Intensity Transformations

Image Processing
CSE 166
Lecture 3
Announcements

• Assignment 1 is due Oct 11, 11:59 PM
• Assignment 2 will be released Oct 11
  – Due Oct 18, 11:59 PM
• Reading
  – Chapter 3: Intensity Transformations and Spatial Filtering
    • Sections 3.1, 3.2, and 3.3
Image coordinates

Origin

Width $N$
$N$ columns

Height $M$
$M$ rows

Image $I(x, y)$
Intensity transformations

\[ g(x, y) = T(f(x, y)) \]

where

- \( f(x, y) \) is input image
- \( g(x, y) \) is output image
- \( T(\cdot) \) is intensity transformation

\[ g(x, y) = T(f(x, y)) \]
\[ s = T(r) \]

where

- \( r = f(x, y) \) is intensity or gray level value of input image at \((x, y)\)
- \( s = g(x, y) \) is intensity or gray level value of output image at \((x, y)\)
Intensity transformations

Contrast stretching function

Thresholding function
Intensity transformations

for 8 bits per pixel (bpp)/channel images, $L = 2^8 = 256$

Some basic transformation functions
Negative transformation

\[ s = (L - 1) - r, \text{ where } r \in [0, L - 1] \]
Gamma transformation

\[ s = cr^\gamma \quad c \text{ is often } 1 \]
Gamma transformation

Dark image

\[ \gamma < 1 \]

\[ S = c r^\gamma \]
Gamma transformation

Light image

$\gamma > 1$

$s = cr^{\gamma}$
Piecewise-linear transformations

• Contrast stretching
• Intensity-level slicing
• Bit-plane slicing
Contrast stretching

A low-contrast electron microscope image of pollen, magnified 700 times

Input image

Piecewise linear transformation function

Output image

FIGURE 3.10
(Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)
Contrast stretching

- Solve for the parameters of each piece

\[ r_1 \mapsto s_1 \]
\[ r_2 \mapsto s_2 \]

\[ s_1 = ar_1 + b \]
\[ s_2 = ar_2 + b \]

In matrix form:

\[
\begin{bmatrix}
  s_1 \\
  1
\end{bmatrix} = \begin{bmatrix}
  a & b \\
  0 & 1
\end{bmatrix} \begin{bmatrix}
  r_1 \\
  1
\end{bmatrix}
\]

Similarly:

\[
\begin{bmatrix}
  s_2 \\
  1
\end{bmatrix} = \begin{bmatrix}
  a & b \\
  0 & 1
\end{bmatrix} \begin{bmatrix}
  r_2 \\
  1
\end{bmatrix}
\]
Contrast stretching

Solve for $a$ and $b$

\[ s_1 = a r_1 + b \]
\[ s_2 = a r_2 + b \]

\[
\begin{bmatrix}
  s_1 & s_2 \\
  1 & 1
\end{bmatrix}
= \begin{bmatrix}
  a & b \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  r_1 & r_2 \\
  1 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
  s_1 & s_2 \\
  1 & 1
\end{bmatrix}
\begin{bmatrix}
  r_1 & r_2 \\
  1 & 1
\end{bmatrix}^{-1}
= \begin{bmatrix}
  a & b \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  a & b \\
  0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
  \frac{s_1 - s_2}{r_1 - r_2} & \frac{r_1 s_2 - r_2 s_1}{r_1 - r_2} \\
  0 & \frac{r_1 s_2 - r_2 s_1}{r_1 - r_2}
\end{bmatrix}
= \begin{bmatrix}
  a & b \\
  0 & 1
\end{bmatrix}
\]

\[
a = \frac{s_1 - s_2}{r_1 - r_2}
\]

\[
b = \frac{r_1 s_2 - r_2 s_1}{r_1 - r_2}
\]
Intensity-level slicing

**FIGURE 3.11** (a) This transformation function highlights range \([A, B]\) and reduces all other intensities to a lower level. (b) This function highlights range \([A, B]\) and leaves other intensities unchanged.

**FIGURE 3.12** (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected range set near black, so that the grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)
Bit-plane slicing

• Example: 8 bpp/channel images
  – Digits of binary number

\[
b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0
\]

Most significant bit  Least significant bit

– Binary to decimal

\[
d = b_7 2^7 + b_6 2^6 + b_5 2^5 + b_4 2^4 + b_3 2^3 + b_2 2^2 + b_1 2^1 + b_0 2^0
\]
Bit-plane slicing

• Example: 8 bpp/channel images

<table>
<thead>
<tr>
<th>Bit-plane</th>
<th>Binary</th>
<th>Hexadecimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2^7, 2^8 – 1]</td>
<td>r&amp;10000000_b</td>
<td>r&amp;0x80</td>
</tr>
<tr>
<td>8</td>
<td>r&amp;01000000_b</td>
<td>r&amp;0x40</td>
</tr>
<tr>
<td>7</td>
<td>r&amp;00100000_b</td>
<td>r&amp;0x20</td>
</tr>
<tr>
<td>6</td>
<td>r&amp;00010000_b</td>
<td>r&amp;0x10</td>
</tr>
<tr>
<td>5</td>
<td>r&amp;00001000_b</td>
<td>r&amp;0x08</td>
</tr>
<tr>
<td>4</td>
<td>r&amp;00000100_b</td>
<td>r&amp;0x04</td>
</tr>
<tr>
<td>3</td>
<td>r&amp;00000010_b</td>
<td>r&amp;0x02</td>
</tr>
<tr>
<td>2</td>
<td>r&amp;00000001_b</td>
<td>r&amp;0x01</td>
</tr>
</tbody>
</table>

Bitwise AND
Bit-plane slicing

Input image

Bit plane 8

Bit plane 7

Bit plane 6

Bit plane 5

Bit plane 4

Bit plane 3

Bit plane 2

Bit plane 1

(most significant bit)

Least noise

Bit plane (least significant bit)
Bit-plane slicing

Input image

Reconstructed images

FIGURE 3.15
Image reconstructed from bit planes:
(a) 8 and 7;
(b) 8, 7, and 6;
(c) 8, 7, 6, and 5.
Histogram processing

• Histogram equalization
• Histogram matching
• Local histogram equalization
**Histogram**

**Figure 3.16** Four image types and their corresponding histograms. (a) dark; (b) light; (c) low contrast; (d) high contrast. The horizontal axis of the histograms are values of $r_k$ and the vertical axis are values of $p(r_k)$.

Example: 8 bpp/channel images
Normalized histogram

• Probability density function (pdf) \( 0 \leq p(x) \leq 1 \)
  – Area under curve is 1

• Histogram
  – Area under curve is total number of pixels = \( MN \)

• Normalized histogram

\[
\text{normalized histogram} = \frac{1}{MN} \text{histogram} \approx \text{pdf}
\]

\( p_f(r) \) is approximate pdf of input image \( f(x, y) \)

\( p_g(s) \) is approximate pdf of output image \( g(x, y) \)
Cumulative distribution function

- Cumulative distribution function

\[ cdf_f(r) = \int_0^r p_f(w)dw \quad \text{Continuous} \]
\[ cdf_f(r) = \sum_{k=0}^r p_f(k) \quad \text{Discrete} \]

- Monotonically increasing
- Last value is maximum value = 1

\[ cdf_f(r) \text{ is cdf of input image } f(x, y) \]
\[ cdf_g(s) \text{ is cdf of output image } g(x, y) \]
Histogram equalization

• Conditions
  – Output intensity values will never be less than corresponding input values
    \[ s = T(r) \text{ is monotonic increasing in } 0 \leq r \leq L - 1 \]
  – The range of output intensities is the same as the input
    \[ 0 \leq T(r) \leq L - 1 \quad 0 \leq r \leq L - 1 \]
  – If we further require the inverse mapping from \( s \) back to \( r \)
    \[ r = T^{-1}(s) \quad 0 \leq s \leq L - 1 \]
    be one-to-one, then
    \[ s = T(r) \text{ is strictly monotonic increasing in } 0 \leq r \leq L - 1 \]
Histogram equalization

- Objective: after (forward) mapping, the output image pdf is flat

\[ p_f(r) \quad \text{Continuous} \]

\[ p_g(s) \quad \text{Continuous} \]

Input image pdf

Output image pdf
Histogram equalization

- This objective is satisfied by the histogram equalization intensity transformation

\[ s = T(r) = (L - 1) \int_{0}^{r} p_{f}(w)dw \]  

\[ p_{f}(r) \] Continuous

\[ \frac{1}{L - 1} \] Continuous

Input image pdf

Output image pdf
Histogram equalization

Proof of \( s = T(r) = (L - 1) \int_0^r p_f(w)dw \)
From calculus, recall the derivative of a definite integral with respect to its upper limit is the integrand evaluated at the limit, i.e.,

\[
\frac{ds}{dr} = \frac{dT(r)}{dr} \\
\frac{ds}{dr} = (L - 1) \frac{d}{dr} \left[ \int_0^r p_f(w)dw \right] \\
\frac{ds}{dr} = (L - 1)p_f(r)
\]

From probability theory, recall if \( p_f(r) \) and \( T(r) \) are known, and \( T(r) \) is continuous and differentiable over the range of values of interest, then the pdf of the transformed (i.e., mapped) variable \( s = T(r) \) can be obtained as

\[
p_g(s) = p_f(r) \left| \frac{dr}{ds} \right| \\
p_g(s) = p_f(r) \left| \frac{1}{(L - 1)p_f(r)} \right| \\
p_g(s) = \frac{1}{L - 1} \quad 0 \leq s \leq L - 1
\]
Histogram equalization

- Objective: for discrete images, after (forward) mapping, desired output image approximate pdf is flat

\[ p_f(r) \]

\[ p_g(s) = \frac{1}{L} \]

Input image normalized histogram (approximate pdf)

Desired output image normalized histogram (approximate pdf)
Histogram equalization

\[ p_g(s) = \frac{1}{L} \quad s = 0, 1, 2, \ldots, L - 1 \]

**Desired output image normalized histogram (approximate pdf)**

\[ cdf_g(s) = \sum_{k=0}^{s} p_g(k) = \frac{i + 1}{L} \quad s = 0, 1, 2, \ldots, L - 1 \]

**Desired output image cdf**

\[ \frac{1}{L} \]

\[ 0 \to L - 1 \]

\[ 1 \]

\[ 0 \to L - 1 \]
Histogram equalization

Discrete

\[ s = T(r) = (L - 1) \text{cdf}_f(r) \quad r = 0, 1, 2, \ldots, L - 1 \]

In general, discrete pdf will not be flat.
In general, discrete pdf will not be flat
Histogram matching

Specified histogram to match

Intensity transformation

Histogram of intensity transformed image
Local histogram equalization

Input image

Global

Local

Neighborhoods

FIGURE 3.32
(a) Original image, (b) Result of global histogram equalization, (c) Result of local histogram equalization.
Next Lecture

• Spatial filtering

• Reading
  – Chapter 3: Intensity Transformations and Spatial Filtering
    • Sections 3.4, 3.5, 3.6, and 3.8