

Recursion

Introduction to Programming and
Computational Problem Solving - 2

CSE 8B

Lecture 16

Announcements

- Assignment 8 is due Nov 23, 11:59 PM
- Assignment 9 will be released today
 - Due Nov 30, 11:59 PM
- Educational research study
 - Nov 25, weekly survey (tentative)
 - Nov 28, post-test (**next lecture**)
- Reading
 - Liang
 - Chapter 18

Recursion

- Recursion is a technique that leads to elegant solutions to problems that are difficult to program using simple loops
- A recursive method is one that invokes itself directly or indirectly

Computing factorials

- Example

$$4! = 4 * 3 * 2 * 1 = 24$$

- Remember, $0! = 1$ (and $1! = 1$)
- As a (non-recursive) method

```
public static long factorial(int n) {  
    long nfactorial = 0 == n ? 1 : n;  
    for (int i = n - 1; 1 < i; --i) {  
        nfactorial *= i;  
    }  
    return nfactorial;  
}
```

Computing factorials

- Alternatively, think recursively

$$0! = 1$$

- *Base case or stopping condition*

$$n! = n * (n - 1)!; n > 0$$

- $(n - 1)!$ is a *subproblem* of $n!$ and is a *recursive call*

- Example

$$4! = 4 * 3!$$

$$4! = 4 * (3 * 2!)$$

$$4! = 4 * (3 * (2 * 1!))$$

$$4! = 4 * (3 * (2 * (1 * 0!)))$$

$$4! = 4 * (3 * (2 * (1 * 1)))$$

$$4! = 4 * (3 * (2 * 1))$$

$$4! = 4 * (3 * 2)$$

$$4! = 4 * 6$$

$$4! = 24$$

Computing factorials

$$0! = 1$$

$$\text{factorial}(0) = 1$$

$$n! = n * (n - 1)!; n > 0$$

$$\text{factorial}(n) = n * \text{factorial}(n - 1)$$

- As a recursive method

```
public static long factorial(int n) {  
    if (0 == n) {  
        // Base case  
        return 1;  
    }  
    else {  
        // Recursive call  
        return n * factorial(n - 1);  
    }  
}
```

Computing factorials

- Example

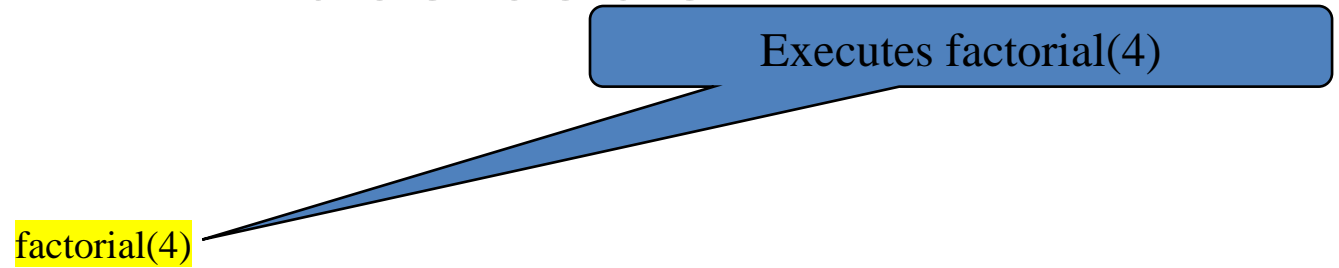
```
0! = 1
n! = n * (n - 1)!; n > 0
```

```
4! = 4 * 3!
4! = 4 * (3 * 2!)
4! = 4 * (3 * (2 * 1!))
4! = 4 * (3 * (2 * (1 * 0!)))
4! = 4 * (3 * (2 * (1 * 1)))
4! = 4 * (3 * (2 * 1))
4! = 4 * (3 * 2)
4! = 4 * 6
4! = 24
```

```
factorial(0) = 1
factorial(n) = n * factorial(n - 1)
```

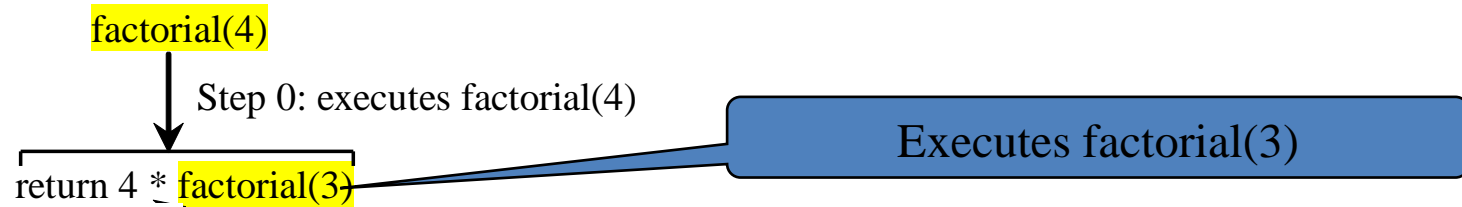
```
factorial(4) = 4 * factorial(3)
factorial(4) = 4 * (3 * factorial(2))
factorial(4) = 4 * (3 * (2 * factorial(1)))
factorial(4) = 4 * (3 * (2 * (1 * factorial(0))))
factorial(4) = 4 * (3 * (2 * (1 * 1)))
factorial(4) = 4 * (3 * (2 * 1))
factorial(4) = 4 * (3 * 2)
factorial(4) = 4 * 6
factorial(4) = 24
```

Trace code



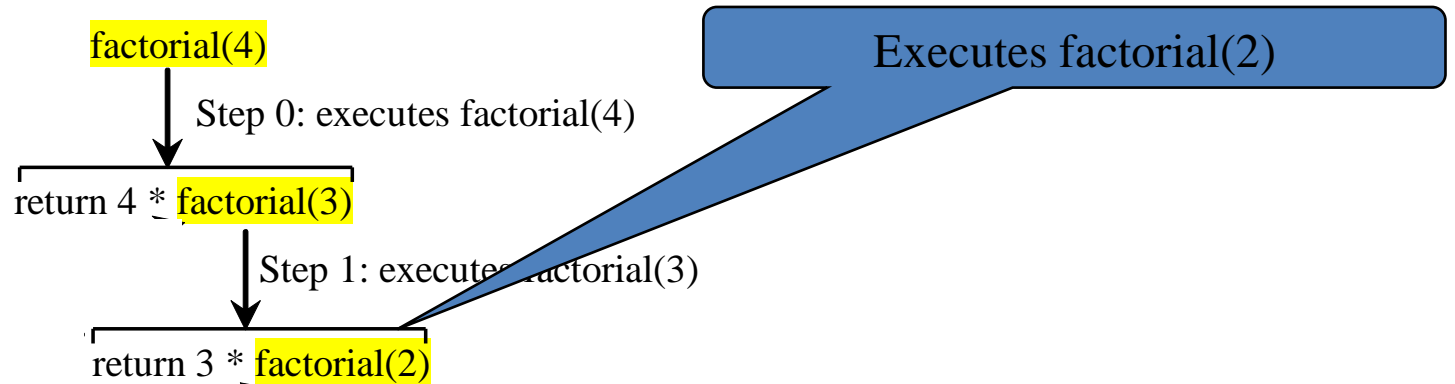
Stack
Space Required for factorial(4)
Main method

Trace code



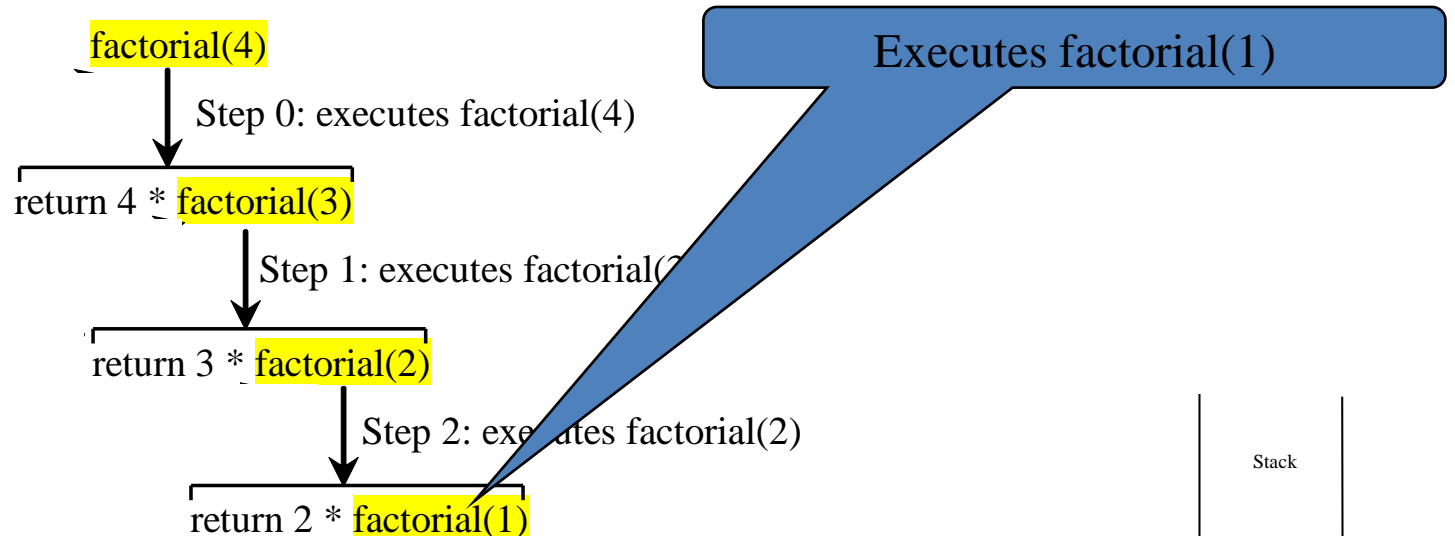
Stack
Space Required for factorial(3)
Space Required for factorial(4)
Main method

Trace code



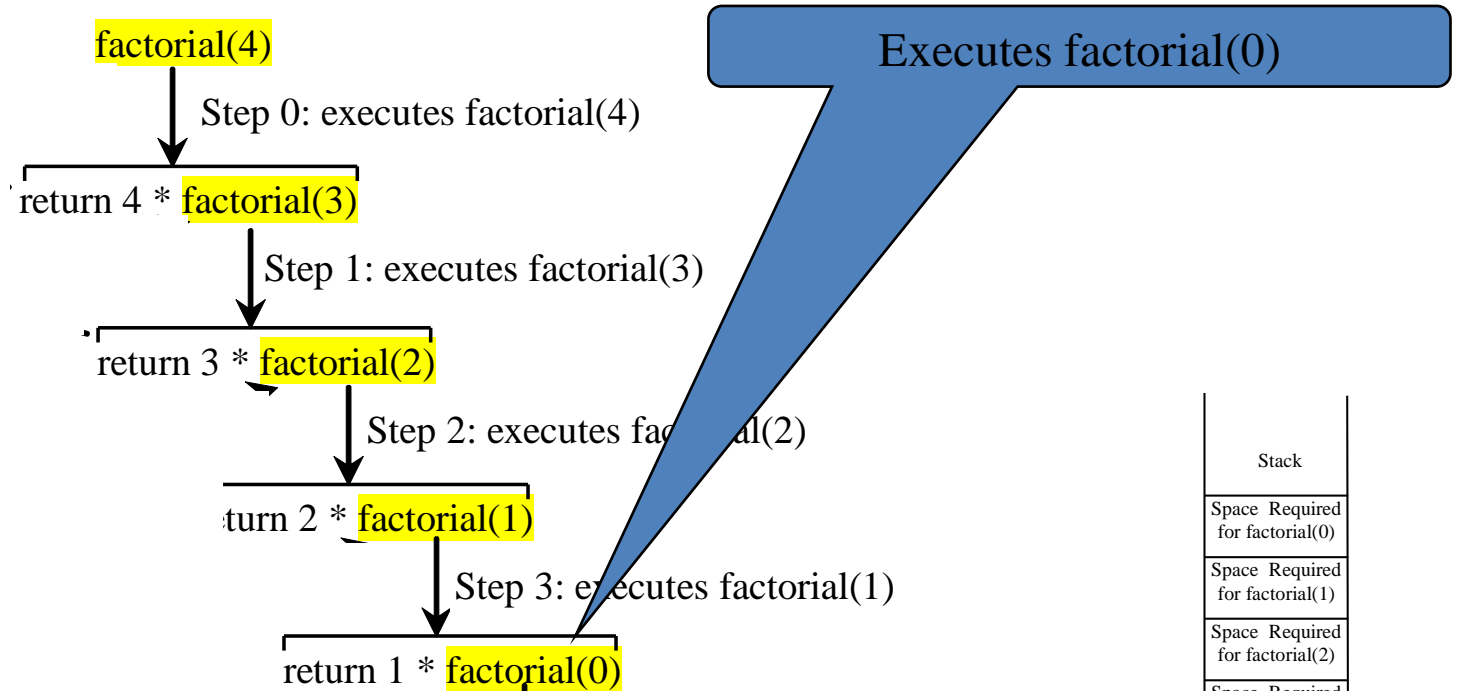
Stack
Space Required for factorial(2)
Space Required for factorial(3)
Space Required for factorial(4)
Main method

Trace code



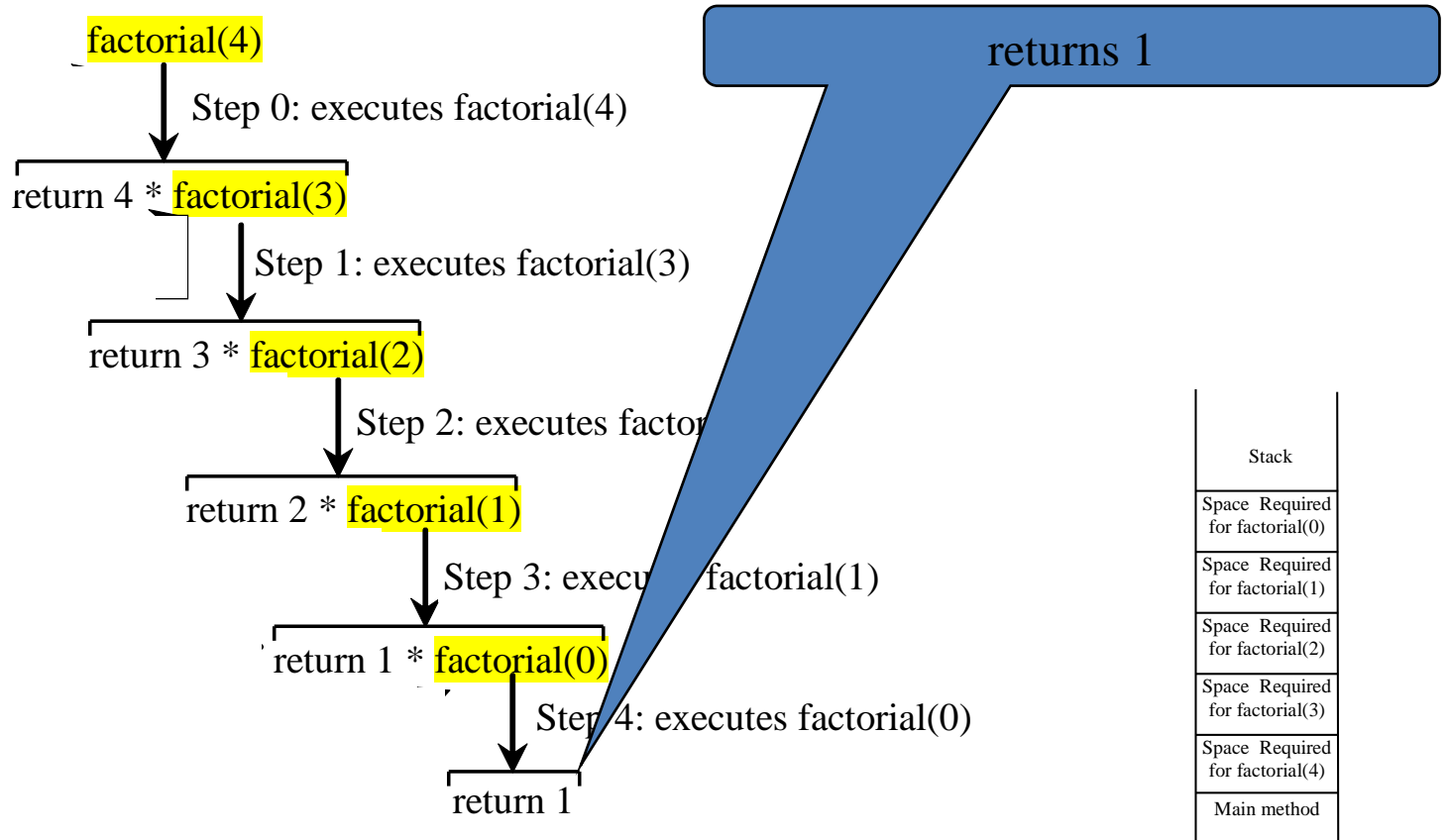
Stack
Space Required for factorial(1)
Space Required for factorial(2)
Space Required for factorial(3)
Space Required for factorial(4)
Main method

Trace code

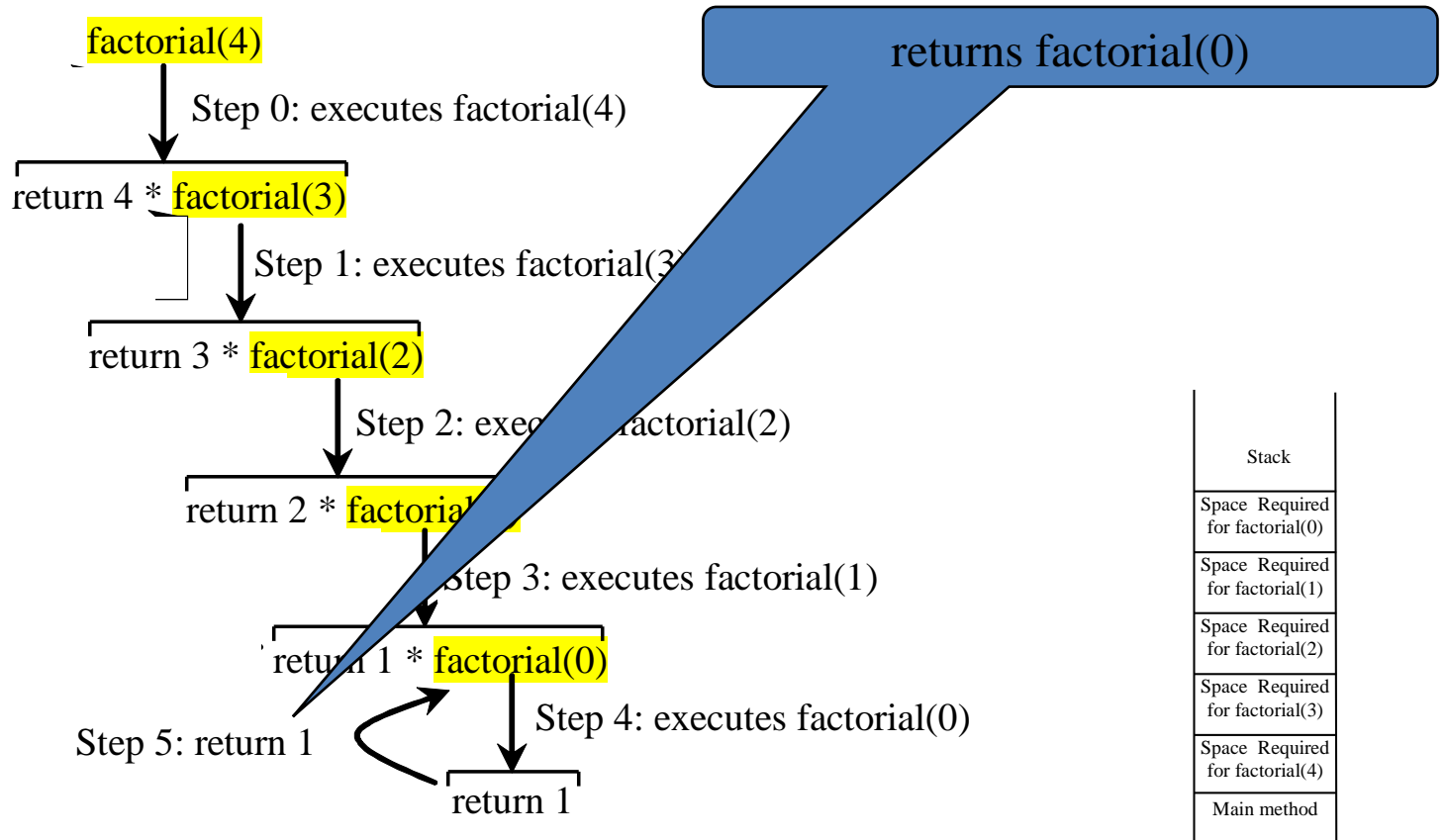


Stack
Space Required for factorial(0)
Space Required for factorial(1)
Space Required for factorial(2)
Space Required for factorial(3)
Space Required for factorial(4)
Main method

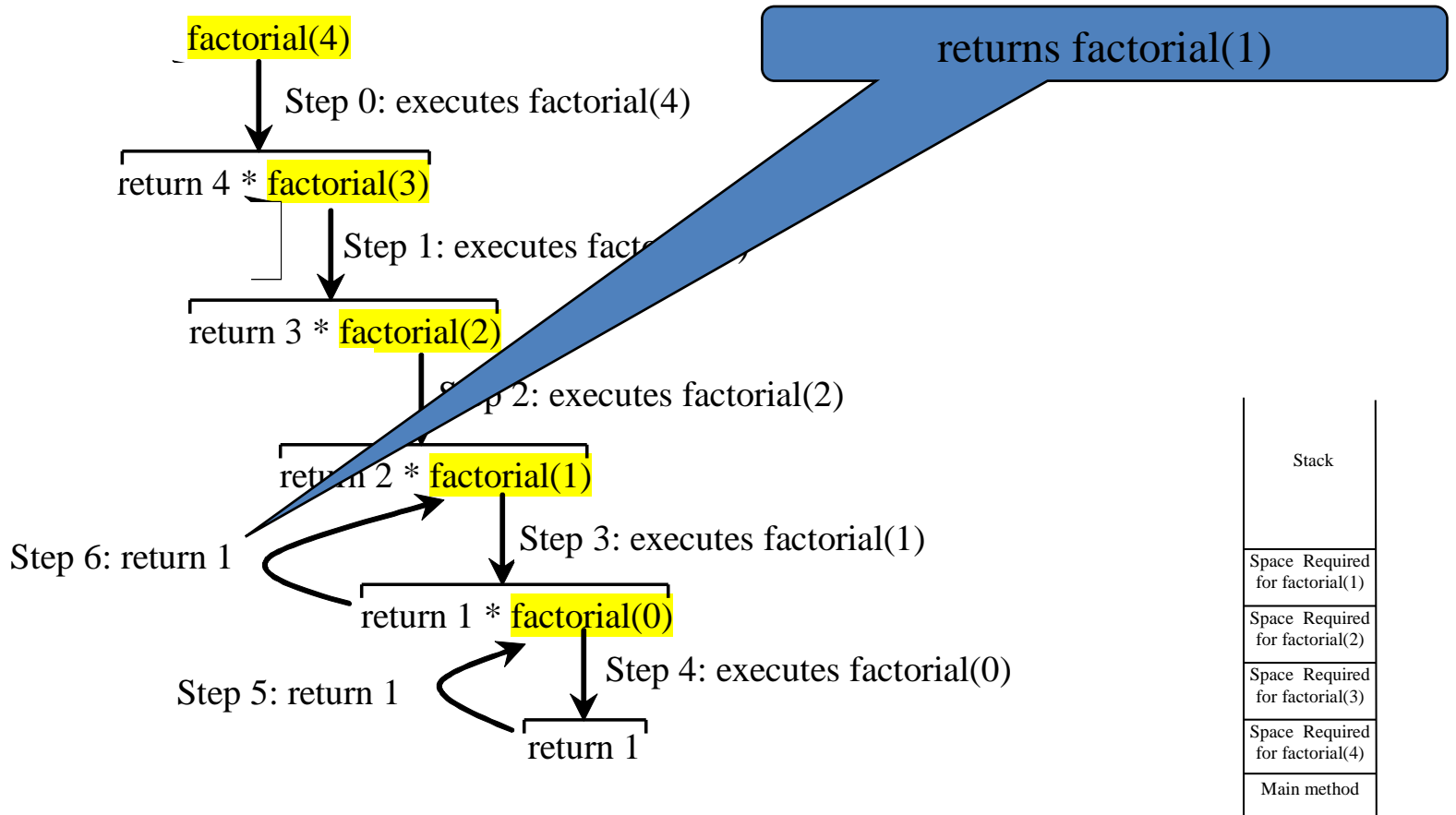
Trace code



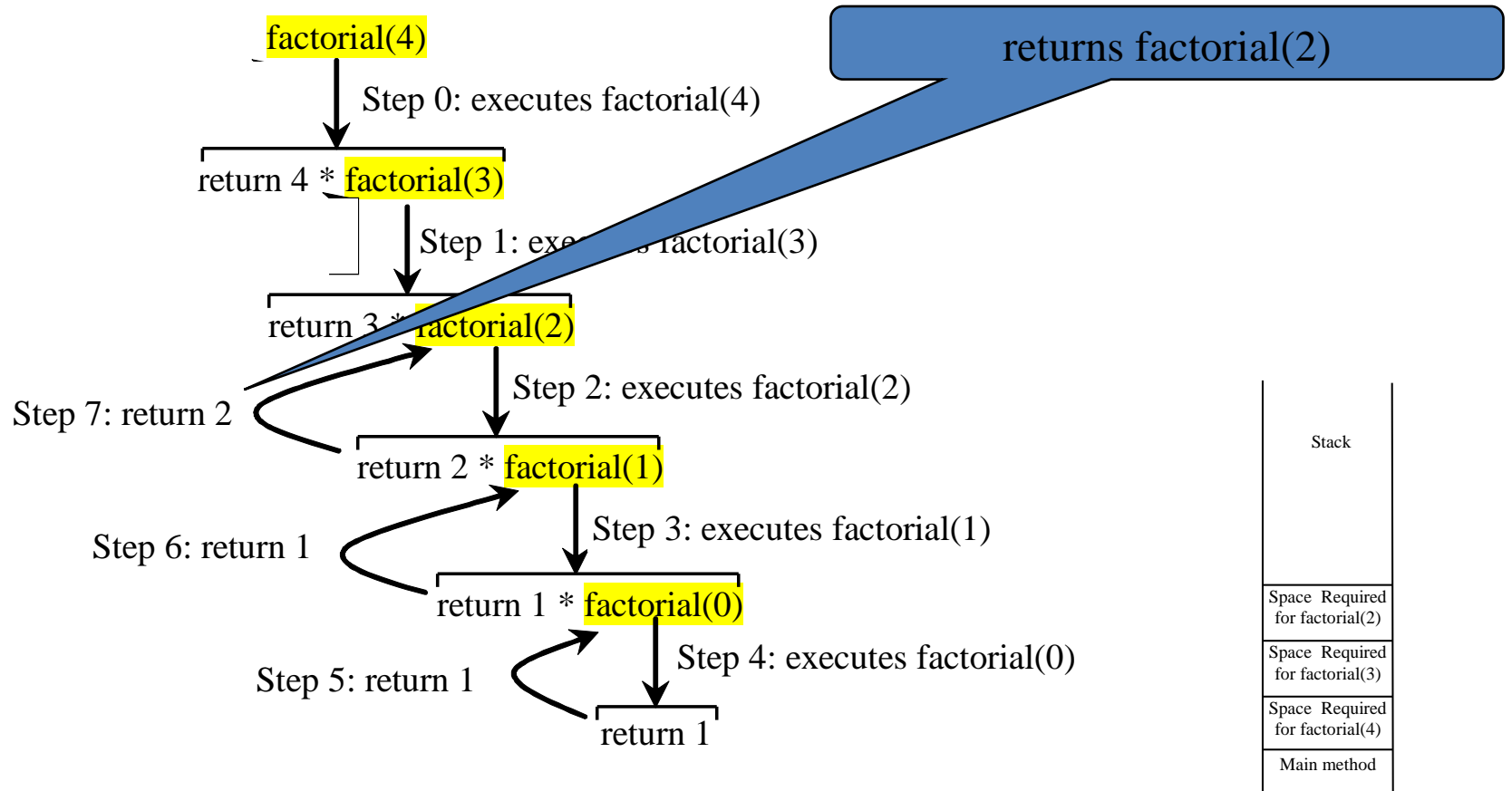
Trace code



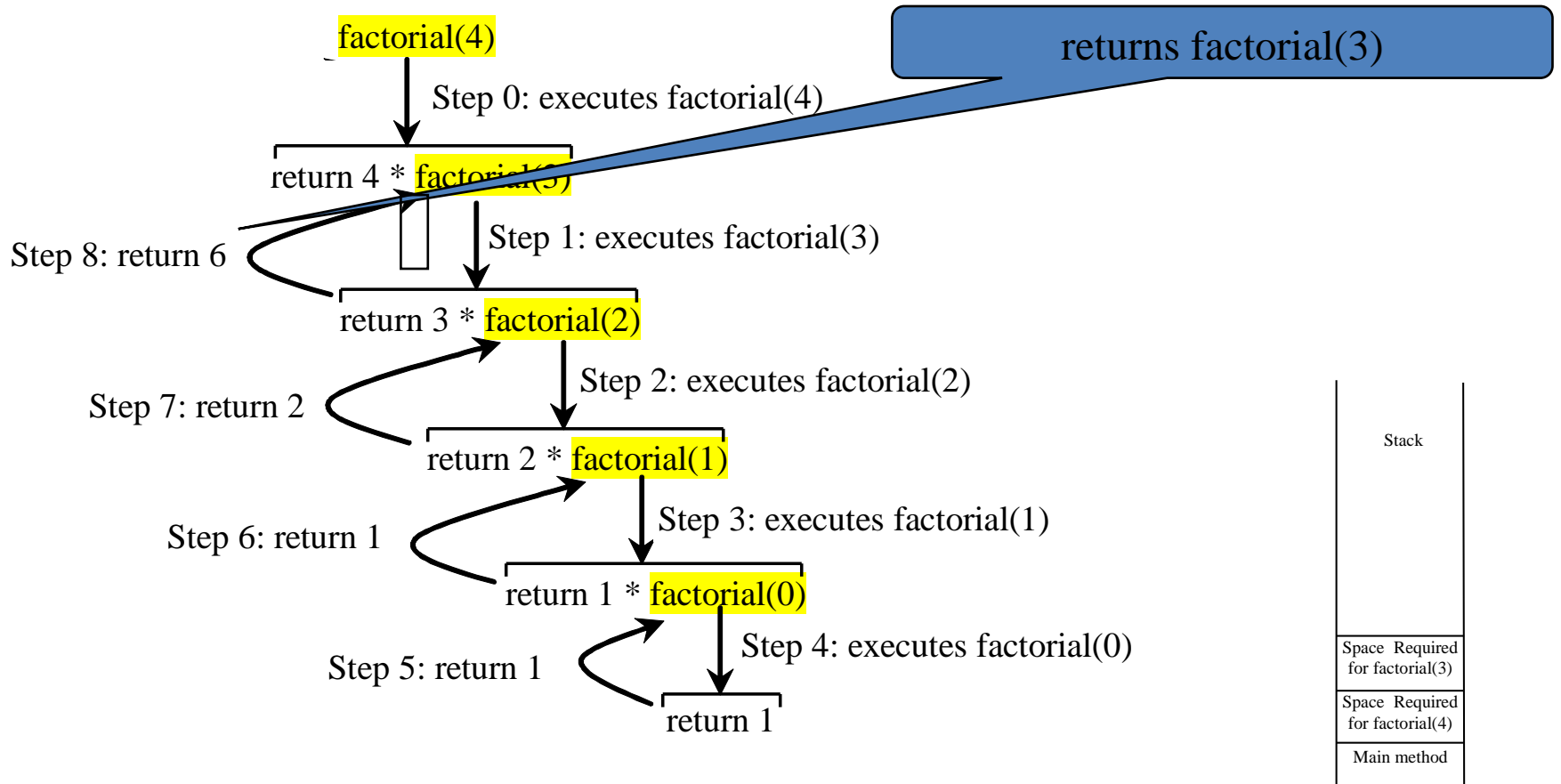
Trace code



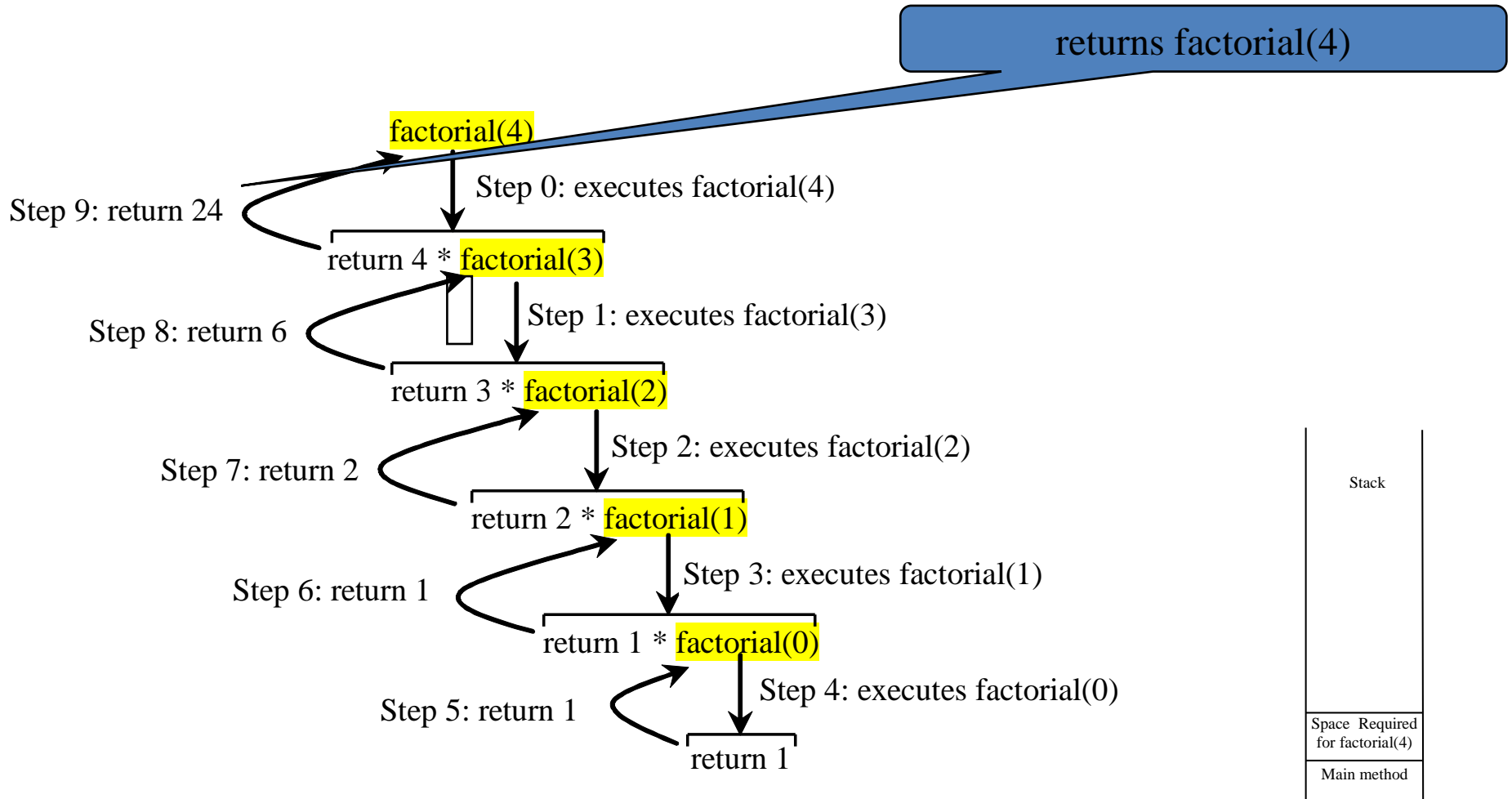
Trace code



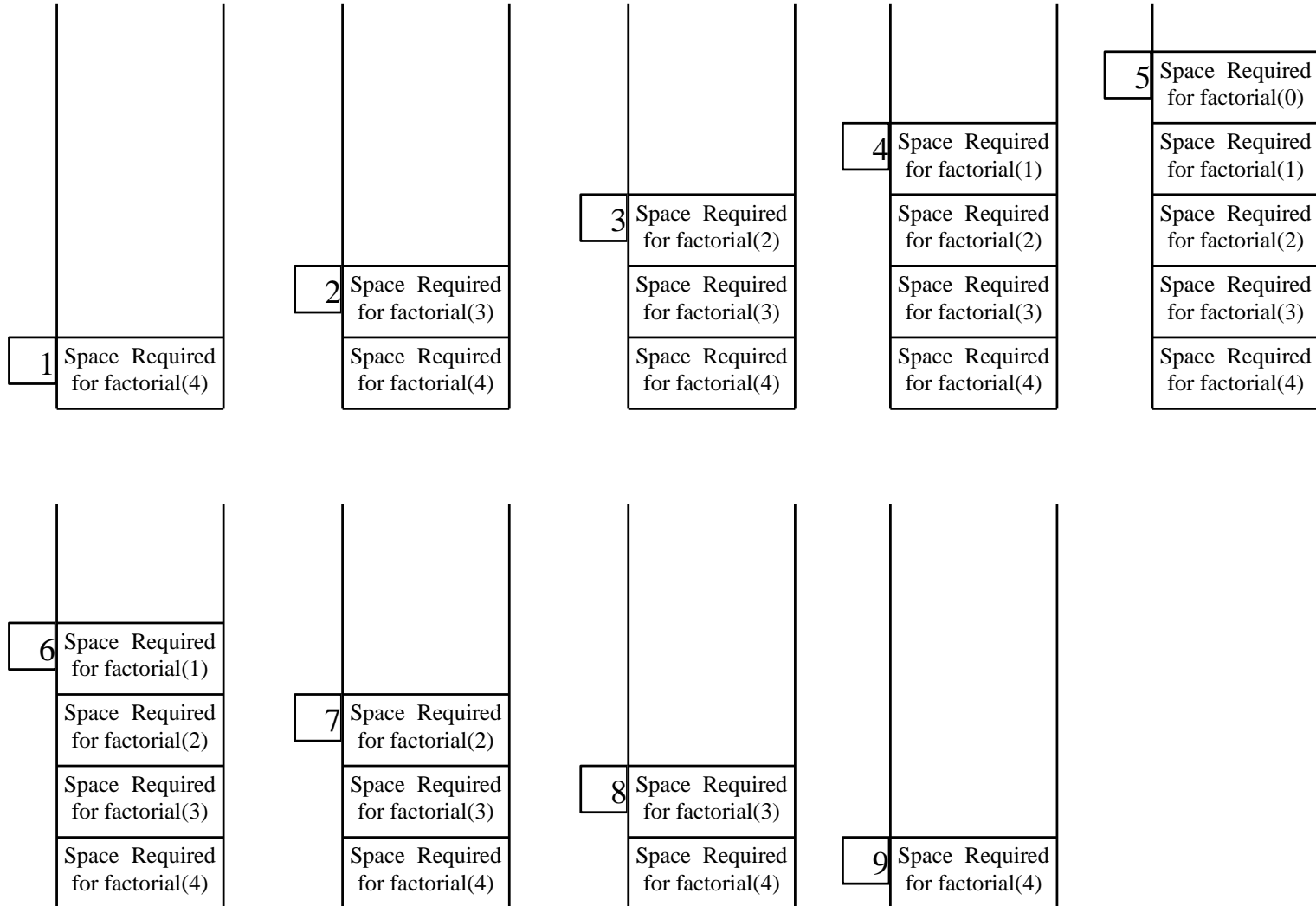
Trace code



Trace code



Trace stack



Stack overflow

- Deep recursion may result in stack overflow
- If recursion does not reduce the problem in a manner that allows it to eventually converge into the base case or a base case is not specified, *infinite recursion* can occur

- Example

```
public static long factorial(int n) {  
    // Mistakenly omit base case  
    return n * factorial(n - 1);  
}
```

- Results in stack overflow

Computing factorials

- As a recursive method

```
public static long factorial(int n) {
    if (0 == n) {
        // Base case
        return 1;
    }
    else {
        // Recursive call
        return n * factorial(n - 1);
    }
}
```

Direct recursion

A recursive method is one that invokes itself directly or indirectly

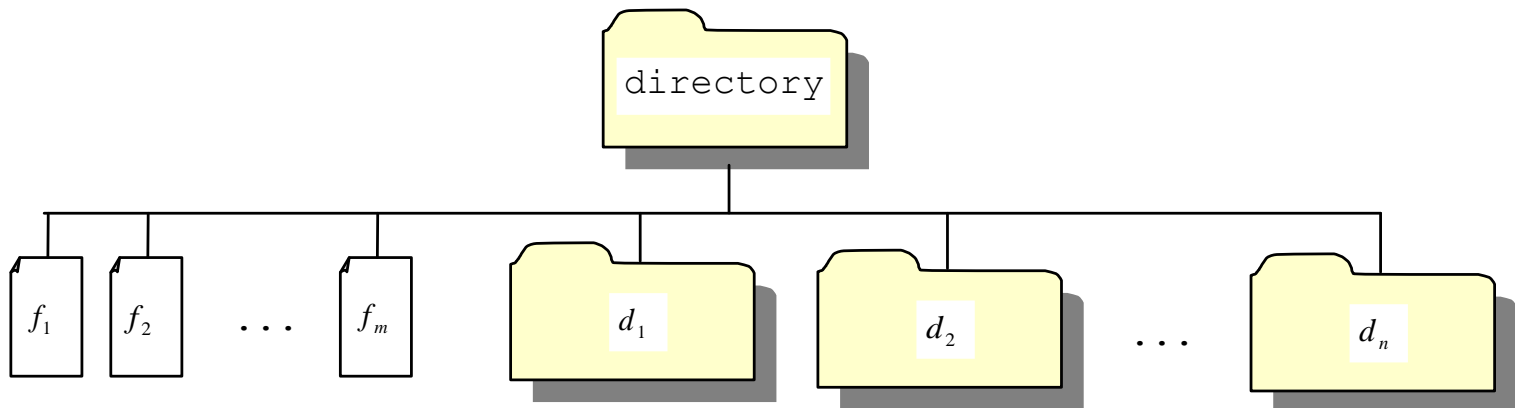
- As a non-recursive method

```
public static long factorial(int n) {
    long nfactorial = 0 == n ? 1 : n;
    for (int i = n - 1; 1 < i; --i) {
        nfactorial *= i;
    }
    return nfactorial;
}
```

Recursive algorithms can be replaced with non-recursive counterparts. However, some problems are inherently recursive, and difficult to solve without using recursion.

Recursion

- Recursive methods are efficient for solving problems with recursive structures
 - Example problem: find the size of a directory
 - The size of a directory is the sum of the sizes of all files in the directory
 - A directory may contain subdirectories
 - Suppose a directory contains files and subdirectories

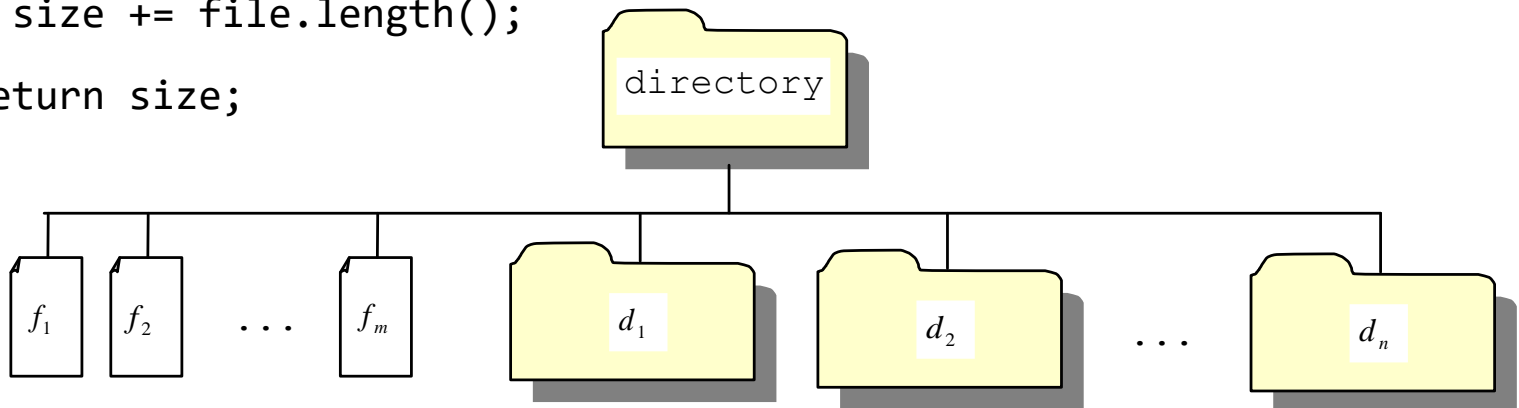


Finding the directory size

- The size of the directory can be defined recursively as
 $size(d) = size(f_1) + size(f_2) + \dots + size(f_m) + size(d_1) + size(d_2) + \dots + size(d_n)$

- Recursive method

```
public static long getSize(File file) {  
    long size = 0; // Store the total size of all files  
    if (file.isDirectory()) {  
        File[] files = file.listFiles(); // All files and subdirectories  
        for (int i = 0; files != null && i < files.length; i++) {  
            size += getSize(files[i]); // Recursive call  
        }  
    }  
    else { // Base case  
        size += file.length();  
    }  
    return size;  
}
```



Recursion

- In some cases, recursion enables you to create an intuitive, straightforward, simple solution to a problem

- Example

- Fibonacci numbers

$$f(0) = 0$$

$$f(1) = 1$$

$$f(n) = f(n-1) + f(n-2)$$

Fibonacci series: 0 1 1 2 3 5 8 13 21 34 55 89 ...
Indices: 0 1 2 3 4 5 6 7 8 9 10 11 ...

Fibonacci numbers

- Think recursively

Fibonacci series: 0 1 1 2 3 5 8 13 21 34 55 89 ...

fib(0) = 0;

Indices: 0 1 2 3 4 5 6 7 8 9 10 11 ...

fib(1) = 1;

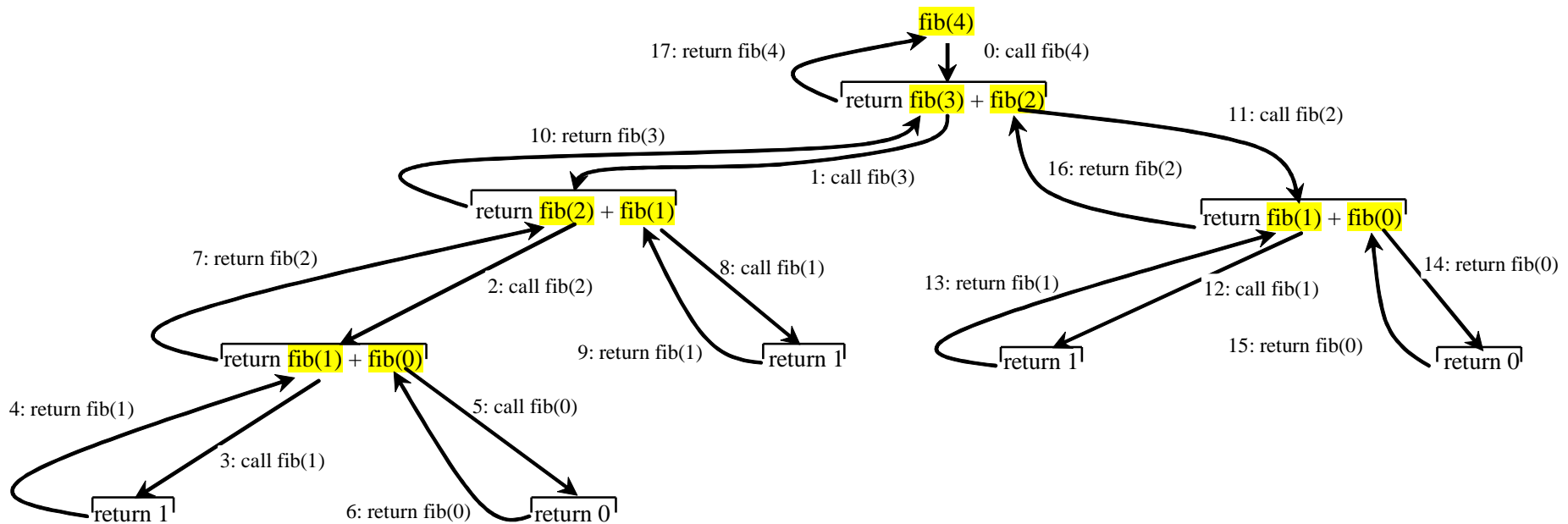
fib(index) = fib(index - 1) + fib(index - 2); index >= 2

```
public static long fib(long index) {  
    if (index == 0) // Base case  
        return 0;  
    else if (index == 1) // Base case  
        return 1;  
    else // Reduction and recursive calls  
        return fib(index - 1) + fib(index - 2);  
}
```

Reduction brings the problem closer to the base case

Fibonacci numbers

- Intuitive, straightforward, and simple, but inefficient
 - Many duplicate calls



Characteristics of recursion

- All recursive methods have the following characteristics
 - The method is implemented using an if-else (or a switch) statement that leads to **different cases**
 - One or more **base cases** (the simplest case) are used to stop recursion
 - Every recursive call **reduces** the original problem, bringing it increasingly **closer to a base case** until it becomes that case
- In general, to solve a problem using recursion, you break it into subproblems
 - If a subproblem resembles the original problem, you can apply the same approach to solve the subproblem recursively
 - This subproblem is almost the same as the original problem in nature with a smaller size

Recursive helper methods

- Sometimes you can find a solution to the original problem by defining a recursive function to a problem similar to the original problem
- This new method is called a recursive helper method
- The original problem can be solved by invoking the recursive helper method

The palindrome problem

- A string is a palindrome if it reads the same from the left and from the right
 - Subproblems
 - Check whether the first character and the last character of the string are equal
 - Ignore the two end characters and check whether the rest of the substring is a palindrome
 - Same as the original problem, but smaller in size (i.e., reduction)
 - Base cases
 - The two end characters are not the same
 - The string size is 0 or 1

The palindrome problem

- Without a recursive helper method

```
public static boolean isPalindrome(String s) {
    if (s.length() <= 1) {
        // Base case
        return true;
    }
    else if (s.charAt(0) != s.charAt(s.length() - 1)) {
        // Base case
        return false;
    }
    else {
        // Reduction and recursive call
        return isPalindrome(s.substring(1, s.length() - 1));
    }
}
```

- Inefficiently creates a new string for every recursive call

The palindrome problem

- With a recursive helper method

```
public static boolean isPalindrome(String s) {  
    return isPalindrome(s, 0, s.length() - 1);  
}
```

```
public static boolean isPalindrome(String s, int low, int high) {  
    if (high <= low) // Base case  
        return true;  
    else if (s.charAt(low) != s.charAt(high)) // Base case  
        return false;  
    else  
        return isPalindrome(s, low + 1, high - 1);  
}
```

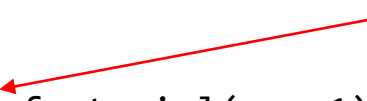
Tail recursion

- A recursive method is said to be *tail recursive* if there are no pending operations to be performed on return from a recursive call
- A tail-recursive method is more efficient than a nontail-recursive method
 - Compilers can optimize to reduce stack size

Computing factorials

- Without tail recursion

```
public static long factorial(int n) {  
    if (n == 0) // Base case  
        return 1;  
    else  
        return n * factorial(n - 1); // Recursive call  
}
```

 Pending operation

- With tail recursion

```
public static long factorial(int n) {  
    return factorial(n, 1); // Call auxiliary method  
}  
  
// Auxiliary tail-recursive method for factorial  
private static long factorial(int n, int result) {  
    if (n == 0)  
        return result;  
    else  
        return factorial(n - 1, n * result); // Recursive call  
}
```

Recursion vs. iteration

- Recursion is an alternative form of program control
- It is essentially repetition without a loop
- Recursion bears substantial overhead
 - Each time the program calls a method, the system must assign space for all of the method's local variables and parameters
 - This can consume considerable memory and requires extra time to manage the additional space

Recursion vs. iteration

- Recursive algorithms can be replaced with non-recursive counterparts
 - If performance is a concern, then avoid using recursion
 - However, some problems are inherently recursive, and difficult to solve without using recursion
- Use whichever approach can best develop an intuitive solution that naturally mirrors the problem
 - If an iterative solution is obvious, then use it

Next Lecture

- Event-driven programming
- Reading
 - Liang
 - Chapter 15