldots, v_{m}\right\}$ represent distinct cubes of $F$. An edge $e=\left(v_{i}, v_{j}\right) \in E, i \neq j$, exists if and only if $v_{i} \cap v_{j} \neq \emptyset$. Such edge $e$ is labeled $v_{i} \cap v_{j}$. Using the kernel structure, it is possible to determine the relationship among cubes of $F$ in order to perform logic sharing. This way, each step of the kernel identification module aims to extract kernels from $F$ that leads to optimized switch count.
The kernel identification module is divided in four steps, as presented in Fig. 3 (left) and in Algorithm 1. Each step is responsible for finding switch networks representing subfunctions of the target function $f$. The NSP kernel finder step aims to obtain optimized NSP networks from an input ISOP $F$. When a switch network is found, the cubes used to achieve such network are removed from $F$. Such removal may lead to a simpler ISOP $F_{1}$.
The SP kernel finder step, in turn, searches for SP networks using as the input $F_{1}$. Similarly to the first step, the cubes of the found SP networks are removed from $F_{1}$, resulting $F_{2}$. Since the remaining cubes of $F_{2}$ were not useful to produce NSP or SP networks, redundant cubes are added into the kernels in order to find NSP arrangements with redundant paths. Therefore, the cubes leading to NSP networks with redundant paths are removed from $F_{2}$, resulting $F_{3}$. The last step produces branched switch networks, which comprises parallel paths corresponding to cubes from $F_{3}$ [18]. Finally, a list of switch networks is produced as output of the


Fig. 4. (a) NSP kernel template. (b) Resulting switch network.
kernel identification module. Each step of this first module is detailed presented below.

1) Nonseries-Parallel Kernel Finder: Let $f$ be a Boolean function given in ISOP form $F=c_{1}+\cdots+c_{m}$, where $m$ denotes the number of cubes in $F$. In order to identify NSP kernels, the combination of $m$ cubes are taken four at a time, i.e., four-combination of cubes. The sum of such four cubes results in an ISOP $H$, which represents $h$ that is a subfunction of $f$. A kernel with four vertices is obtained from $H$. To ensure that the generated kernel results in a NSP switch network, two rules must be checked.

Rule 1: Let $E_{0}$ be the set of edges connected to the vertex $v \in V$. For each cube (vertex) $v \in V$, all literals from $v$ must be shared through the edges $e \in E_{v}$. This rule is satisfied if and only if the following equation results the value 1 :

$$
\begin{equation*}
\prod_{v \in V}\left(\left(\bigcup_{e \in E_{v}} e\right)=v\right) \tag{2}
\end{equation*}
$$

Rule 2: The kernel obtained from $H$ must be isomorphic to the graph shown in Fig. 4(a). Such a graph template is referred as NSP kernel.

An NSP kernel is mapped to a switch network by applying an edge swapping over three edges of the kernel. For instance, let us consider the generic NSP kernel shown in Fig. 4(a). To map this kernel to a network, the edge $e_{2}$ is moved to the place of $e_{4}, e_{4}$ is moved to the place of $e_{3}$, and $e_{3}$ is moved to the place of $e_{2}$. By applying such a reordering, it is possible to achieve the network shown in Fig. 4(b). The reordering procedure is necessary to ensure that each path of the switch network represents a cube from the subfunction $h$.

Example 1: Consider the following ISOP as the input to the NSP kernel finder step:

$$
\begin{equation*}
F=a \cdot b+a \cdot c \cdot e+d \cdot e+b \cdot c \cdot d \tag{3}
\end{equation*}
$$

The resulting kernel $K_{1}$ shown in Fig. 5(a), satisfies Rule 1 and Rule 2, and can be mapped by edge reordering to the switch network $S_{1}$, shown in Fig. 5(b).

Example 2: By combining cubes four at a time, the NSP kernel finder procedure can find more than one kernel per ISOP. For instance, consider the following equation:
$F=a \cdot b+a \cdot c+c \cdot e+a \cdot d+b \cdot c \cdot d+a \cdot g+b \cdot c \cdot g$.

For this ISOP, only two combinations of four cubes satisfy both Rule 1 and Rule 2, resulting in the NSP kernels

(a)

(b)

Fig. 5. (a) NSP kernel $K_{1}$, derived from (3). (b) Resulting switch network $S_{1}$.


Fig. 6. NSP kernels (a) $K_{2}$ and (c) $K_{3}$, obtained from (4). Corresponding switch networks (b) $S_{2}$ and (d) $S_{3}$.

```
Algorithm 2 Pseudocode of the NSP Kernel Finder Step
    NSPKernelFinder ( \(F\) )
        \(C \leftarrow\) getCubeCombinations \((F, 4)\)
        \(S \leftarrow \emptyset\)
    for each combination \(\mathrm{c} \in C\) do
            \(K \leftarrow\) obtainKernel( c )
            if ( \(K\) satisfies Rule 1 and Rule 2 ) then
                \(S \leftarrow S \cup\) edgeReordering \((K)\)
            end if
        end for
        return \(S\)
    end
```

$K_{2}$ and $K_{3}$ shown in Fig. 6(a) and (c), respectively. By applying the edge reordering procedure, these kernels are mapped to the switch networks $S_{2}$ and $S_{3}$ shown in Fig. 6(b) and (d), respectively.

The pseudocode of the NSP kernel finder step is presented in Algorithm 2. Let $C$ be the set of all possible fourcombinations of cubes, generated by getCubeCombinations (line 2) procedure. Considering an ISOP $F$ with $m$ cubes, the getCubeCombinations has a time complexity of $O\left(m^{4}\right)$. Then, for each possible combination of four cubes, a kernel is obtained (line 5). The time complexity of the obtain kernel subroutine is $O\left(m^{2} n\right)$, where $n$ is the number of variables in $F$. Since only four cubes are considered at a time $(m=4)$, the time complexity can be simplified to $O(n)$.

(a)

(b)

(c)

Fig. 7. (a) SP kernel template. (b) Auxiliary template graph. (c) Resulting switch network.


Fig. 8. (a) SP kernel $K_{4}$ derived from (5). (b) Auxiliary template graph $A$. (c) Switch network $S_{4}$ obtained after applying the edge reordering routine.

The time complexity to test Rule 1 is the same of the obtained kernel subroutine. A kernel satisfies Rule 2 if and only if the graph contains five edges. This test is done in constant time $O(1)$. In this sense, the time complexity to check both rules is bounded by $O(n)$. If the kernel satisfies both Rule 1 and Rule 2 , the edge reordering subroutine is executed. Such a reordering is done in constant time. The resulting NSP switch network is then added to the found switch networks. This way, the time complexity of the NSP kernel finder procedure is bounded by $O\left(m^{4}\right)$.
2) Series-Parallel Kernel Finder: Let $F_{1}$ be an ISOP form that represents all the cubes of $F$ that were not used to build switch networks in the NSP kernel finder step. To identify SP kernels, combination of $m_{1}$ cubes from $F_{1}$ are taken four at a time. A kernel with four vertices is then obtained. To ensure that the obtained kernel results in a valid SP network, Rule 1 and the following Rule 3 must be checked.
Rule 3: The obtained kernel must be isomorphic to the graph shown in Fig. 7(a). Such a graph template is referred as SP kernel.
Similarly to previous step, the SP kernel finder step must apply some transformations over the kernel in order to achieve a switch network. First, the kernel edges shown in Fig. 7(a) are mapped to an auxiliary template graph, as shown in Fig. 7(b). Afterward, a switch network is obtained by applying the edge reordering subroutine over the auxiliary template graph, as shown in Fig. 7(c).

Example 3: Consider the kernel $K_{4}$ shown in Fig. 8(a), obtained from the following equation:

$$
\begin{equation*}
F=a \cdot c+b \cdot c+b \cdot d+a \cdot d \tag{5}
\end{equation*}
$$

The edge labels are mapped from the kernel $K_{4}$ to the auxiliary graph $A_{4}$ shown in Fig. 8(b). Consequently, by applying the edge reordering over the graph $A_{4}$, the kernel $K_{4}$ is mapped to the switch network $S_{4}$ shown in Fig. 8(c).

(a)

(c)

(b)

(d)

Fig. 9. SP kernels (a) $K_{5}$ and (c) $K_{6}$, obtained from (6). Corresponding switch networks (b) $S_{5}$ and (d) $S_{6}$.

```
Algorithm 3 Pseudocode of the SP Kernel Finder Step
    SPKernelFinder \(\left(F_{1}\right)\)
    \(C \leftarrow \operatorname{getCubeCombinations}\left(F_{1}, 4\right)\)
    \(S \leftarrow \emptyset\)
    for each combination \(\mathrm{c} \in C\) do
        \(K \leftarrow\) obtainKernel( c )
        if ( \(K\) satisfies Rule 1 and Rule 3 ) then
            \(S \leftarrow S \cup\) edgeReordering \((K)\)
        end if
    end for
    return \(S\)
    end
```

Example 4: To demonstrate that multiple kernels can also be found during the SP kernel finder step, let us consider the following equation:

$$
\begin{align*}
F= & !a \cdot!b \cdot!c \cdot d+!a \cdot!b \cdot c \cdot!d+!a \cdot b \cdot!c \cdot!d \\
& +!a \cdot b \cdot c \cdot d+a \cdot!b \cdot!c \cdot!d+a \cdot!b \cdot c \cdot d \\
& +a \cdot b \cdot!c \cdot d+a \cdot b \cdot c \cdot!d . \tag{6}
\end{align*}
$$

For this function, the SP kernel finder procedure is able to find two SP kernels $K_{5}$ and $K_{6}$, shown in Fig. 9(a) and (c), respectively. Such kernels are remapped to the corresponding switch networks $S_{5}$ and $S_{6}$, as shown in Fig. 9(b) and (d), respectively.

The pseudocode of the SP kernel finder step is described in Algorithm 3. This pseudocode is quite similar to the NSP kernel finder one, presented in Algorithm 2. The main difference is that, in the line 6 of Algorithm 3, Rule 3 is checked instead of Rule 2. Basically, Rule 3 verifies if all vertices of the kernel have degree equals to two, as shown in Fig. 7(a). This test is done in constant time. This way, it is easy to see that the worst case time complexity of the SP kernel finder procedure is bounded by the cost of combining $m_{1}$ cubes from $F_{1}$, i.e., $O\left(\left(m_{1}\right)^{4}\right)$
3) Redundant Cube Insertion: In some cases, it is useful to build NSP arrangements with redundant cubes instead of using

(a)

(b)

(c)

Fig. 10. (a) Graph $G_{1}$ obtained from (8). (b) Used to determine a NSP kernel $K_{7}$ with a redundant cube. (c) Resulting switch network $S_{7}$.

SP associations. Thus, when there still cubes not represented through NSP and SP networks, the redundant cube insertion step tries to build NSP kernels by combining remaining cubes with redundant cubes.

Let $F$ be an ISOP representing the Boolean function $f$. A cube $c$ is redundant if $F+c=f$. Consider a switch network representing an ISOP $f$. An implementation of a redundant cube $c$ in such a network leads to a redundant logic path, i.e., the path does not contribute to the logic behavior of the network. Even though, redundant paths allow efficient logic sharing in NSP networks.

The redundant cube insertion step works over an ISOP $F_{2}$ representing the cubes that were not implemented by NSP and SP kernel finder steps. To obtain NSP kernels with redundant cubes, combinations of $m_{2}$ cubes are taken three at a time, where $m_{2}$ is the number of cubes in $F_{2}$. A kernel with three vertices is then obtained for each combination. Thus, a fourth cube (vertex) $v_{z}$ is inserted into the kernel according to the following rule.

Rule 4: Let $E_{v}$ be the set of edges connected to the vertex $v \in V$. For each cube (vertex) $v \in V$, the literals from $v$ that were not shared through the edges $e \in E_{0}$ are inserted in $v_{z}$. Hence, the literals of the new vertex $v_{z}$ are obtained by

$$
\begin{equation*}
v_{z}=\prod_{v \in V}\left(v-\bigcup_{e \in E_{v}} e\right) \tag{7}
\end{equation*}
$$

where minus signal $(-)$ denotes relative complement. Therefore, after building the redundant cube $v_{z}$, Rule 1 and Rule 2 are applied over the resulting kernel in order to check if the cubes share all their literals through the edges.

Example 5: To demonstrate an NSP kernel with a redundant cube, let us consider the following equation:

$$
\begin{equation*}
F=!a \cdot!d+!a \cdot!b \cdot c+!b \cdot!c \cdot!d \tag{8}
\end{equation*}
$$

The graph $G_{1}$ obtained from (8) is shown in Fig. 10(a). This graph contains three edges, representing common literals between the cubes. Notice that the literals circled by dashed lines are not shared through any edge. In this sense, by applying Rule 4, these literals can be merged into a single

```
Algorithm 4 Pseudocode of the Redundant Cube
Insertion Step
    redundantCubeInsertion ( \(F_{2}\) )
    \(C \leftarrow\) getCubeCombinations \(\left(F_{2}, 3\right)\)
    \(S \leftarrow \emptyset\)
    for each combination \(\mathrm{c} \in C\) do
        \(K \leftarrow\) obtainKernel ( c )
        \(K \leftarrow\) insertRedundantCube \((K)\)
        if ( \(K\) satisfies Rule 1 and Rule 2 ) then
            \(S \leftarrow S \cup\) edgeReordering \((K)\)
        end if
    end for
    return \(S\)
    end
```

vertex $v_{z}=!\boldsymbol{c} \cdot \boldsymbol{c}$. By adding the vertex $v_{z}$ into the graph, a valid NSP kernel $K_{7}$ is obtained, as presented in Fig. 10(b). As can be seen, $v_{z}$ represents a redundant cube, i.e., $F+!c \cdot c=f$. Hence, by applying the edge reordering routine over $K_{7}$, the method provides the optimized network $S_{7}$ comprising five switches, as shown in Fig. 10(c). Notice that the exact factoring for (8) comprises six literals, as shown in the following equation, resulting in a SP network with six switches:

$$
\begin{equation*}
F=(!d+c \cdot!b) \cdot(!a+!c \cdot!b) \tag{9}
\end{equation*}
$$

Algorithm 4 presents the procedure to determine an NSP kernel with a redundant cube. There are two differences between this algorithm and the Algorithm 2 of the NSP kernel finder step. The first one is that, instead of generating four-combinations, Algorithm 4 generates three-combinations of cubes (line 2). The second difference is that, after obtaining the kernel from the selected cubes, the procedure inserts a redundant cube $v_{z}$ into the kernel (line 6). Since only three cubes are considered at a time, the time complexity to build the redundant cube is $O\left(n_{2}\right)$, where $n_{2}$ is the number of variables in $F_{2}$. The time complexity of the redundant cube insertion step is bounded by the cost to perform cube combinations, i.e., $O\left(\left(m_{2}\right)^{3}\right)$.
4) Branched Network Generation: Cubes from ISOP $F$ are removed when a network implementation representing it is found. Even though previous steps are very efficient in finding logic sharing, there may still cubes not represented through any of the found networks. In this sense, the remaining cubes in $F_{3}$ are implemented as a single switch network. Therefore, the branched network generation step translates each remaining cube in $F_{3}$ to a branch of switches associate in series.

Example 6: To demonstrate how a branched network is obtained, consider the following equation:

$$
\begin{equation*}
F=a \cdot b \cdot c \cdot!d+!a \cdot b \cdot!c+a \cdot!b \cdot d \tag{10}
\end{equation*}
$$

In this case, there are three remaining cubes to be implemented. The obtained network is demonstrated in Fig. 11, where each literal in a cube was directly translated to a switch in the network.

Algorithm 5 presents the pseudocode of the branched network generation step. Basically, a switch network $N$ starts empty and each cube is placed into the network. After placing


Fig. 11. Branched switch network obtained from (10).

```
Algorithm 5 Pseudocode of the Branched Network
Generation Step
    branchedNetworkGeneration \(\left(F_{3}\right)\)
        \(N \leftarrow \emptyset\)
        for each cube \(\in F_{3}\) do
            \(N \leftarrow\) placeCube \((N\), cube )
    end for
    return \(N\)
    end
```

```
Algorithm 6 Pseudocode of the Network Composition Module
    networkComposition \((F, S)\)
        network \(\leftarrow \emptyset\)
        for each \(\mathrm{s} \in S\) do
            network \(\leftarrow\) makeParallelAssociation ( network, \(s\) )
            network \(\leftarrow\) simpleSwitchSharing \((F\), network )
            network \(\leftarrow\) complexSwitchSharing ( \(F\), network )
    end for
    return network
    end
```

all cubes, the network is returned. The branched generation is a quite simple process with time complexity of $O\left(m_{3} n_{3}\right)$, where $m_{3}$ is the number of the remaining cubes and $n_{3}$ is the number of variables in $F_{3}$.

## B. Network Composition

The network composition module receives the function $F$ and a list of partial switch networks $S$, generated during the kernel identification module. This module composes the networks from $S$ in an iterative process by performing logic sharing among such networks. The target network starts empty and, for each network $s \in S$ a parallel association is performed together with simple and complex sharing strategies. The simple and the complex switch sharing are applied in order to remove redundant switches in the target network. The pseudocode of the network composition is presented in Algorithm 6. The makeParallelAssociation subroutine, in line 4 , just places two networks in parallel. This way, this subroutine runs in constant time $O(1)$. The simple and the complex switch sharing steps are presented in the following sections 1) Simple Sharing and 2) Complex Sharing together with their respective time complexities.

1) Simple Sharing: The simple sharing step implements the edge sharing technique presented in [13]. Basically, the method traverses the switch network searching for equivalent switches, i.e., switches that are controlled by the same literal. The network is then restructured in such a
```
Algorithm 7 Pseudocode of the Simple Sharing Step
    simpleSharing ( \(F\), network )
        do
            initialSwitchCount = getSwitchCount( network )
            \(E \leftarrow\) findEquivalentSwitches( network)
            if \((E \neq \emptyset)\) then
                network \(\leftarrow\) switchSwapping (network, \(E\) )
                network \(\leftarrow\) switchSharing( network, E)
                network \(\leftarrow \operatorname{logicalEquivalenceChecking(~} F\), network )
            end if
            finalSwitchCount \(=\) getSwitchCount ( network )
        while ( initialSwitchCount > finalSwitchCount)
        return network
    end
```


(a)

(b)

Fig. 12. Networks obtained from (11). (a) SP network $S_{9}$. (b) NSP network $S_{10}$ with a redundant cube $!\boldsymbol{a} \cdot!\boldsymbol{b} \cdot \boldsymbol{a} \cdot \boldsymbol{c} \cdot \boldsymbol{d}$.
way that one common node between equivalent switches is available. In some cases, the equivalent switches must be swapped in the networks in order to share a common node. When a common node between equivalent switches is available, only one switch is necessary, leading to a reduction in the number of switches.

After performing a switch sharing, the logic behavior of the network must be checked to ensure an accurate implementation of the target function. The switch sharing is accepted only if the logic behavior of the network is maintained. This optimization and validation process is applied iteratively over the network until there is no more feasible switch sharing to be applied.
A high level description of the simple sharing step is presented in Algorithm 7. Among all operations and subroutines needed to perform simple switch sharing, the highest time complexity is given by the logicalEquivalence Checking subroutine, in line 8 . This procedure verify all logic paths of the network, requiring a time complexity of $O\left(2^{e / 2}\right)$, where $e$ is the number of switches (edges) in the network. Thus, the simple sharing step is bounded by $O\left(2^{e / 2}\right)$.
Example 7: As an example of simple switch sharing, consider the following input ISOP:

$$
\begin{align*}
F= & !a \cdot!b \cdot!c \cdot!d+!a \cdot b \cdot d+!a \cdot b \cdot c+a \cdot!b \cdot d \\
& +a \cdot!b \cdot c+a \cdot b \cdot!c \cdot!d+b \cdot c \cdot d . \tag{11}
\end{align*}
$$

In this case, the SP kernel finder step was able to find the SP network $S_{9}$ shown in Fig. 12(a). Moreover, the redundant cube insertion step was able to find the NSP network $S_{10}$ shown in Fig. 12(b). In order to compose a network corresponding to the given function described in (11), these partial switch networks $S_{9}$ and $S_{10}$ are associated in parallel, as shown in Fig. 13(a).


Fig. 13. (a) Parallel association of networks $S_{9}$ and $S_{10}$. (b) Intermediate network. (c) Final network to implement (11).

(a)

(b)

(c)

Fig. 14. Networks obtained from (4). (a) NSP network $S_{11}$. (b) NSP network $S_{12}$. (c) Branched network $S_{13}$ implementing the remaining cube $\boldsymbol{e} \cdot \boldsymbol{c}$.

Notice that the network shown in Fig. 13(a) has some redundant switches, circled by dashed. Therefore, the simple sharing step is applied to remove such redundancies, resulting in the network shown in Fig. 13(b). The arrows in Fig. 13 indicate a swap between two switches. The optimization process is repeated until there are no more redundant switches in the network. Finally, the network shown in Fig. 13(c) represents the target function from (11).

Example 8: Let us reconsider Example 2, where the NSP kernel finder step was able to generate the networks $S_{11}$ and $S_{12}$ shown in Fig. 14(a) and (b), respectively. Observe that the cube $\boldsymbol{e} \cdot \boldsymbol{c}$ was not implemented in $S_{11}$ and $S_{12}$. Hence, this cube was implemented as an independent branch $S_{13}$ in the branched network generation step, as shown in Fig. 14(c). When the network composition starts, the first two networks, $S_{11}$ and $S_{12}$, are associated in parallel as shown in Fig. 15(a). Then, by applying the simple switch sharing over $S_{11}$ and $S_{12}$,

(a)

(c)

(b)

(d)

Fig. 15. (a) Parallel association of networks $S_{11}$ and $S_{12}$ obtained from Example 2. (b) Merged networks. (c) Parallel association of the branched network $S_{13}$. (d) Resulting switch network.

(a)

(b)

Fig. 16. Networks obtained from (6). SP networks (a) $S_{14}$ and (b) $S_{15}$.
the equivalent switches are shared, as shown in Fig. 15(b). Thus, the branded network $S_{13}$ of the cube $\boldsymbol{e} \cdot \boldsymbol{c}$ is inserted into the network, as shown in Fig. 15(c). Finally, by reapplying the switch sharing procedure, the final solution is found, as shown in Fig. 15(d).

Example 9: In order to demonstrate the potential of the simple sharing procedure even when applied over SP networks, let us revisit Example 4. In this case, the SP kernel finder step was able to generate two optimized switch networks $S_{14}$ and $S_{15}$ shown in Fig. 16(a) and (b), respectively. By arranging $S_{14}$ and $S_{15}$ in parallel, the switch network shown in Fig. 17(a) is obtained. Then, the simple sharing technique is applied to remove redundant switches from the network, resulting in the NSP solution shown in Fig. 17(b). Notice that the method is able to start from SP arrangements and achieve more optimized networks.
2) Complex Sharing: The complex sharing step receives a preprocessed network provided by the previous step and tries to perform additional optimizations. As mentioned in the simple sharing step, after finding equivalent switches, the procedure checks if the candidate switches have a common node that enables sharing. However, there are some cases where a common node is not directly found due to the position of the switches in the network. Hence, in order to improve the switch sharing, straightforward SP switch compressions are performed, as shown in Fig. 18(a) and (b), respectively. Then, simple switch sharing is applied over the compressed network.

Algorithm 8 presents the pseudocode of the complex sharing step. The main idea of the method is to execute the

(a)

(b)

Fig. 17. (a) Parallel association of the switch networks $S_{14}$ and $S_{15}$. (b) Optimized NSP switch network obtained by applying simple sharing.


$$
x=a \cdot b
$$

(a)


$\boldsymbol{x}=\boldsymbol{a}+\boldsymbol{b}$
(b)

Fig. 18. (a) Series switch compression. (b) Parallel switch compression.

```
Algorithm 8 Pseudocode of the Complex Sharing Step
    complexSharing ( \(F\), network)
    do
        SPCompression( network )
        initialSwitchCount \(=\) getSwitchCount ( network )
        simpleSharing ( \(F\), network)
        finalSwitchCount = getSwitchCount ( network)
        while ( initialSwitchCount > finalSwitchCount)
        SPExpansion (network)
    return network
    end
```

SP compression and the simple sharing procedure in an iterative way, as presented in lines 3 and 5. One level of SP compression is applied in each step, allowing that multiple switches can be simultaneously swapped in the network. Consequently, it allows the sharing of multiple switches at the same time. At the end of this iterative process, the optimized network is expanded, as presented in line 8 of Algorithm 8. This is necessary to normalize the network, i.e., expand SP compressions in such a way that each switch is controlled by a single literal. Both the SP compression and SP expansion subroutines traverse the graph finding switches for compressing and expanding, respectively. The time complexity to traverse a switch network with $c$ contact nodes and $s$ switches is $O(c+s)$. In this sense, the time complexity of the complex sharing is bounded by the complexity of the simple sharing step, which is $O\left(2^{e / 2}\right)$.


Fig. 19. (a) Switch network received as the input to the complex sharing step when processing the function described in (12). (b) Resulting network after performing series compressions.

Example 11: Consider the following ISOP:

$$
\begin{align*}
F= & !a \cdot!c+!d \cdot!e \cdot!i+!a \cdot!d+!b \cdot!d \cdot!g \cdot!h \\
& +!b \cdot!c \cdot!g \cdot!h+!b \cdot!c \cdot!e+!d \cdot!i \cdot!g \cdot!h \\
& +!c \cdot!i \cdot!g \cdot!h+!c \cdot!e \cdot!i+!b \cdot!d \cdot!e \tag{12}
\end{align*}
$$

After running the kernel identification module and performing the simple sharing step, the network shown in Fig. 19(a) is obtained. As can be seen, there are still some redundant switches in this network. The simple sharing step cannot remove such redundancies due to the position of switches. In this case, a common node between the redundant switches is not available. Moreover, none of these switches can be moved to other node without changing the logic behavior of the network. In this sense, a succession of SP compressions are performed. For instance, a series compression is applied over the switches $!\boldsymbol{g}$ and $!\boldsymbol{h}$, resulting in a single switch $\boldsymbol{w}=!\boldsymbol{g} \cdot!\boldsymbol{h}$, as shown in Fig. 19(b).

Afterward, a parallel switch compression is applied over the switches $!\boldsymbol{e}$ and $\boldsymbol{w}$ resulting in a single switch $\boldsymbol{x}=!\boldsymbol{e}+!\boldsymbol{g} \cdot \boldsymbol{\prime} \boldsymbol{h}$. The parallel compression is also applied over the switches $!\boldsymbol{b}$ and $!\boldsymbol{i}$, resulting in $\boldsymbol{y}=!\boldsymbol{b}+!\boldsymbol{i}$, and over switches $!\boldsymbol{c}$ and $!\boldsymbol{d}$, resulting in $z=!\boldsymbol{c}+!\boldsymbol{d}$. The obtained network is shown in Fig. 20(a). A swap operation is applied over the compressed switches, resulting in the arrangement shown in Fig. 20(b). As can be seen, there is at least one common node between redundant switches. Hence, running the simple sharing procedure once more, it is possible to achieve the network shown in Fig. 20(c). Finally, an SP expansion is performed over the network, as shown in Fig. 20(d).

(a)

(c)

(b)

(d)

$$
x=!e+!g \cdot!h \quad y=!b+!i \quad z=!c+!d
$$

Fig. 20. Obtained networks. (a) Performing parallel compressions. (b) Swapping $\boldsymbol{x}$ and $\boldsymbol{y}$. (c) After sharing the equivalent switches $\boldsymbol{x}$ and $\boldsymbol{y}$. (d) Final network after applying the SP expansion.

## IV. Transistor Stack Bounding

Switch networks can be exploited by switch-based technologies, which present some restrictions or guidelines to be followed by designers. For example, in the conventional CMOS design technology, the maximum number of stacked transistors is usually limited to four. Such restriction is done in order to avoid performance degradation. Notice that there is a lower bound on the stacked transistors in switch networks. This lower bound corresponds to the minimum decision chain (MDC) property of the represented Boolean functions [19]. In this sense, an interesting feature to control (or to limit) the number of stacked transistors was included in our method. The method can operate in two execution modes, bounded and unbounded, as described below.

## A. Bounded Mode

In this execution mode, a bound variable is used as reference to control the maximum number of transistors in series. The bound value must be equal or greater than the number of literals of the smallest cube from $F$, i.e., the maximum number of literals in a single cube. When the method is running in the bounded mode, the kernel identification module accepts only switch networks in which maximum stacked transistors do not exceed the bound value. Hence, the networks satisfying such a bound are added to the list $S$ of found networks. This control is also performed during the network composition module when applying switch sharing, since it can increase the transistor stack.

## B. Unbounded Mode

When running in the unbounded mode, there is no restriction of transistor stacking, i.e., the bound variable is not considered. Basically, just the total transistor count of the network is taken as metric cost. Hence, there are cases that the

(a)

(b)

Fig. 21. Two possible transistor networks generated by the proposed method to implement the function described in (13), considering both execution modes. (a) Unbounded. (b) Bounded.
networks generated through the unbounded mode result fewer transistors when compared with bounded solutions. Moreover, these different modes are quite useful to explore the tradeoff between circuit area and performance.
Example 12: In order to demonstrate these two execution modes, consider the following equation:

$$
\begin{equation*}
F=!a \cdot!b \cdot d+!a \cdot!c \cdot d+!b \cdot!c \cdot!d \tag{13}
\end{equation*}
$$

The network shown in Fig. 21(a) is obtained when running the method in the unbounded mode. Notice that, in this network, the size of the transistor stack $!\boldsymbol{a} \cdot \boldsymbol{d} \cdot!\boldsymbol{b} \cdot \boldsymbol{d}$ is $>3$, which is the number of literals of the smallest cube from (13). When running the method in the bounded mode, it is possible to ensure transistor stacks with at most three devices, as shown in Fig. 21(b). On one hand, the bounded network presents an overhead of one transistor in comparison with the unbounded solution. On the other hand, the bounded solution has smallest transistor stacks. In this sense, one can consider the bounded solution when targeting performance or the unbounded solution for smaller area.

## V. Experimental Results

For evaluation and validation, the proposed method was applied over different sets of representative functions in order to provide a fair comparison with other available solutions [10]-[14]. Four different set of functions were considered: 1) the set of 4-input P-class of functions; 2) a set of handcrafted networks that do not present transistors in SP associations [20]; 3) a given function with 11 variables as a more complex case study; and 4) the functions and transistor networks described in the Ninomiya's catalog [21].

## A. Transistor Count Evaluation

The first experiment was carried out over Boolean functions up to four variables, representing 65536 functions. These functions were grouped into a set of equivalent classes by considering input permutation called, herein, 4-input P-class. This set comprises 3982 representative functions. The corresponding CMOS gate for each function of this set was built, by generating both the pull-up and pull-down networks through the methods in evaluation. Table I shows the results obtained considering the total transistor count used to build all gates, including required input inverters. Notice that, only input inverters are allowed.

The second experiment was carried out over a set of 53 functions obtained from handmade networks where there

TABLE I
Transistor Count for the 4-Input P-Class Functions

|  | $[11]$ | $[12]$ | $[13]$ | $[14]$ | Bounded | Unbounded |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Transistor <br> count | 102,668 | 103,049 | 96,804 | 97,174 | $\mathbf{9 6 , 8 2 4}$ | $\mathbf{9 5 , 5 9 5}$ |

TABLE II
Transistor Count for the 53 Handmade Networks [20]

|  |  | [20] <br> (optimum) | [11] | [12] | [13] | [14] | Bounded |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | Unbounded

Fig. 22. Histogram comparing the proposed method (unbounded) with other approaches.
are neither series nor parallel associations [20]. These networks have at most four transistors in series, being suitable for CMOS gate implementation. It represents a worthy benchmark set taken as reference to evaluate the proposed approach and other related methods [11]-[14]. The results are summarized in Table II.
Fig. 22 shows the distribution of gains and losses when synthesizing transistor networks for the set of 53 handmade functions. In this histogram, horizontal axis corresponds to the transistor count overhead in respect to the proposed method (unbounded), used herein as reference. The vertical axis relates the number of functions for each increase on transistor count. As can be seen, our unbounded method was able to reduce up to eight transistors in some cases. In general, the gains are around two to three transistors per network. Although there is an increasing in transistor count, the proposed method (bounded) is near to the optimum transistor count [20].
A more complex case study is presented using an 11-input function, comprising 99 literals as follows:

$$
\begin{align*}
F= & a \cdot i+c \cdot k+b \cdot d \cdot i+b \cdot m \cdot k+a \cdot g \cdot j+b \cdot e \cdot j \\
& +c \cdot h \cdot j+c \cdot m \cdot d \cdot i+b \cdot e \cdot g \cdot i+c \cdot h \cdot g \cdot i \\
& +a \cdot d \cdot m \cdot k+a \cdot g \cdot h \cdot k+b \cdot e \cdot h \cdot k+b \cdot d \cdot g \cdot j \\
& +a \cdot d \cdot e \cdot j+c \cdot m \cdot e \cdot j+b \cdot m \cdot h \cdot j \\
& +c \cdot h \cdot e \cdot d \cdot i+c \cdot m \cdot e \cdot g \cdot i+b \cdot m \cdot h \cdot g \cdot i \\
& +a \cdot g \cdot e \cdot m \cdot k+b \cdot d \cdot g \cdot h \cdot k+a \cdot d \cdot e \cdot h \cdot k \\
& +c \cdot m \cdot d \cdot g \cdot j+a \cdot d \cdot m \cdot h \cdot j \tag{14}
\end{align*}
$$



Fig. 23. Transistor network related to (14), provided by our method.

TABLE III
Transistor Count Used to Implement (14)

|  | $[11]$ | $[12]$ | $[13]$ | $[14]$ | Bounded | Unbounded |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Transistor <br> count | 31 | 31 | 25 | 22 | $\mathbf{1 1}$ | $\mathbf{1 1}$ |


(a)

(b)

Fig. 24. Network related to (15). (a) Solution provided in the Ninomiya's catalog-function N12' [21]. (b) Arrangement delivered by our method.

In this case, the proposed method provides the transistor network presented in Fig. 23. This is the optimum solution with only 11 transistors. Table III presents the transistor count obtained from other methods, showing that our approach is able to deliver a network with at least $50 \%$ of reduction.
Finally, we have evaluated the proposed method using the set of 402 functions/networks presented in the Ninomiya's catalog [21], also adopted as benchmarking in other works. Kagaris and Haniotakis [14] and Tanaka and Kambayashi [22] claim that their methods have obtained networks with the same number of transistors as presented in this catalog, excepting for function $N 58^{\prime}$ where they achieved a reduction of one transistor. Our method is not capable to achieve the best solution for all functions provided in the catalog. However, in this experiment, our contribution was to achieve a reduction of one transistor for the function $N 12^{\prime}$, represented in the catalog by the following equation:

$$
\begin{equation*}
F=w \cdot!x+!w \cdot!y+!w \cdot!z+!y \cdot z+x \cdot y \cdot z \tag{15}
\end{equation*}
$$

The Ninomiya's network for this function is shown in Fig. 24(a), whereas our solution with one transistor saving is shown in Fig. 24(b).
Moreover, we observed that the function N54 from this catalog presents an inconsistency between the given Boolean expression

$$
\begin{align*}
F= & w \cdot x \cdot!y+w \cdot x \cdot z+w \cdot!x \cdot y \cdot z \\
& +!w \cdot!x \cdot y \cdot z \tag{16}
\end{align*}
$$



Fig. 25. (a) Network provided in the Ninomiya's catalog [21] for function N54, which is not logically equivalent to the catalog expression represented in (16). (b) Arrangement delivered by our method for this expression.
and the transistor network presented, shown in Fig. 25(a), said to be logically equivalent. The right solution, delivered by our method, is presented in Fig. 25(b).

## B. Performance and Area Evaluation

In terms of electrical and physical characteristics of CMOS gates, the reduction in the number of transistors may lead to improvements on speed, power dissipation, and area. However, there are other parameters that impact circuit quality such as transistor sizing, layout compaction, and waveform of input stimuli.

The experiments were carried out over the 4-inputs P-class set of functions. Then, three libraries were built, being the gates generated by applying the exact factorization method Functional Composition (FC)-MDC [10], as well as the kernel finder, bounded, and unbounded modes, proposed in this paper. Each library comprises 3982 gates. Notice that the network can be built considering the lower bound of transistors in stack for a given Boolean function [19]. In this sense, the FC-MDC and the bounded cell libraries were generated considering such lower bound. It is known that the longest transistor path is the main responsible for the worst case of delay propagation through the gate.

Electrical characterization of the libraries were carried out using the Cadence Encounter Library Characterizer tool, considering the $32-\mathrm{nm}$ CMOS predictive technology model typical parameters [23] and the nominal power supply voltage of 1 V . First, transistors were sized according to the logical effort method [24] that considers the transistor stacking in the network. The channel length of transistors is 32 nm . The nMOS transistor width is 64 nm while the pMOS transistor width is defined using the PN ratio equals to two, in the inverter gate used as reference for the logical effort method. The input slopes and output load applied in the characterization process were defined considering the usual fan-out four.

Gate delay is mainly affected by the maximum stacked transistors in a network. Considering two networks with the same transistor stack, the network comprising fewer transistors usually has a better performance. In this sense, performance improvements can be observed in the comparative analysis


Fig. 26. Delay evaluation of networks, comparing the bounded execution mode to (a) exact factorization generation (FC-MDC) [10] and (b) unbounded mode generation.
presented in Fig. 26. When comparing the bounded and the FC-MDC methods, only the cases where the proposed method reduced transistor count were considered. This analysis corresponds to 1680 gates, as shown in Fig. 26(a). The comparison between bounded and unbounded modes is shown in Fig. 26(b), where 2922 gates present the same topology and, consequently, the same behavior.

A decrease in device count tends to present a significant reduction in gate power dissipation. Fewer transistors in the gate represent less capacitance to be charged during the signals switching. The power reduction analysis comparing the bounded and the FC-MDC methods is shown in Fig. 27(a). Only the cases where the proposed method reduced the number of transistors is considered. The results confirmed that reducing transistor count leads to a power dissipation reduction.

A power-delay analysis was performed comparing bounded and unbounded execution modes. The power-delay product evaluation is shown in Fig. 27(b). This analysis corresponds to the dynamic power component. The short-circuit component was not considered, since it usually represents $<10 \%$ of the total power dissipation in well-designed circuits.

In terms of physical area evaluation, it is intuitive to expect area saving when the number of devices is reduced. One could think in building carefully the layout of a set of networks. However, such a task is impractical and may result even in an inconclusive analysis due to many other factors involved.


Fig. 27. (a) Power dissipation analysis of the networks provided by bounded mode and exact factorization methods (FC-MDC) [10]. (b) Power-delay product of networks generated by bounded and unbounded modes.

The network layout dimensions can be estimated considering the usual standard cell template, where pMOS transistors of the pull-up network are placed over nMOS transistors of the pull-down plane. Such a transistor placement is made side-byside, in line, exploring layout techniques like Euler paths [25]. The Euler path analysis gives a good idea of layout length, whereas the layout height can be estimated considering the power lines, the P and N active areas, and the signals wire congesting. Notice that the layout compaction is a very hard and handmade time consuming task not explored in this paper. The Euler path (and eventual active area breaks) and the signal routing were extracted for each generated network. An area estimative evaluation, considering only cases that the proposed method reduced transistor count, is shown in Fig. 28. Such cases comprise 1680 gates. For 254 gates, an area increase was observed due to breaks insertion in order to match Euler paths. For 1186 gates, an area reduction was observed, demonstrating the tendency of the circuit area saving as a result of transistor count reduction.

## C. Execution Time

The total execution time of the proposed method to generate the networks described on all the experiments presented in this paper was 1.2 s . The platform was an Intel Core i5 processor at 2.8 GHz with 4 GB of RAM. It demonstrates the feasibility of


Fig. 28. Comparison between the networks provided by the bounded mode and the exact factorization generation (FC-MDC) [10] methods in terms of area estimation
the proposed method to generate optimized transistor network, increasing design quality.

## VI. Conclusion

This paper described an efficient graph-based method to generate optimized transistor (switch) networks. Our approach generates more general arrangements than the usual SP associations. Experimental results demonstrated a significant reduction in the number of transistor needed to implement logic networks, when compared with the ones generated by existing related approaches. It is known that the transistor count minimization in CMOS gates may improve the performance, power dissipation, and area of digital ICs. In a general point-of-view, the proposed method produces efficient switch arrangements quite useful to be explored by different IC technologies based on switch theory.

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